A LAGRANGEAN/SURROGATE HEURISTIC FOR THE MAXIMAL COVERING LOCATION PROBLEM USING HILLSMAN´S EDITION

Luiz A. N. Lorena*, Marcos A. Pereira#.

Laboratório Associado de Computação e Matemática Aplicada - LAC.
Instituto Nacional de Pesquisas Espaciais - INPE.
PO Box 515, São José dos Campos, 12201-970, Brazil.
Email: {*lorena, #marcos}@lac.inpe.br

The Maximal Covering Location Problem (MCLP) deals with the location of facilities in order to attend the largest subset of a population within a service distance. Many successful heuristic approaches have been developed to solve this problem. In this work we use the Unified Linear Model developed by Hillsman to adapt the distance coefficients of a p-median problem to reflect the demand information of a population. This transformation permits the application of a Lagrangean/surrogate heuristic developed for solving p-median problems to solve the MCLP. In previous works this heuristic proved to be very affordable, providing good quality solutions in reduced computational times. Computational tests for random generated scenarios ranging from 100 to 900 vertices and GIS-referenced instances of São José dos Campos city (Brazil) were conducted, showing the effectiveness of the combined approach.

Significance: In addition to the economic relevance of decisions related to facility location problems, applications in computer network design and flexible manufacturing systems can benefit with the use of MCLP models.

Keywords: Maximal covering location problems, p-Median problems, Hillsman´s edition, Lagrangean and surrogate relaxation, Heuristic solution, Geographic Information Systems.

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1. INTRODUCTION

Location-allocation problems deals with decisions of finding the best (or optimal) configuration for the installation of one or more facilities in order to attend the demand of a population (Daskin (1995), Drezner (1995)). In the private sector the term *facility* can be replaced by plants, warehouses, telecommunication antennas etc. The applications in the public sector can be divided between public services (schools, libraries, hospitals, bus stops) and emergency services (fire and police stations, ambulance posts). Facility location analysis can be improved if geo-referenced data, as provided by Geographic Information Systems (GIS), is available.

Despite the possible different nature of the applications, location-allocation models present the same basic structure. Based in the *p*-median models of Hakimi (1965) and ReVelle and Swain (1970), Hillsman (1974) developed an Unified Linear Model (ULM) that can be adapted to model other location-allocation problems. Given a network, the p-median problem (*p*MP) is the problem of locating *p* facilities minimizing the sum of the distances of each demand point to its nearest facility. The maximal covering location problem (MCLP) is the problem of locating *p* facilities on a network such that the maximal population is attended (or *covered*) within a given service distance (Church and ReVelle (1974)).

In his work, Hillsman proposed a change in the distance coefficients of a *p*MP to obtain a new set of coefficients, based in the population information and the *service distance* of a MCLP. Since no changes are made in the structure of the *p*MP model, existent solution procedures for solving *p*MP's can be applied to the new data set and obtain solutions for the corresponding MCLP. This paper assess the quality of the combined approach using the Lagrangean/surrogate heuristic of Senne and Lorena (2000) for solving *p*MP's in problem instances with both random generated and real world data.

In the next section we present the unified linear model and the proper change in problem coefficients to model a maximal covering location problem from data of a *p*-median problem. Section 3 presents the Lagrangean/surrogate relaxation of Senne and Lorena for solving the *p*-median problem. Section 4 contains an interchange algorithm for improvement of primal solutions. In section 5 we report the computational tests for random generated and real world data, ranging from 100 to 900 vertices. The integration of the heuristic to *ArcView*, a GIS software developed by Environmental Systems Research Institute Inc., is also presented, using geo-referenced data of São José dos Campos city, Brazil. Section 6 presents the conclusions and future extensions.

2. THE UNIFIED LINEAR MODEL FOR THE P-MEDIAN PROBLEM AND THE MAXIMAL COVERING LOCATION PROBLEM

For a n vertex network and a symmetric distance matrix $\mathbf{D} = [d_{ij}]_{n\times n}$, the ULM adapted for the pMP can be stated as the following binary integer programming problem:

$$v(p\text{MP}) = \min \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij} \qquad \text{... (1)}$$

$$(p\text{MP}) \quad \text{s.t.} \quad \sum_{i=1}^{n} x_{ij} = 1,\ \forall j \in N. \qquad \text{... (2)}$$

$$\sum_{i=1}^{n} x_{ii} = p \qquad \text{... (3)}$$

$$x_{ij} \le x_{ii},\ \forall i, j \in N. \qquad \text{... (4)}$$

$$x_{ij} \in \{0,1\},\ \forall i, j \in N. \qquad \text{... (5)}$$

Constraint (3) is obtained assuming $k = p$ and the equality relation in the generalized form of the corresponding inequality of the ULM:

$$\sum_{i=1}^{n} x_{ii} \begin{matrix}\le\\=\\\ge\end{matrix} k \qquad \text{... (6)}$$

