Computational Simulation of MHD dynamos

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OUTLINE

- Motivation: the intermittent solar-cycle
- Description of the problem: the ABC dynamo
- Dynamical systems approach:
  - Transition to intermittent dynamos: Blowout bifurcation
  - Coherence/incoherence intermittency
  - Characterizing spatial complexity: the spectral entropy
- Concluding remarks

DYNAMO

Dynamo process: a weak (seed) magnetic field is amplified due to the conversion of kinetic energy into magnetic energy

Dynamos can be classified as large-scale (or **mean-field**) dynamos, or small-scale (or **fluctuation**) dynamos, according to weather the magnetic fields grow in spatial scales larger or smaller than the energy carrying scale of the fluid motion

Source: Baker, JASTP 62, 1669 (2000)

THE SOLAR DYNAMO

- Differential rotation in the tachocline converts weak Bp into Bt;
- Bt rises due to buoyancy and is is shredded in the convective layer, where Bp is strenghtened;
- Bp is carried back to the tachocline;
- If Bt is trong enough, it can go through the convective layer without being shredded, reaching the photosphere, where it is seen as a pair of sunspots

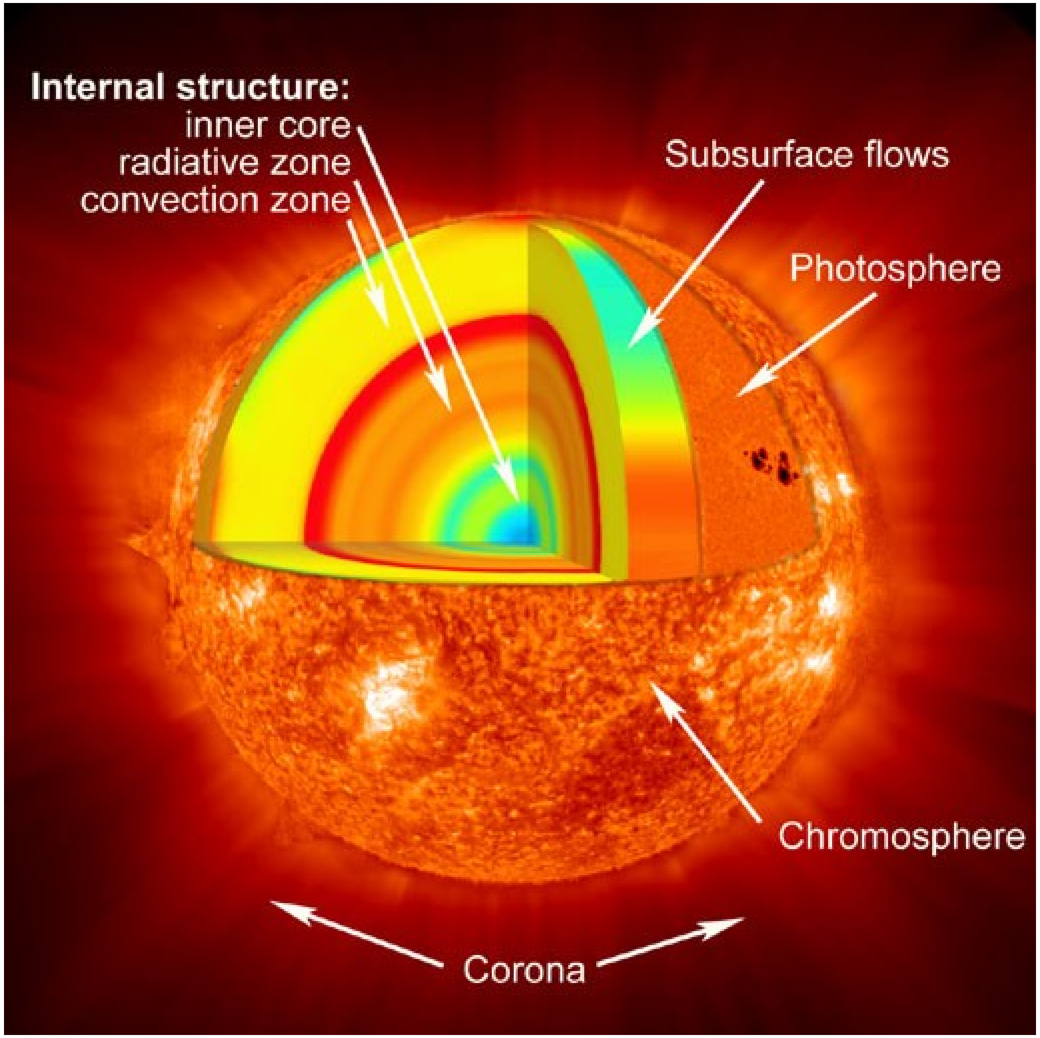
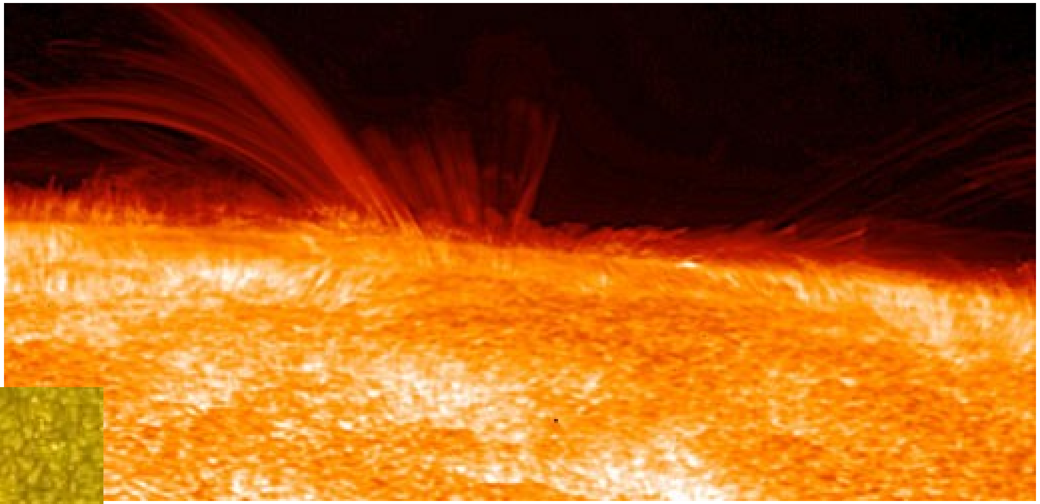
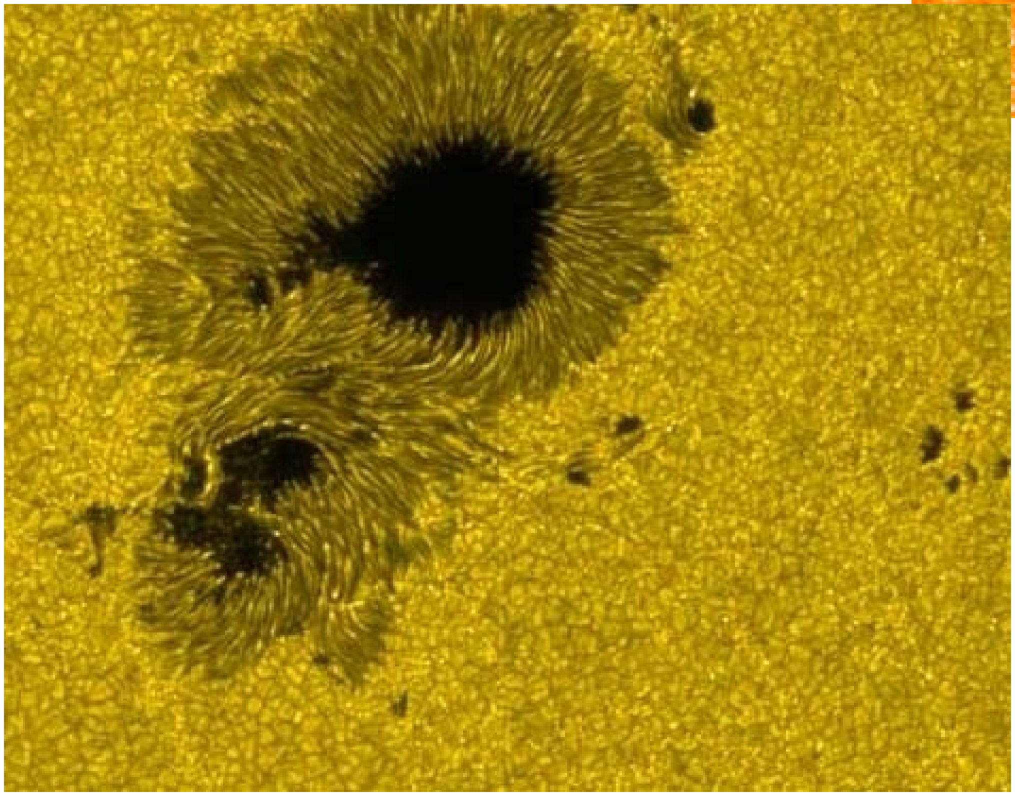


Image credit: NASA

FLUCTUATION X MEAN-FIELD DYNAMO

Chromosphere,
Hinode's Solar Optical
Telescope

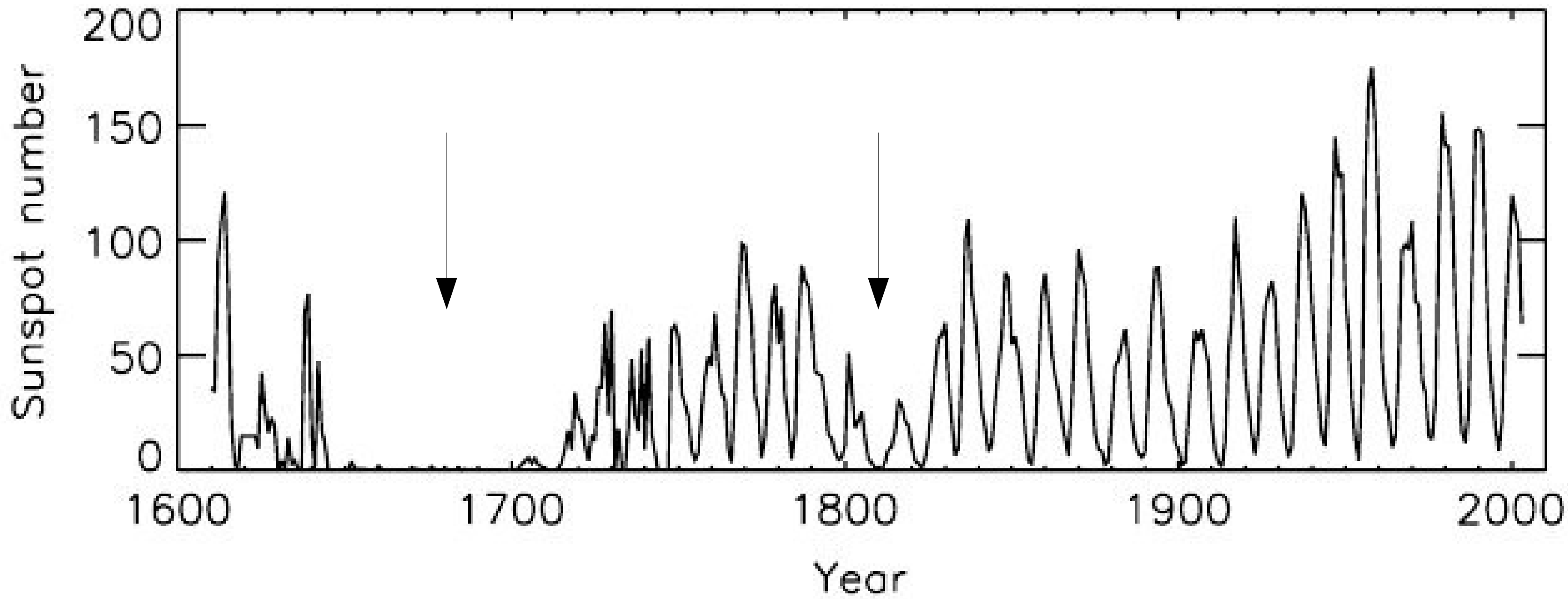




Sunspot,
Hinode's Solar Optical
Telescope

Images credit: Hinode JAXA/NASA

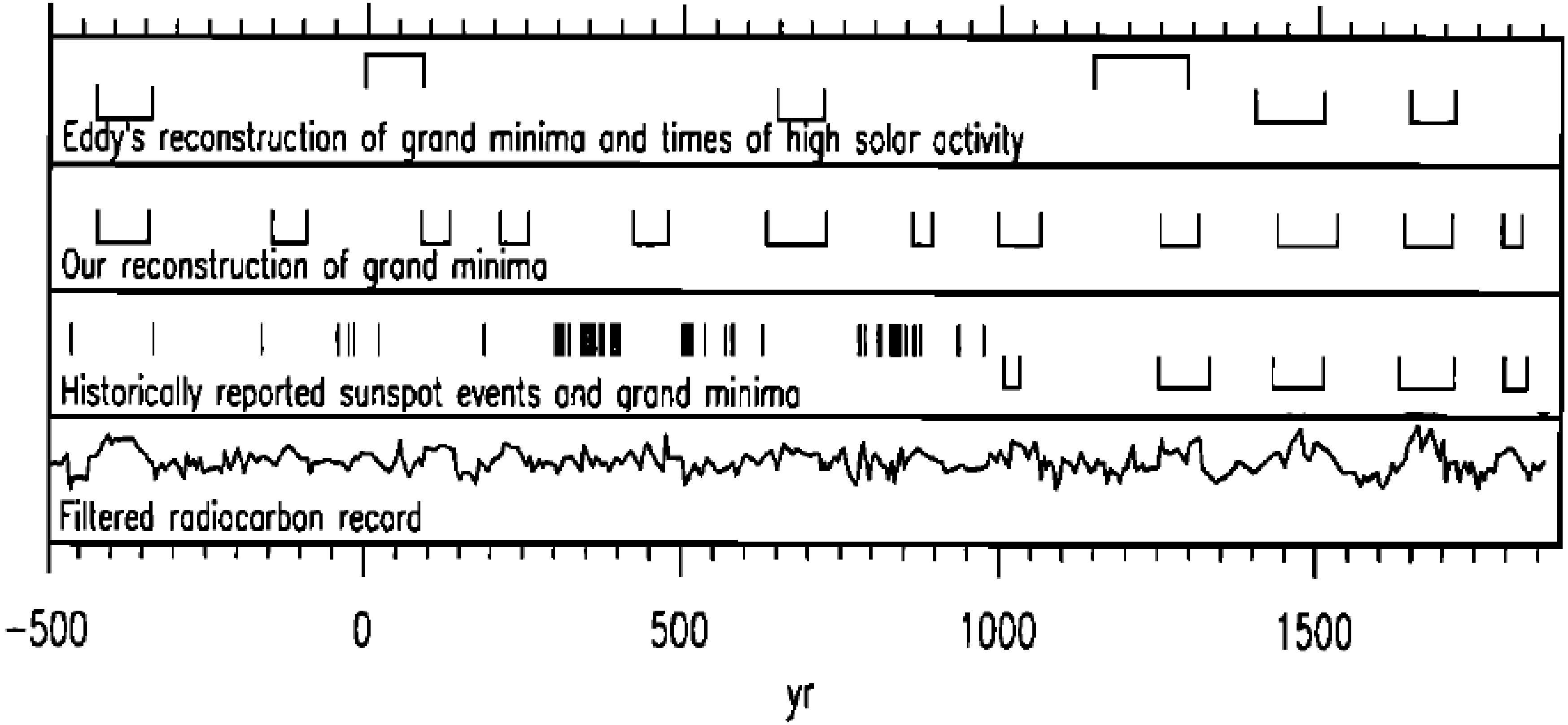
INTERMITTENT DYNAMO?



Intermittency is characterized by time series with random switching between phases of "laminar" and "bursty" behaviors

Image source: Solanki et al., Rep. Prog. Phys. 69 (2006) 563

INTERMITTENT DYNAMO?



Reconstruction of grand minima from historical record of sunspots (Eddy, Sci Am. 1977) and from proxy data (filtered ^{14}C measured from tree-rings of very old trees, Voss et al., JGR 1996); Historical data from Wittmann, A&A 1978 and minima by Krivsky, Sol. Phys. 1984.