The variables x_{ij}, $i, j \in N = \{1, ..., n\}$, indicate if node j is served by the facility located in candidate node i ($x_{ij} = 1$) or not ($x_{ij} = 0$) and if candidate node i is chosen for the installation of a facility ($x_{ii} = 1$) or not ($x_{ii} = 0$). The objective function (1) represents the total distance from every node to its nearest facility. Constraints (2) and (4) specify that every node must be served by only one installed facility. Constraint (3) indicates that exactly p nodes are to be chosen as candidates for installation of facilities. The binary condition over the variables are given in constraint (5).

An optimal solution to the model (1)-(5) is a solution that yields the minimum value of equation (1) for some matrix of coefficients $\mathbf{C} = [c_{ij}]_{n\times n}$. When population information is available for every node $j \in N$, say w_j, then a new set of coefficients can be calculated from the distance matrix $\mathbf{D}$ of the pMP in the following way:

$$c_{ij} = \begin{cases} 0, & \text{if } d_{ij} \le S; \\ w_j, & \text{if } d_{ij} > S. \end{cases} \qquad \text{... (7)}$$

If these coefficients are used in equation (1), then the problem switches to determine the best candidates for the installation of p facilities, minimizing the unattended population of the nodes that are more than S distance units away from any facility or, equivalently, maximizing the attendance of the population of the nodes within S distance units of any facility. The model (1)-(5), with coefficients calculated as in (7), is the ULM correspondent of MCLP. If $\mathbf{D}$ contains information about time, then S should be chosen accordingly to represent the limit time to reach a served node from an installed facility.

In both problems, x_{ij} represents the solution of the location-allocation problem, with $x_{ii} = 1$ representing the chosen candidates for the installation of the facilities. Although constraint (2) forces every node to be allocated to exactly one facility, the fact that $x_{ij} = 1$ in the MCLP does not guarantee attendance: only the nodes within S distance (or time) units from a facility will be considered *covered*. Another particularity of this transformation is that the value $v(p\text{MP})$ of the objective function using the coefficients from matrix $\mathbf{C}$ gives the total unattended population: the value of the corresponding solution to the MCLP is calculated as:

$$v(\text{MCLP}) = \sum_{j=1}^{n} w_j - v(p\text{MP}) \qquad \text{... (8)}$$

3.THE LAGRANGEAN/SURROGATE RELAXATION

To shorten the notation, we refer to the model defined as in (1)-(5), with coefficients taken from the calculated matrix $\mathbf{C}$, as P. Problem P can be solved using relaxation heuristics. Narciso and Lorena (1999) developed a Lagrangean/surrogate heuristic to approximately solve problem P. As proposed by Glover (1968), for a given $\lambda \in R_+^n$, a surrogate relaxation of P can be defined by:

$$v(\mathrm{SP}^{\lambda}) = \min \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij} \qquad \text{... (9)}$$

$$(\mathrm{SP}^{\lambda}) \quad \text{s.t.} \quad \sum_{j=1}^{n}\sum_{i=1}^{n} \lambda_j x_{ij} = \sum_{j=1}^{n} \lambda_j \qquad \text{... (10)}$$

and (3)-(5).

The optimal value of $v(\mathrm{SP}^{\lambda})$ is less than or equal to $v(\mathrm{P})$, and results from the solution of the dual surrogate problem $\max_{\lambda\geq 0}\{v(\mathrm{SP}^{\lambda})\}$. Problem SP^{λ} is an integer linear problem with no special structure to explore. In addition, the surrogate function $s: R_{+}^{n} \rightarrow R$, $(\lambda, v(\mathrm{SP}^{\lambda}))$ has some properties that make it difficult to find a dual solution. Methods for find approximated solutions of the surrogate dual were proposed by Karwan and Rardin (1979) and Dyer (1980).

Due to the difficulties with relaxation SP^{λ} we proposed to relax again the problem, now in the Lagrangean way. For a given $t \geq 0$, constraint (10) is relaxed, and the *Lagrangean/surrogate* relaxation is given by:

$$v(\mathrm{L}_t\mathrm{SP}^{\lambda}) = \min \sum_{j=1}^{n}\sum_{i=1}^{n} (c_{ij} - t\lambda_j) x_{ij} + t\sum_{j=1}^{n}\lambda_j \qquad \text{... (11)}$$

$(\mathrm{L}_t\mathrm{SP}^{\lambda})$ s.t. (3), (4) and (5).