Source: Voss et al., J. Geophys. Res. 101 (1996) 637

THE SOLAR CYCLE AS AN INTERMITTENT CHAOTIC EVENT

ODE Low-D models:

- Covas, Ashwin &¨Tavakol, Phys. Rev. E 56, 6451 (1997)
- Wilmot-Smith et al., MNRAS 363, 1167 (2005)

PDE Mean-Field models:

- Covas & Tavakol, Phys. Rev. E 60, 5435 (1999) - multiple-intermittency hypothesis
- Ossendrijver, A&A 359, 364 (2000)
- Ossendrijver & Covas, Int. J. Bifurcation Chaos 13, 2327 (2003) crisis
- Moss & Brooke, MNRAS 315, 521 (2000)
- Charbonneau, ApJ, 616, L183 (2004)
- Brandenburg & Spiegel, Astron. Nachr. 329, 351 (2008)
- Spiegel, Space Sci. Rev. 144, 25 (2009) - review

NUMERICAL SIMULATION OF A NONLINEAR DYNAMO

We consider a compressible gas ($\nabla \cdot \mathbf{u} \neq 0$) with constant sound speed c_s, constant dynamical viscosity μ, constant magnetic diffusivity η, and constant magnetic permeability μ_0

Compressible, resistive MHD equations:

$\partial_t \ln\rho + \mathbf{u} \cdot \nabla \ln\rho + \nabla \cdot \mathbf{u} = 0$ (Continuity eq.)

$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -c_s^2 \nabla \ln\rho + (\mathbf{J} \times \mathbf{B}) / \rho + \mu/\rho(\nabla^2 \mathbf{u} + \nabla\nabla \cdot \mathbf{u}/3) + \mathbf{f}$ (Momentum eq.)

$\partial_t \mathbf{A} = \mathbf{u} \times \mathbf{B} - \eta\mu_0 \mathbf{J}$ (Induction eq.)

where $\mathbf{J} = \nabla \times \mathbf{B}/\mu_0$ is the current density, $\mathbf{B} = \nabla \times \mathbf{A}$, and $c_s^2 = p/\rho$.

NUMERICAL SIMULATION OF A NONLINEAR DYNAMO

PENCIL CODE[1] in a box with sides $\mathbf{L = 2\pi}$ and periodic boundary conditions.
Initial condition: $\ln\rho = \mathbf{u} = 0$, and **A** is a set of normally distributed, uncorrelated random numbers with standard deviation of 10^{-3}.

Forcing function **f** - **ABC flow** (Arnold-Beltrami-Childress):

$$\mathbf{f(x)} = A_f/\sqrt{3}(\sin k_f z + \cos k_f y, \sin k_f x + \cos k_f z, \sin k_f y + \cos k_f x)$$

Beltrami flow: $\nabla\times\mathbf{u} \propto \mathbf{u}$, provides maximum **Helicity** ($H^2 = \langle\mathbf{u}\cdot\nabla\times\mathbf{u}\rangle^2 = \langle|\mathbf{u}|^2\rangle\langle|\nabla\times\mathbf{u}|^2\rangle$), and Lagrangean **chaos**.

We use $\mathbf{A_f = 0.1}$ and a resolution of $\mathbf{64^3}$ grid points. We choose $\mathbf{k_f = 5}$ in order to be able to see the emergence of a large scale magnetic field, with spatial scales larger than the energy injection scale.

[1]http://www.nordita.org/software/pencil-code

THE PENCIL CODE

PENCIL CODE[1] is useful for weakly compressible, non-conservative simulations of driven MHD turbulence, with or without shear.

It uses 6th order explicit (for better parallelization) centered finite differences squeme in space:

$$f_i' = (-f_{i-3} + 9f_{i-2} - 45f_{i-1} + 45f_{i+1} - 9f_{i+2} + f_{i+3})/(60\delta x),$$

$$f_i'' = (2f_{i-3} - 27f_{i-2} + 270f_{i-1} - 490f_i + 270f_{i+1} - 27f_{i+2} + 2f_{i+3})/(180\delta x^2),$$

and a third order Runge-Kutta scheme for time integration.

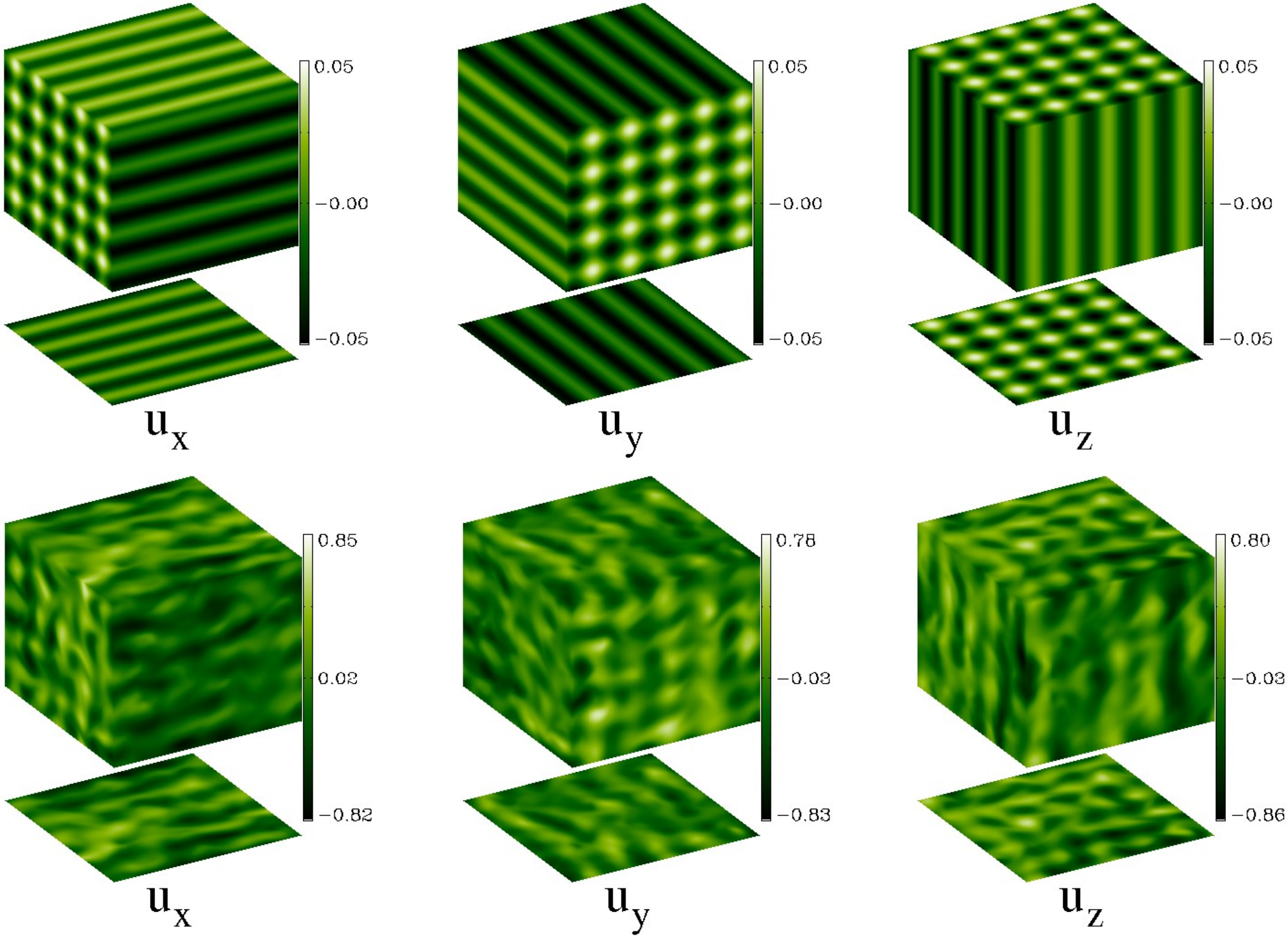
MPI parallelization is implemented.