For given $t \geq 0$ and $\lambda \in R_{+}^{n}$, $v(\mathrm{L}_t\mathrm{SP}^{\lambda}) \leq v(\mathrm{SP}^{\lambda}) \leq v(\mathrm{P})$. Problem $\mathrm{L}_t\mathrm{SP}^{\lambda}$ is solved considering implicitly constraint (3) and decomposing for index i, obtaining the following n problems:

$$\min \sum_{j=1}^{n} (c_{ij} - t\lambda_j) x_{ij} \qquad \text{... (12)}$$

s.t. (4) and (5).

that can be easily solved calculating:

$$\beta_i = \sum_{j=1}^{n} \left[\min\{0, c_{ij} - t\lambda_j\}\right], \ \forall i \in N, \qquad \text{... (13)}$$

and defining I as the index set of the p smallest β_i (here constraint (3) is considered implicitly). Then, a solution x_{ij}^{λ} to problem $\mathrm{L}_t\mathrm{SP}^{\lambda}$ is:

$$x_{ii}^{\lambda} = \begin{cases} 1, & \text{if } i \in I \\ 0, & \text{otherwise} \end{cases} \qquad \text{... (14)}$$

and for $i \neq j$:

$$x_{ij}^{\lambda} = \begin{cases} 1, & \text{if } i \in I \text{ and } c_{ij} - t\lambda_j < 0 \\ 0, & \text{otherwise} \end{cases} \qquad \text{... (15)}$$

The Lagrangean/surrogate solution is given by:

$$v(\mathrm{L}_t\mathrm{SP}^{\lambda}) = \min \sum_{i=1}^{n} \beta_i x_{ii} + t\sum_{j=1}^{n}\lambda_j \qquad \text{... (16)}$$

The interesting characteristic of relaxation $\mathrm{L}_t\mathrm{SP}^{\lambda}$ is that for $t = 1$ we have the usual Lagrangean relaxation using the multiplier λ. For a fixed multiplier λ, the best value for t can be found by solving the Lagrangean dual:

$$(\mathrm{D}_t^{\lambda}) \quad v(\mathrm{D}_t^{\lambda}) = \max_{t\geq 0} v(\mathrm{L}_t\mathrm{SP}^{\lambda}) \qquad \text{... (17)}$$

resulting $v(\mathrm{SP}^{\lambda}) \geq v(\mathrm{D}_t^{\lambda}) \geq v(\mathrm{L}_1\mathrm{SP}^{\lambda})$.

The Lagrangean function $l: R_{+} \rightarrow R$, $(t, v(\mathrm{L}_t\mathrm{SP}^{\lambda}))$, is concave and piecewise linear (Parker and Rardin (1988)). The best Lagrangean/surrogate relaxation value gives an improved bound over the usual Lagrangean relaxation. An exact solution to D_t^{λ} may be obtained by a search over different values of t (Minoux (1975) and Handler and Zang (1980)). However, in general, we have an interval of values $t_0 \leq t \leq t_1$ (with $t_0 = 1$ or $t_1 = 1$) which also produces improved bounds. So, in order to obtain an improved bound to the usual Lagrangean relaxation it is not necessary to find the best value t^*, as is enough to find a value T such that $t_0 \leq T \leq t_1$. Senne and Lorena (2000) describe a search heuristic that is used to find approximated best values of T. If the values of T remains unchanged for an *a priori* fixed number of iterations then this value is assumed for the upcoming iterations and the search procedure is no longer executed.

3.1 The Subgradient Heuristic

The following general subgradient heuristic is used as base to the relaxation heuristic used in this work. In this algorithm, $C = \{i \in N \mid x_{ii} = 1\}$ is the set of nodes already fixed as medians:

Given $\lambda \geq 0$, $\lambda \neq 0$;
Set $lb = -\infty$, $ub = +\infty$, $C = \varnothing$;
Repeat
  Solve relaxation (L_tSP^λ) obtaining x^λ and $v(L_tSP^\lambda)$;
  Obtain a feasible solution x_f and the respective v_f;
  Update $lb = \max\{lb, v(L_tSP^\lambda)\}$;
  Update $ub = \min\{ub, v_f\}$;
  Fix $x_{ii} = 1$ if $v(L_tSP^\lambda \mid x_{ii=0}) \geq ub$, $i \in N - C$;
  Update the set C;
  Set $g_j^\lambda = 1 - \sum_{i=1}^{n} x_{ij}^\lambda$, $j \in N$;
  Update the step size θ;
  Set $\lambda_j = \max\{0, \lambda_j + \theta g_j^\lambda\}$, $j \in N$;
Until (stopping tests).

The initial value for λ is set as $\lambda_j = \min_{i \in N}\{c_{ij}\}$, $j \in N$. The step sizes used are:

$$\theta = \frac{\pi(ub - lb)}{\left\| g^\lambda \right\|^2}. \qquad \text{... (18)}$$

The control of the parameter π is the same proposed in Held and Karp (1971). Beginning with $\pi = 2$, its value is halved whenever ub does not decrease for 15 consecutive iterations. The stopping testes used are:

a) $\pi \leq 0.005$;
b) $ub - lb < 1$;
c) $\left\| g^\lambda \right\|^2 = 0$
d) every median was fixed.

Solutions x^λ are not necessarily feasible to P, but feasible solutions can be produced at each iteration, by assigning non-median nodes to the nearest median node in I. The objective value of a feasible solution x_f is calculated as:

$$v_f = \sum_{j=1}^{n} [\min_{i \in I}\{c_{ij}\}] \qquad \text{... (19)}$$

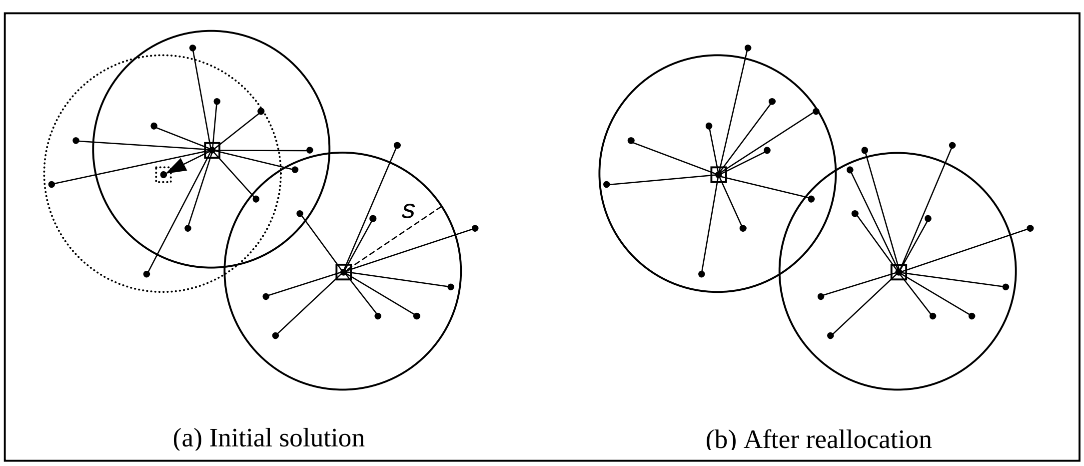

Figure 1: Reallocation of nodes for overlapping clusters.

4. IMPROVING PRIMAL SOLUTIONS

Primal solutions are calculated whenever *lb* increases. Then, set *C* is updated to store the indexes of the nodes chosen as new medians for *p*MP and exact *p* clusters can be identified, corresponding to the *p* medians and their allocated non-median nodes. Primal solutions x_f can be improved searching for a new median in each cluster, swapping the current median with a non-median node of the same cluster, changing the allocation solution.

As shown in Figure 1, this change may alter both allocation and covering configuration of the current *p*MP and MCLP solutions, respectively, so an algorithm for recalculating the coverage is needed:

```
While (v_f decreases)
    For k = 1, ..., p;
        Interchange median and non-median nodes in cluster C^k;
        Calculate the corresponding value v of the best reallocation;
        If v < v_f
            Update the median node for cluster C^k;
            Set v_f = v;
        End If;
    End For;
End While;
```

The interchange procedure for nodes in each cluster C^k, $k = 1,..., p$, can be performed for:

a) all allocated non-median nodes in cluster C^k, or;
b) only served non-median nodes in cluster C^k, or;
c) only the non-median nodes within $R < S$ distance (or time) units from median node of cluster C^k.

5. COMPUTATIONAL TESTS

The Lagrangean/surrogate heuristic with cost coefficients adapted to solve the MCLP was tested with random and real world data. For the random generated data, we used the distance matrices of the 100 and 150-vertex network of Galvão and ReVelle (1996) and Galvão et al. (2000) and the data sets pmed32.txt and pmed39.txt (with 700 and 900 vertices, respectively) from Beasley (1990). The number of facilities *p*, the service distance *S* and the demand information were considered as of Galvão and ReVelle (1996) and Galvão et al. (2000). The demand values used were not identical, but generated in the same way: the population of each node were sampled from a uniform distribution in the range [20, 30] for the 100-vertex network and from a normal distribution with mean equal to 80 and standard deviation equal to 15 for the other networks. The real world data for 324, 402, 500, 708 and 818-vertex networks were taken from a geo-referenced database for São José dos Campos city, Brazil. The nodes represent the blocks of some downtown and uptown portions of the city and the number of houses, apartments, commercial and government buildings in each block provides the demand information needed (the data files are available in http://www.lac.inpe.br/~lorena/instancias.html). For these problems we simulated the installation of radio antennas for Internet service, with short, medium and long range values (800, 1200 and 1600 m., respectively). A summary of the test problems used is given in Table 1.