[1] http://www.nordita.org/software/pencil-code

HYDRODYNAMIC SIMULATIONS

ν = 0.02
Re = L U / ν = 12.38
L = $2\pi/k_f$

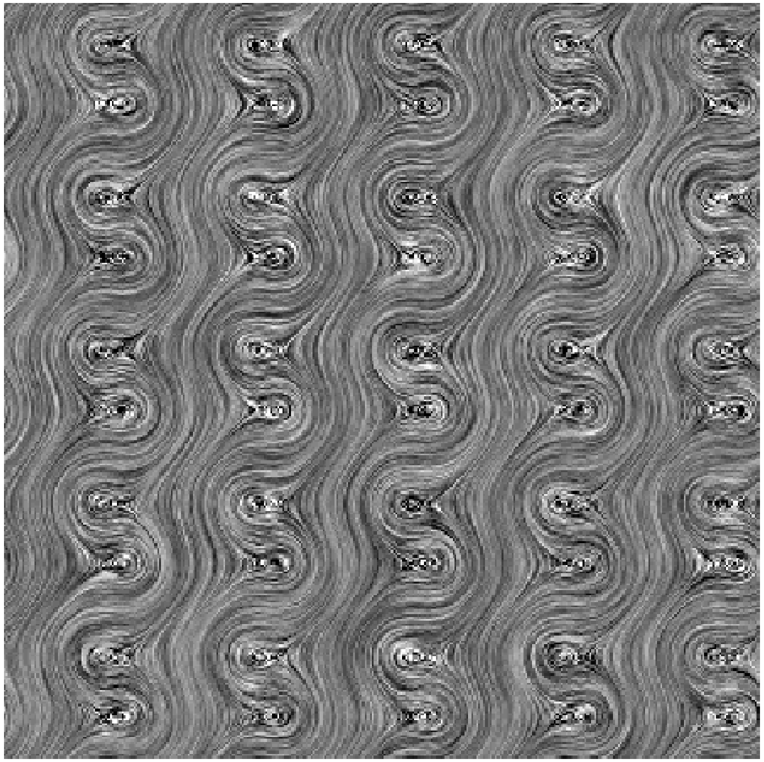
ν = 0.005
Re ~ 100
(Box scale Re ~ 500)



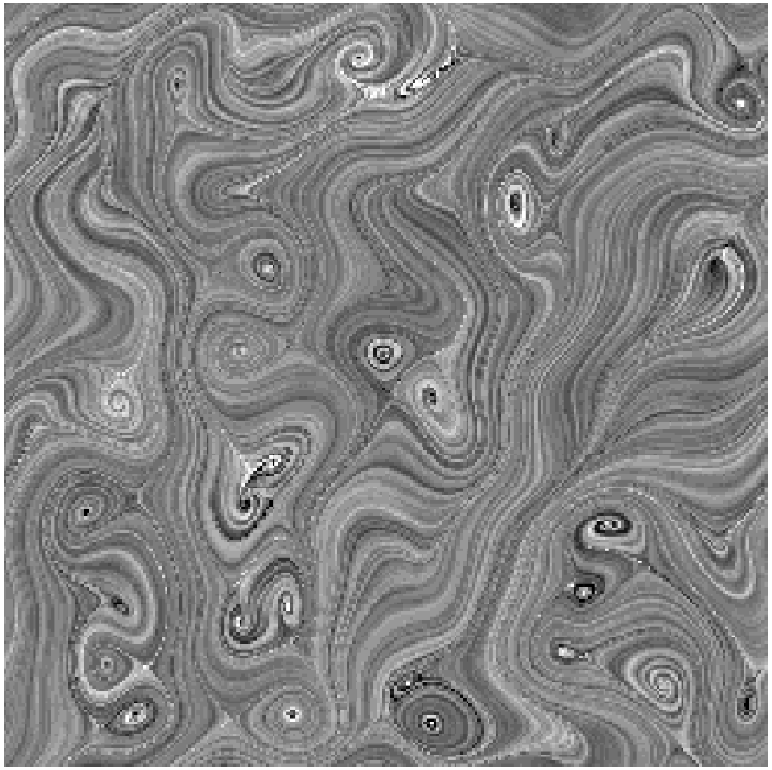
Source: Rempel et al., MNRAS, 400, 509-517 (2009)

LINE INTEGRAL CONVOLUTION PLOTS

$\nu = 0.02$
$Re = L\,U / \nu = 12.38$
$L = 2\pi/k_f$

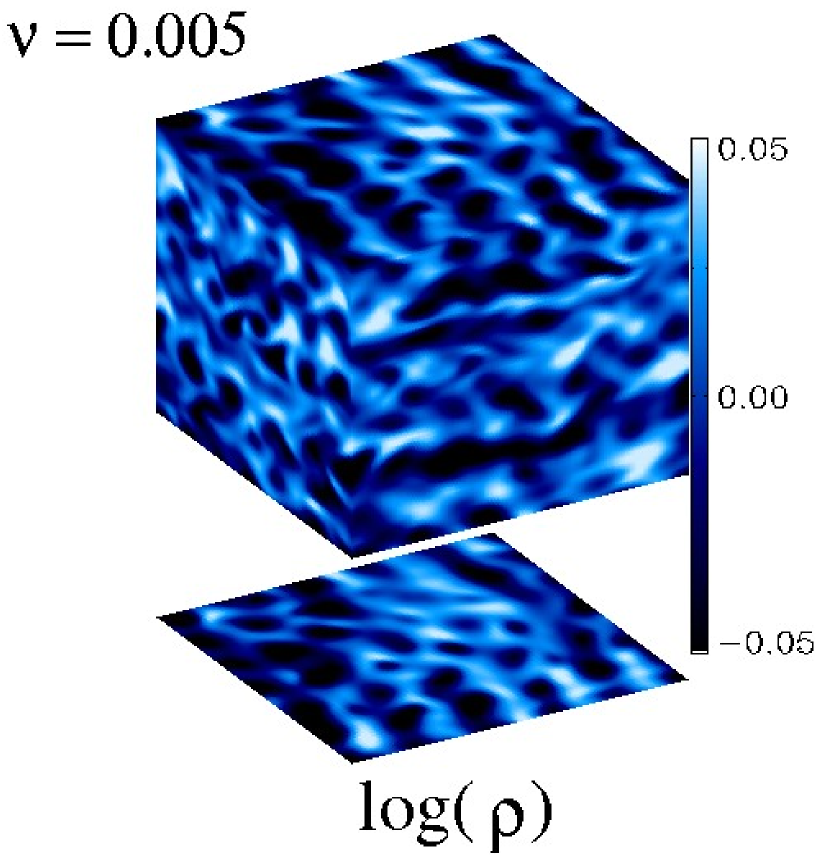


$\nu = 0.005$
$Re \sim 100$
(Box scale $Re \sim 500$)



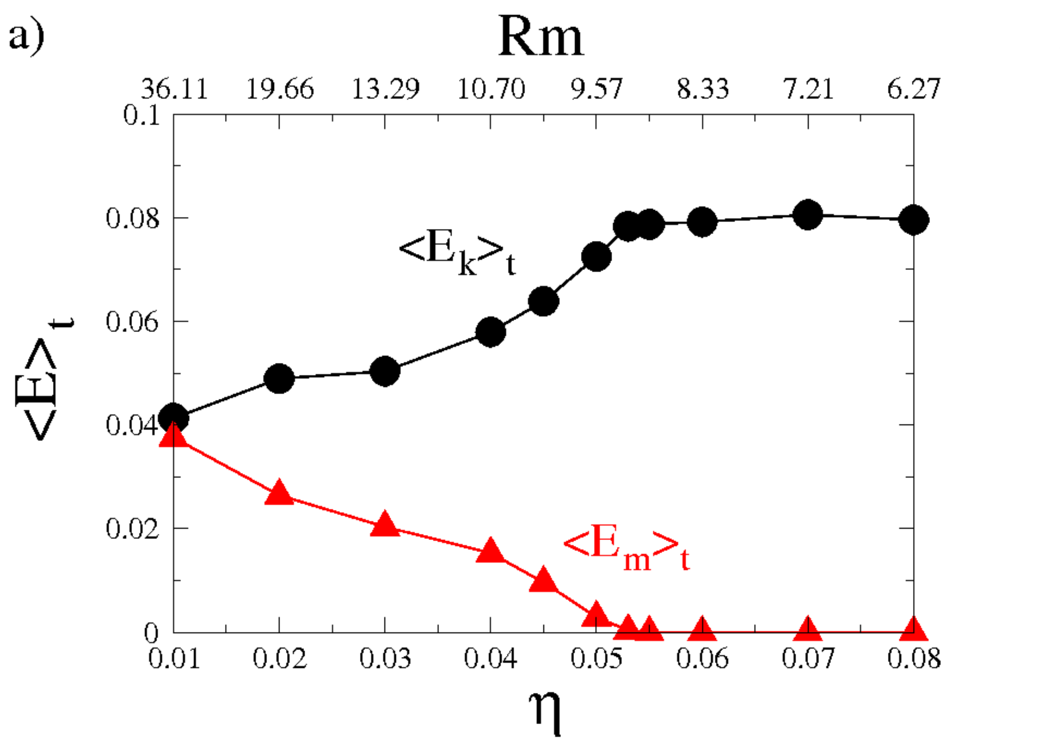
HYDRODYNAMIC SIMULATIONS

Compressible flow



Source: Rempel et al., MNRAS, 400, 509-517 (2009)