Problem set	Number of vertices	Values of *p*	Values of *S*	Source
G&R100	100	[8, 10, 12]	[50, 65, 80]	Galvão and ReVelle (1996)
G&R150	150	[8, 10, 12]	[75, 80, 85, 90]	Galvão and ReVelle (1996)
G150	150	[5, 7, 8, 10, 12, 14, 16, 18, 20]	[70, 80, 90, 95]	Galvão et al. (2000)
SJC324	324	[1, 2, 3, 4, 5]	[800, 1200, 1600]	dmatrix324.txt
SJC402	402	[1, 2, 3, 4, 5, 6]	[800, 1200, 1600]	dmatrix402.txt
SJC500	500	[1, 2, 3, 4, 5, 6, 7, 8]	[800, 1200, 1600]	dmatrix500.txt
SJC708	708	[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]	[800, 1200, 1600]	dmatrix708.txt
SJC818	818	[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]	[800, 1200, 1600]	dmatrix818.txt
B700	700	[20, 24, 28]	[13, 15, 20]	pmed32.txt
B900	900	[20, 24, 28]	[10, 13, 16]	pmed39.txt

Table 1: Summary of test problems.

The Lagrangean/surrogate heuristic were implemented in C and run on an IBM PC equipped with one Intel Pentium III 733 MHz processor and 128 MB RAM. A simple program was developed to generate the demand information for the problems G&R100, G&R150, G150, B700 and B900 and to perform the necessary coefficient transformation, according to (7). After the call to the Lagrangean/surrogate routine the resulting output is interpreted to provide the allocation in terms of the original distance values. For each combination of (n, p, S) we generated 20 instances with random demand values.

Tables 2, 3, 4, and 5 shows the computational results for the problems with 100, 150, 700 and 900-vertex networks. Time values were calculated disregarding I/O operations and data manipulation. The columns Ref. Cov. and Ref. Time inform the best values of coverage reported in Galvão and ReVelle (1996) and Galvão et al. (2000) and the respective solution time. The entries in bold indicates the instances where the Lagrangean/surrogate provides better results of coverage.

The values of coverage obtained for these problems by the Lagrangean/surrogate heuristic are comparable to those reported in Galvão and ReVelle (1996) and Galvão et al. (2000). For the 700 and 900-vertex networks, the improvement of primal solutions were restricted to nodes within $R = 0.7*S$ distance units of the corresponding facility, in order to keep computational times low. There was no significant degradation in the coverage results for the 700-vertex network, but for the 900-vertex network this restriction played a crucial role. In this case, when the search for new facilities was performed for all nodes within S distance units, coverage values are marginally increased but computational times were multiplied by a factor of 5!

n	*p*	*S*	Avg. Population	Ref. Cov. (%)	Avg. Cov. (%)	Max. Cov. (%)	Avg. Iterations	Ref. Time (s)	Avg. Time (s)	Max. Time (s)
100	8	50	2489	69.43	69.19	**70.49**	328	51.69	0.84	0.94
100	10	50	2499	76.23	76.00	**76.94**	309	62.90	1.01	1.15
100	12	50	2505	81.61	81.42	**82.27**	314	64.26	1.22	1.37
100	8	65	2485	87.36	87.09	**87.89**	321	53.81	0.90	1.04
100	10	65	2506	94.33	**94.77**	**95.57**	263	20.40	1.05	1.16
100	12	65	2507	99.18	**99.57**	**100.00**	252	22.03	2.02	3.13
100	8	80	2496	88.46	**88.57**	**88.92**	292	43.56	0.87	1.10
100	10	80	2494	96.21	95.84	**96.41**	277	20.54	1.18	1.37
100	12	80	2506	100.00	99.76	**100.00**	259	7.41	2.00	3.30

Table 2: Computational results for G&R100.

n	*p*	*S*	Avg. Population	Ref. Cov. (%)	Avg. Cov. (%)	Max. Cov. (%)	Avg. Iterations	Ref. Time (s)	Avg. Time (s)	Max. Time (s)
150	10	70	11860	68.86	**69.37**	**70.74**	446	9.00§	2.99	3.35
150	12	70	11949	77.09	**77.91**	**78.69**	433	11.00§	3.69	4.01
150	14	70	11863	83.34	**83.97**	**84.66**	399	12.00§	3.60	4.23
150	16	70	11957	87.75	**88.46**	**89.35**	367	13.00§	3.75	4.40
150	18	70	11910	92.39	92.13	**92.92**	350	12.00§	3.84	4.34
150	20	70	11912	93.95	**95.22**	**96.28**	289	6.00§	4.36	5.11
150	8	75	11914	59.14	**59.63**	**60.46**	437	109.71¶	2.05	2.36
150	10	75	11909	68.86	**69.37**	**70.44**	454	122.35¶	2.97	3.24
150	12	75	11977	77.34	**77.53**	**78.48**	432	127.28¶	3.54	3.96
150	8	80	11874	61.49	**62.36**	**63.63**	430	4.00§	2.14	2.69
150	10	80	11879	70.91	71.67	**72.99**	454	6.00§	3.09	3.57
150	12	80	11914	78.14	**78.87**	**80.10**	418	10.00§	3.46	4.06
150	14	80	11874	84.47	**84.88**	**84.64**	383	12.00§	3.55	4.23
150	8	85	11884	73.94	**74.49**	**74.64**	425	96.39¶	3.05	3.35
150	10	85	11982	81.56	**82.03**	**83.04**	458	127.59¶	3.88	4.40
150	12	85	11846	87.95	**88.51**	**89.09**	428	154.12¶	4.11	4.56
150	6	90	11907	82.47	81.69	**82.88**	439	4.00§	2.75	3.30
150	8	90	11950	89.79	89.51	**90.30**	427	8.00§	3.73	5.05
150	10	90	11869	94.04	**94.55**	**95.07**	402	7.00§	4.14	4.78
150	12	90	11886	96.93	**97.87**	**98.27**	351	5.00§	4.71	5.22
150	14	90	11962	99.03	**99.98**	**100.00**	547	5.00§	10.66	12.52
150	5	95	11935	87.23	**87.83**	**88.95**	396	4.00§	2.15	2.37
150	7	95	11956	93.94	**94.45**	**94.85**	454	8.00§	3.88	4.23

Table 3: Computational results for G&R150 (¶) and G150 (§).

n	*p*	*S*	Avg. Population	Ref. Cov. (%)	Avg. Cov. (%)	Max. Cov. (%)	Avg. Iterations	Ref. Time (s)	Avg. Time (s)	Max. Time (s)
700	20	13	55545	70.03	**70.05**	**70.77**	963	329.00	135.69	149.23
700	24	13	55722	74.44	74.10	**74.86**	676	399.00	164.01	187.90
700	28	13	55483	78.05	77.56	**78.46**	668	536.00	198.17	220.86
700	20	15	55662	79.56	**79.69**	**80.18**	701	592.00	163.98	181.75
700	24	15	55650	83.17	83.06	**83.43**	717	662.00	199.51	223.33
700	28	15	55778	86.18	85.83	**86.21**	699	841.00	235.14	272.27
700	20	20	55495	95.76	**96.19**	**96.45**	1234	1281.00	547.08	652.02
700	24	20	55630	97.01	**97.39**	**97.57**	1105	1641.00	630.82	759.79
700	28	20	55640	98.02	**98.26**	**98.51**	1046	2076.00	737.77	909.94

Table 4: Computational results for B700.

n	*p*	*S*	Avg. Population	Ref. Cov. (%)	Avg. Cov. (%)	Max. Cov. (%)	Avg. Iterations	Ref. Time (s)	Avg. Time (s)	Max. Time (s)
900	20	10	71464	67.72	67.08	67.70	707	763.00	186.20	214.53
900	24	10	71559	71.58	70.78	71.44	725	1099.00	244.17	264.19
900	28	10	71478	75.12	73.73	74.55	730	1272.00	298.30	323.73
900	20	13	71345	88.03	87.55	87.88	820	1272.00	467.75	528.43
900	24	13	71555	90.48	89.75	90.19	848	1656.00	598.15	696.51
900	28	13	71481	92.30	91.58	91.95	931	1989.00	771.27	852.51
900	20	16	71556	96.73	96.64	**96.82**	1544	2725.00	1533.13	1781.15
900	24	16	71481	97.66	**97.76**	**97.98**	1408	3269.00	1873.06	2750.71
900	28	16	71616	98.43	**98.58**	**98.88**	1244	4244.00	2031.29	2575.65

Table 5: Computational results for B900.

The behavior of the Lagrangean/surrogate heuristic is partially illustrated in Figure 2. The scattered points in the upper half portion of the graphic correspond to the primal solutions, calculated for each improved dual solution (the asymptotically ascending set of points in the lower portion of the graphic). Each primal solution is improved, obtaining better values for *v*(*p*MP), represented by the set of points immediately above the horizontal line, which represents the optimal value for this (150, 7, 95) instance.