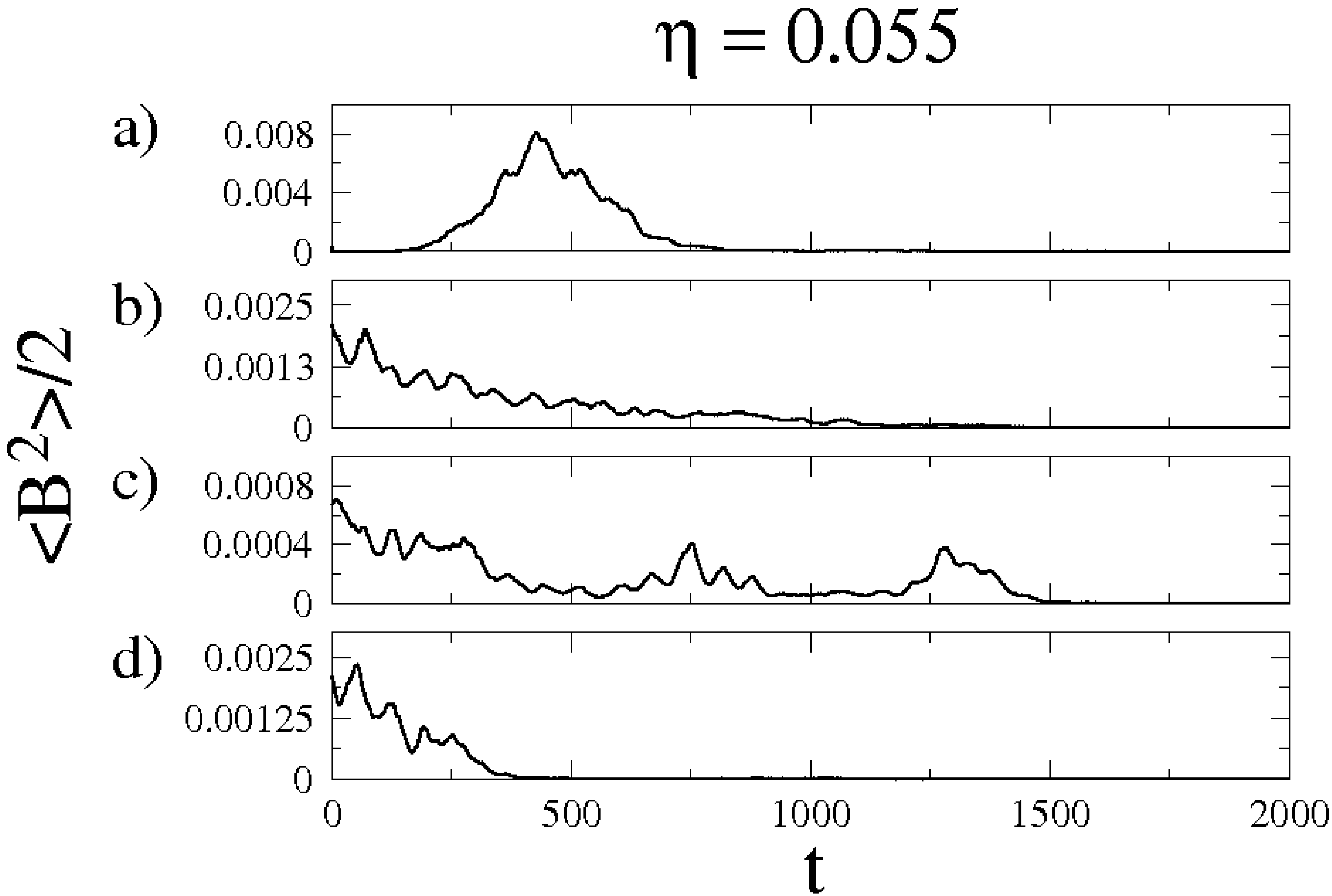
BIFURCATION DIAGRAMS, MHD SIMULATIONS



Bifurcation diagram, time-averaged
Kinetic (black) and magnetic (red) energies

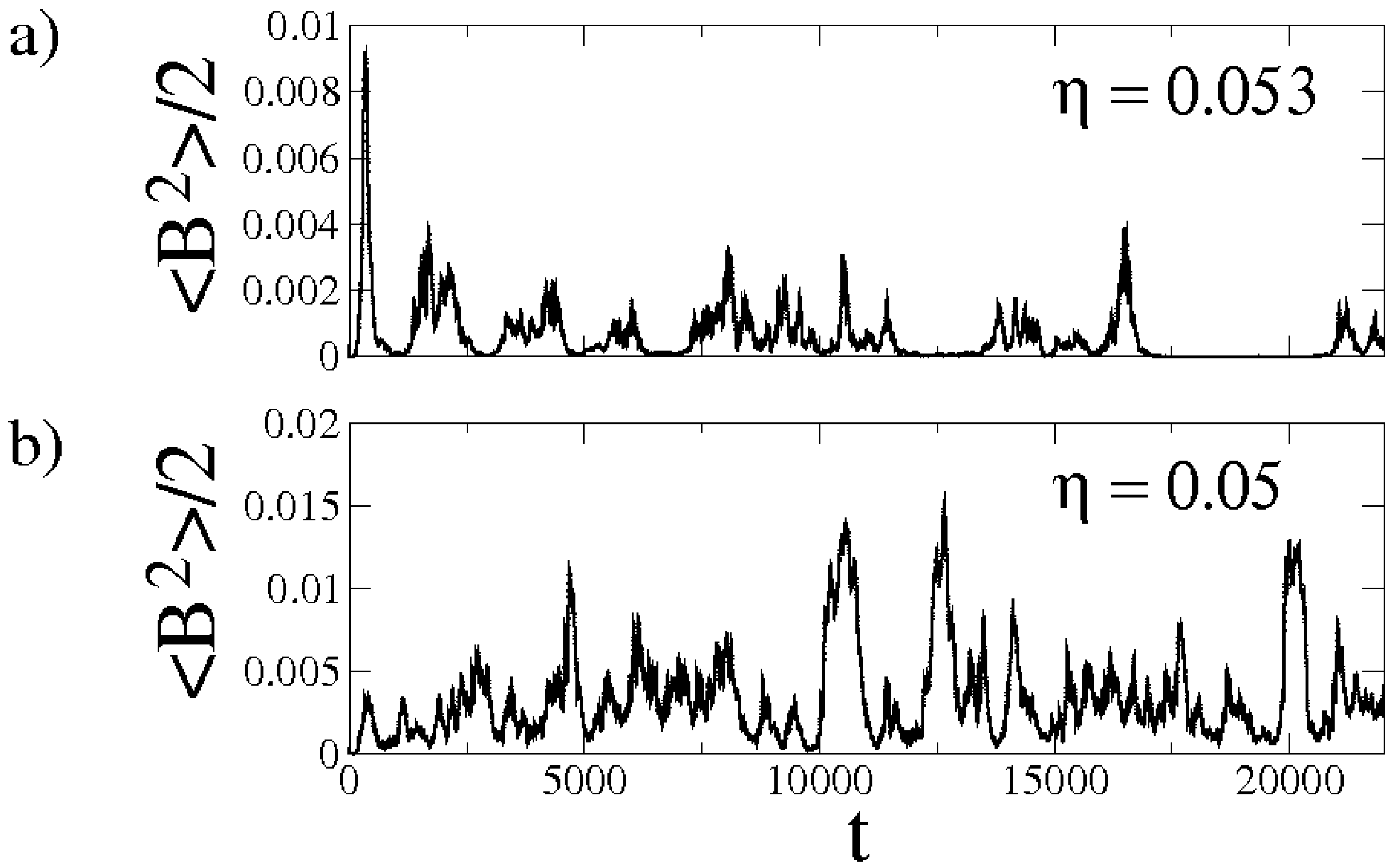
Source: Rempel et al., MNRAS, 400, 509-517 (2009)

TRANSIENT "DYNAMO"



Source: Rempel et al., MNRAS, 400, 509-517 (2009)

INTERMITTENT TIME SERIES



a) On-off intermittent dynamo

b) ?