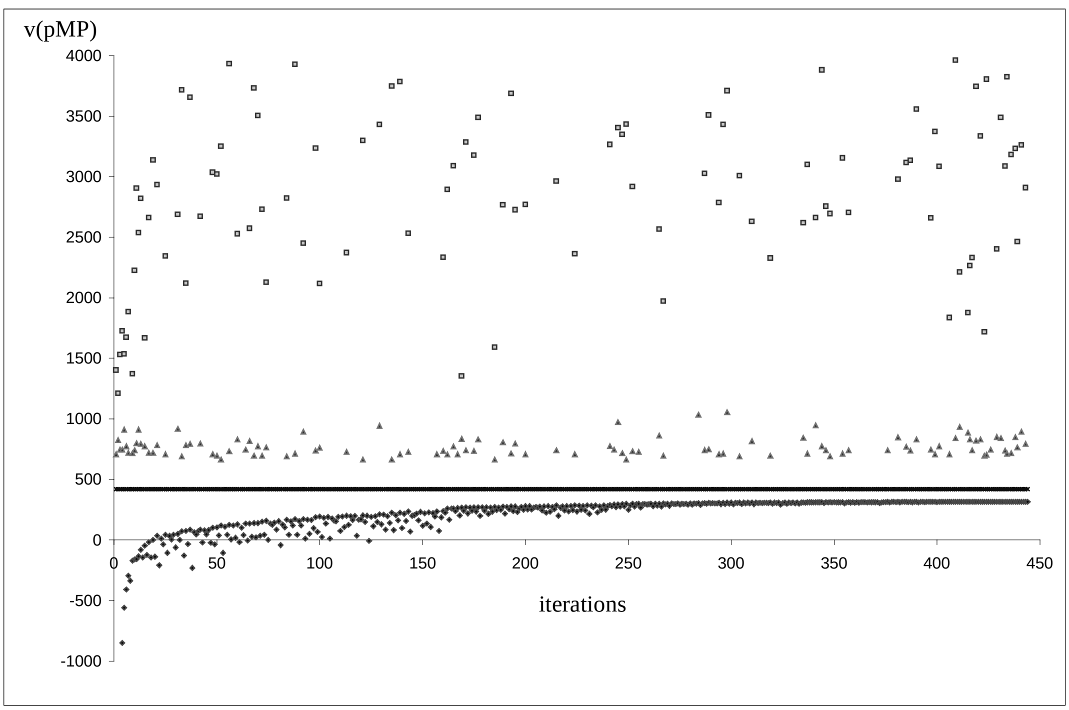

Figure 2: Convergence of the Lagrangean/surrogate heuristic.

Cost coefficients calculated as in (7) introduces a stronger discontinuity in $v(p\text{MP})$ when changing the median node of a cluster with another candidate of the same cluster. The zero or non-zero nature on the cost coefficients, introduced by Hillsman's edition, also affects the calculation of the upper and lower bounds of the Lagrangean/surrogate heuristic. So, in this approach, the values of the duality gaps cannot be used to measure the quality of heuristic solutions. In addition, the random nature of the data and other characteristics of the network can influence the computational effort needed to solve *p*-median location problems (Schilling et al. (2000)). We intend to study these issues in future researches.

n	*p*	*S*	Pop. Attended	Cov. (%)	Iter.	Time (s)
324	1	800	5461	44.94	31	0.28
324	2	800	8790	72.33	465	5.92
324	3	800	11604	95.49	333	5.33
324	4	800	12106	99.62	493	11.92
324	5	800	12152	100.00	448	16.20
324	1	1200	9932	81.73	27	0.27
324	2	1200	11555	95.08	358	5.22
324	3	1200	12152	100.00	428	9.84
324	1	1600	12123	99.76	22	0.27
324	2	1600	12152	100.00	698	15.00

Table 6: Computational results for SJC324.

n	*p*	*S*	Pop. Attended	Cov. (%)	Iter.	Time (s)
402	1	800	6555	41.01	39	0.55
402	2	800	11339	70.94	545	10.16
402	3	800	14690	91.90	486	11.09
402	4	800	15658	97.96	493	13.73
402	5	800	15970	99.91	541	29.11
402	6	800	15984	100.00	567	38.01
402	1	1200	10607	66.36	41	0.71
402	2	1200	14832	92.79	342	7.14
402	3	1200	15984	100.00	405	13.46
402	1	1600	15438	96.58	36	0.77
402	2	1600	15984	100.00	483	11.87

Table 7: Computational results for SJC402.

n	*p*	*S*	Pop. Attended	Cov. (%)	Iter. Iter.	Time (s)
500	1	800	7944	40.31	37	0.77
500	2	800	12454	63.20	368	8.89
500	3	800	15730	79.82	561	16.42
500	4	800	17794	90.29	517	22.79
500	5	800	18859	95.70	550	39.06
500	6	800	19525	99.08	522	47.18
500	7	800	19692	99.92	710	85.58
500	8	800	19707	100.00	719	103.87
500	1	1200	10726	54.43	42	1.04
500	2	1200	18070	91.69	526	20.48
500	3	1200	19393	98.41	525	22.90
500	4	1200	19707	100.00	570	45.92
500	1	1600	14804	75.12	39	1.15
500	2	1600	19668	99.80	780	25.04
500	3	1600	19707	100.00	856	60.74

Table 8: Computational results for SJC500.

n	*p*	*S*	Pop. Attended	Cov. (%)	Iter.	Time (s)
708	1	800	8393	34.69	38	1.48
708	2	800	13306	55.00	489	22.25
708	3	800	17272	71.40	533	26.25
708	4	800	20338	84.07	570	33.84
708	5	800	21486	88.81	581	54.65
708	6	800	22504	93.02	526	66.19
708	7	800	23151	95.70	456	74.65
708	8	800	23667	97.83	572	108.81
708	9	800	24024	99.31	654	139.18
708	10	800	24163	99.88	684	165.26
708	11	800	24192	100.00	750	207.51
708	1	1200	11612	48.00	43	1.98
708	2	1200	20376	84.23	386	31.86
708	3	1200	22422	92.68	485	32.40
708	4	1200	23884	98.73	593	55.97
708	5	1200	24142	99.79	570	84.69
708	6	1200	24192	100.00	515	98.70
708	1	1600	16827	69.56	40	2.04
708	2	1600	23366	96.59	815	64.87
708	3	1600	23888	98.74	649	52.73
708	4	1600	24192	100.00	644	71.40

Table 9: Computational results for SJC708.

n	*p*	*S*	Pop. Attended	Cov. (%)	Iter.	Time (s)
818	1	800	8393	28.77	31	1.48
818	2	800	13306	45.62	461	29.16
818	3	800	17507	60.02	650	37.02
818	4	800	21428	73.46	582	43.83
818	5	800	24531	84.10	503	51.03
818	6	800	25908	88.82	502	73.87
818	7	800	26933	92.34	484	99.80
818	8	800	27783	95.25	540	129.84
818	9	800	28351	97.20	553	158.02
818	10	800	28639	98.19	575	197.79
818	11	800	29019	99.48	638	215.36
818	12	800	29103	99.78	664	283.91
818	13	800	29144	99.92	615	299.89
818	14	800	29168	100.00	704	337.02
818	1	1200	11612	39.81	32	1.71
818	2	1200	20290	69.56	578	49.16
818	3	1200	25211	86.43	446	35.21
818	4	1200	27029	92.67	488	52.95
818	5	1200	28513	97.75	569	89.97
818	6	1200	29137	99.89	526	106.61
818	7	1200	29168	100.00	531	137.31

Table 10: Computational results for SJC818.

n	*p*	*S*	Pop. Attended	Cov. (%)	Iter.	Time (s)
818	1	1600	16827	57.69	41	2.64
818	2	1600	24646	84.50	466	43.72
818	3	1600	27672	94.87	569	53.89
818	4	1600	28862	98,95	581	62.28
818	5	1600	29168	100.00	654	110.40

Table 10: (continued).

Tables 6, 7, 8, 9 and 10 presents the results obtained for problems SJC324, SJC402, SJC500, SJC708 and SJC818. The number of facilities range from 1 to the minimum needed to obtain full coverage.

Figure 4 illustrates the heuristic solution for problem SJC708, with 3 short range antennas, using *ArcView* routines to process the output file containing the allocation solution (straight lines) and to represent the coverage (circles).

Figure 4: Location of 3 short range antennas in São José dos Campos.

6. CONCLUSIONS

In this work we adapted the distance coefficients of a *p*MP model to solve an associated MCLP. This approach permits the use of existent *p*-median location problem algorithms to solve MCLP with minimum changes (if any) in the adopted method. The similarity of the results to that obtained by Galvão and ReVelle (1996) and Galvão et al. (2000) shows the effectiveness of this approach for the Lagrangean/surrogate heuristic of Senne and Lorena (2000), although the quality of the solutions cannot be measured directly with the inherent heuristic mechanisms, due to the non-continuous nature of the cost parameters. Further research will be carried to evaluate the performance of the heuristic for such data.

The utilization of a GIS database permitted the evaluation of this approach with real word data. The low computational times for obtaining solutions permits the study of different scenarios, which is helpful to decision makers in public and private sectors.

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