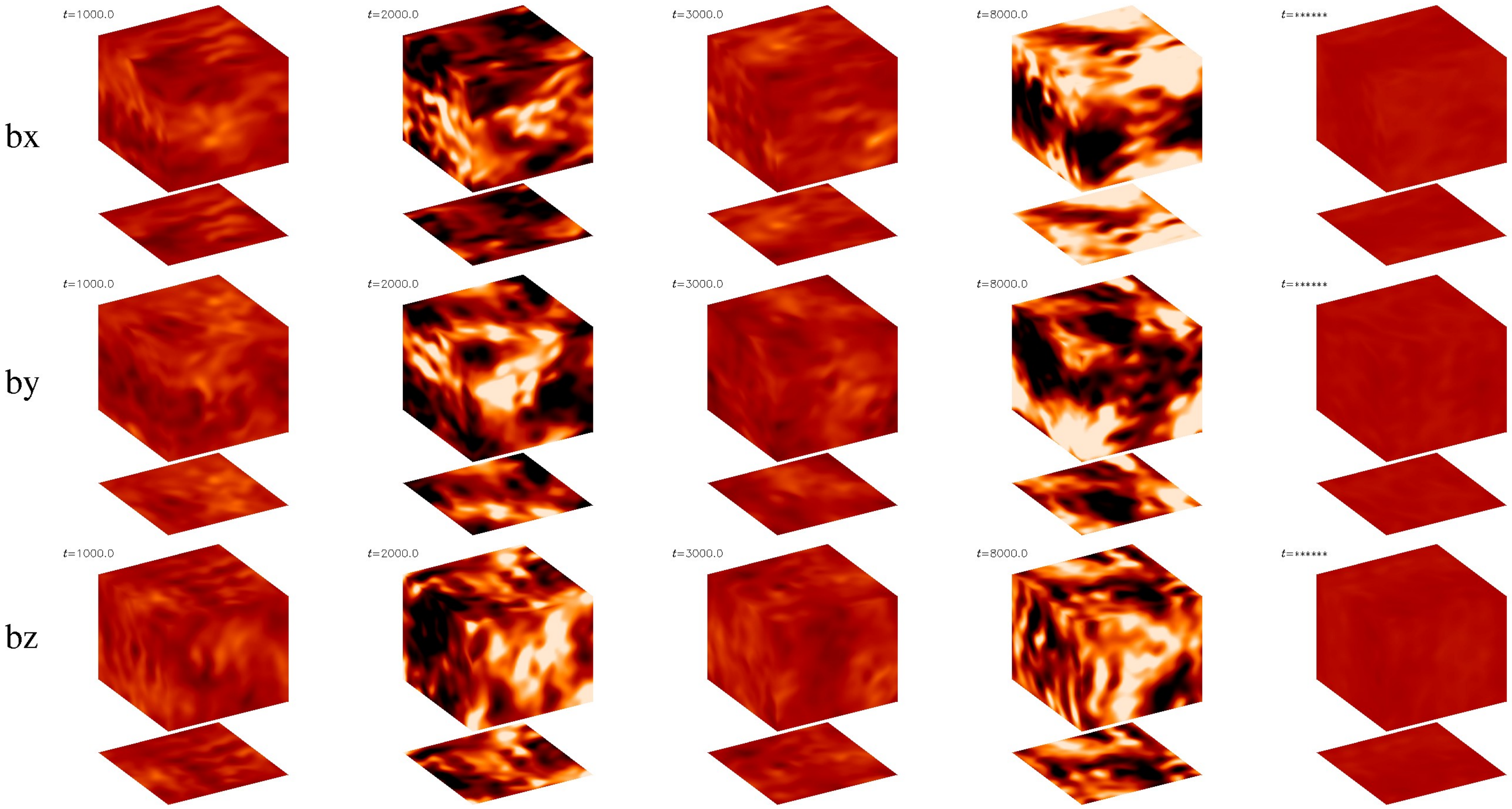
Source: Rempel et al., MNRAS, 400, 509-517 (2009)

BLOWOUT BIFURCATION

- There is an invariant manifold in the phase space (e.g., the **B=0** hydrodynamic state)
- There is a chaotic attractor on this manifold (e.g., chaotic velocity field)
- For $Rm < Rm_c$ the chaotic attractor on the manifold is transversely stable (e.g., "almost all" perturbations in **B** decay to **B=0**)
- For $Rm > Rm_c$ t the chaotic attractor loses its average stability to transverse perturbations

Source: Sweet et al., Phys. Plasmas 8 (2001) 1944

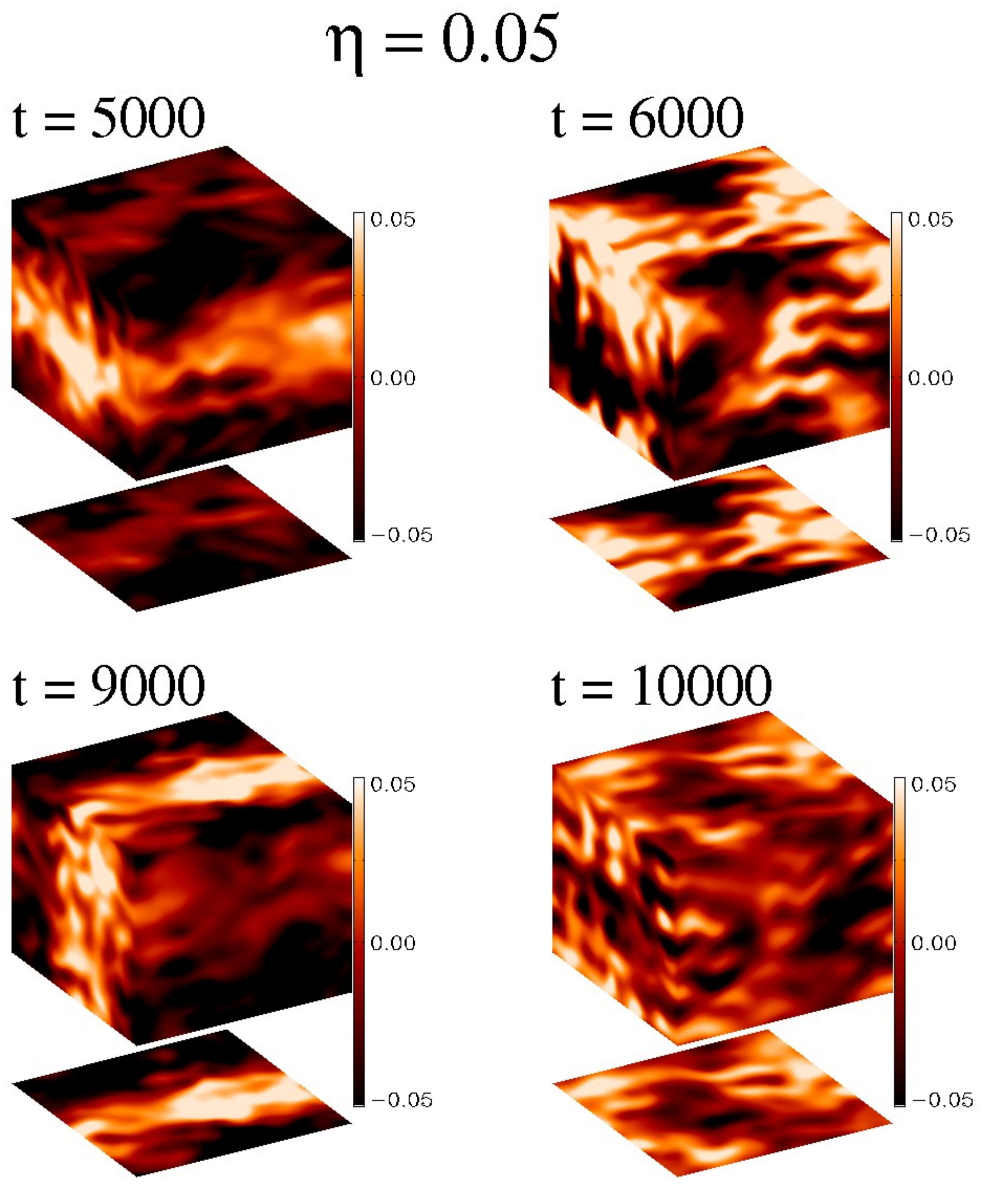
ON-OFF INTERMITTENCY



Bx at η = 0.053, on-off intermittent dynamo

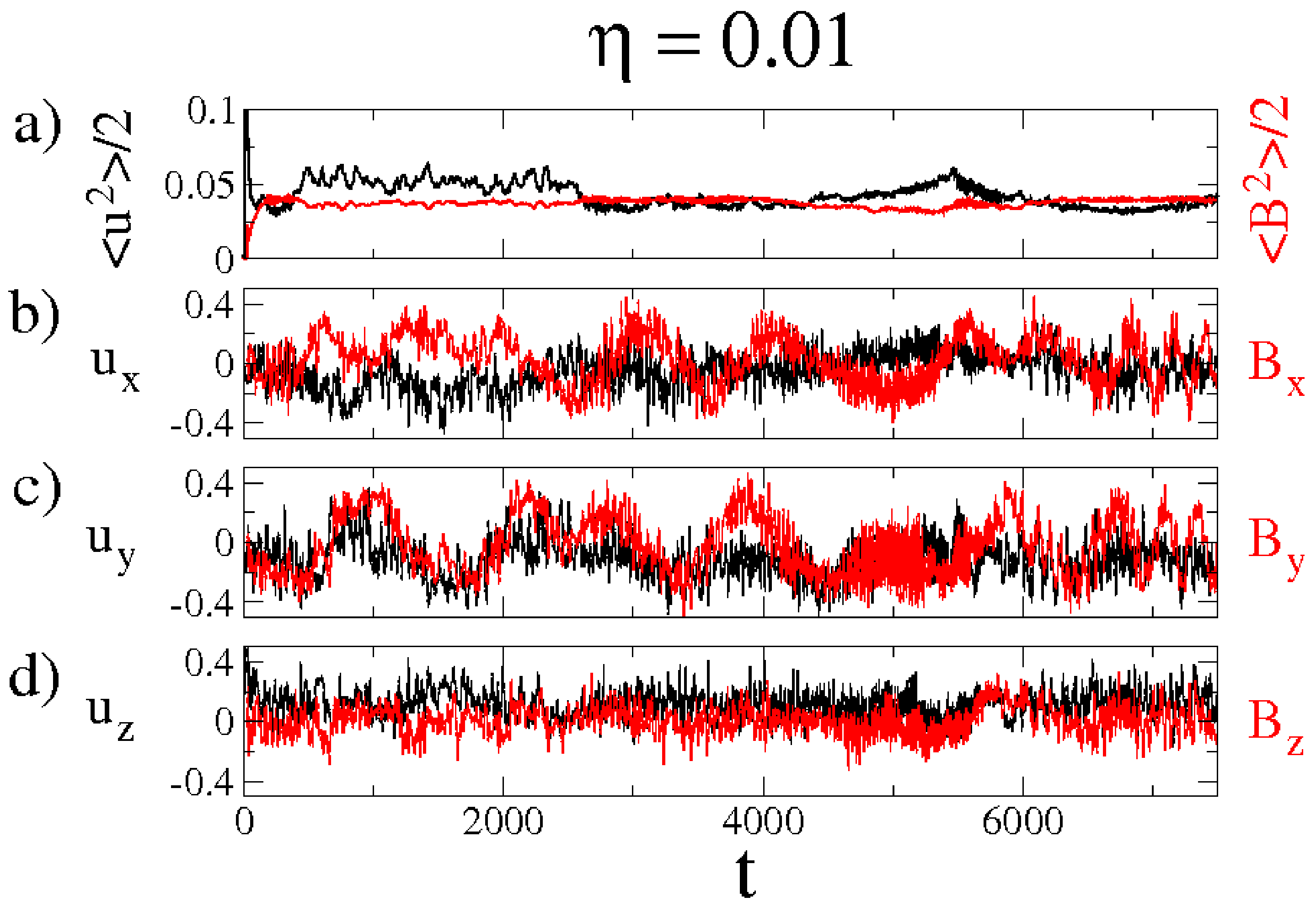
COHERENCE-INCOHERENCE INTERMITTENCY

Bx at $\eta = 0.05$, intermittent dynamo



Source: Rempel et al., MNRAS, 400, 509-517 (2009)

TIME SERIES

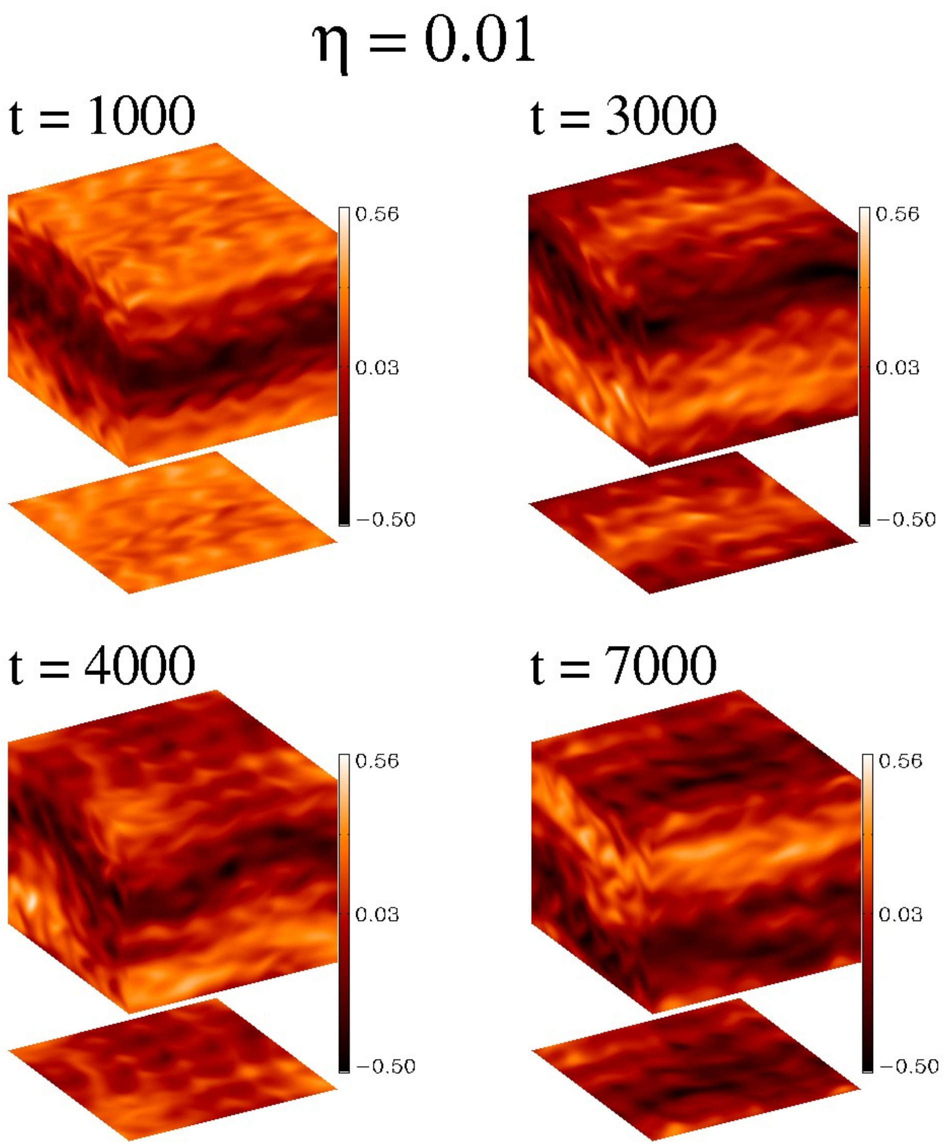


$\eta = 0.01$, Re ~ 75.4 (~ 377 box scale),
Rm ~ 37.7 (~ 188.5 box scale)

Source: Rempel et al., MNRAS, 400, 509-517 (2009)

SINUSOIDAL MEAN-FIELD

$\eta = 0.01$, sinusoidal Bx



Source: Rempel et al., MNRAS, 400, 509-517 (2009)

SHOW MOVIES

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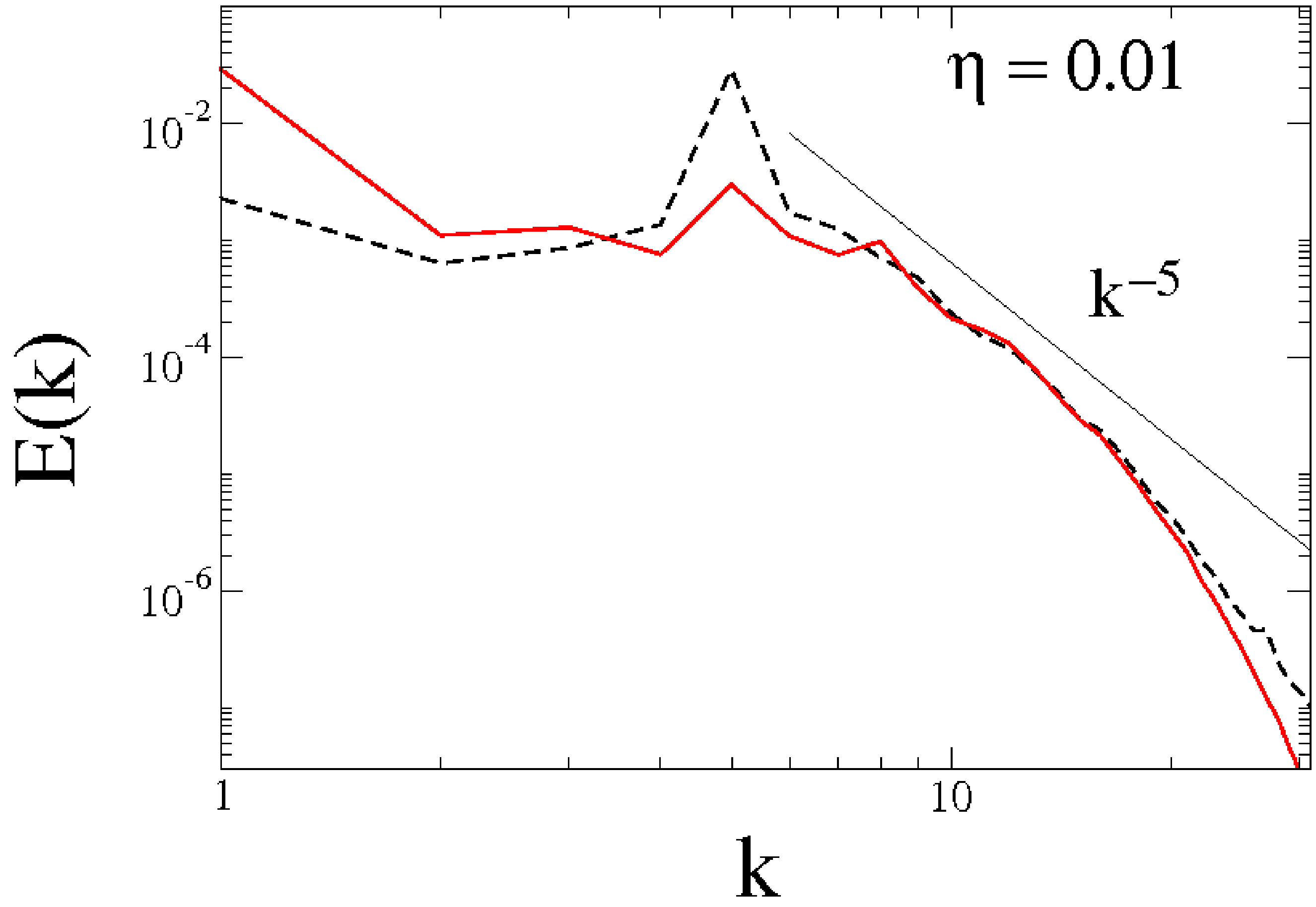
POWER SPECTRUM AND INVERSE CASCADE



$\eta = 0.01$, magnetic (red) and kinetic (black) energy spectra

Source: Rempel et al., MNRAS, 400, 509-517 (2009)

MEASURING SPATIAL COMPLEXITY

Spectral entropy

$S(t) = - \sum_k p_{k,t} \ln p_{k,t}$

$p_{k,t} \ln p_k = 0$ if $p_k = 0$

Relative weight of mode *k*

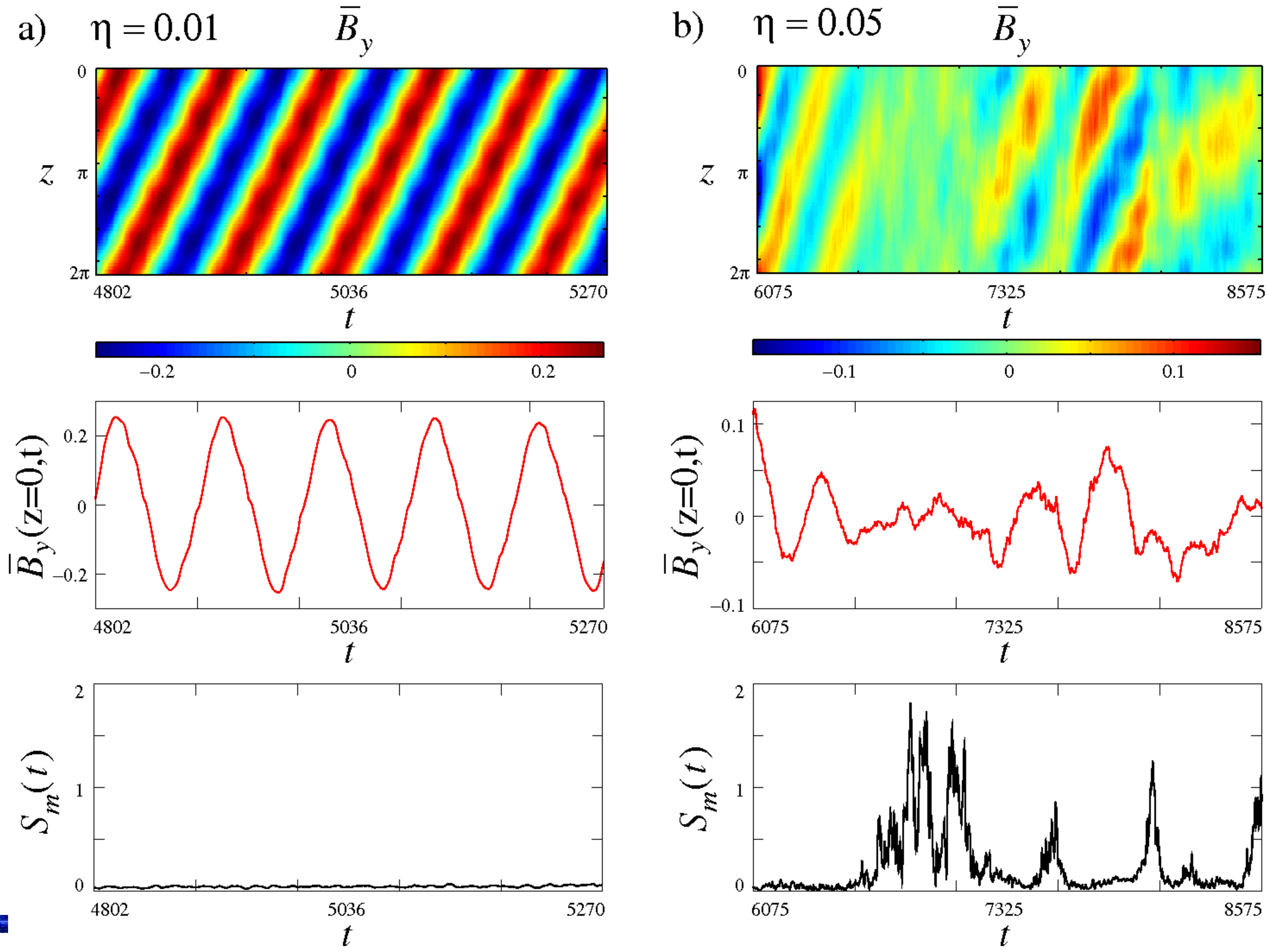
$p_{k,t} = |b_k(t)|^2 / \sum_j |b_j(t)|^2$

$p_{k,t} \in [0,1]$ and $\sum_k p_{k,t} = 1$

Min(S) = 0 (ordered state with $p_{k,t} = 1$ for some *k*)

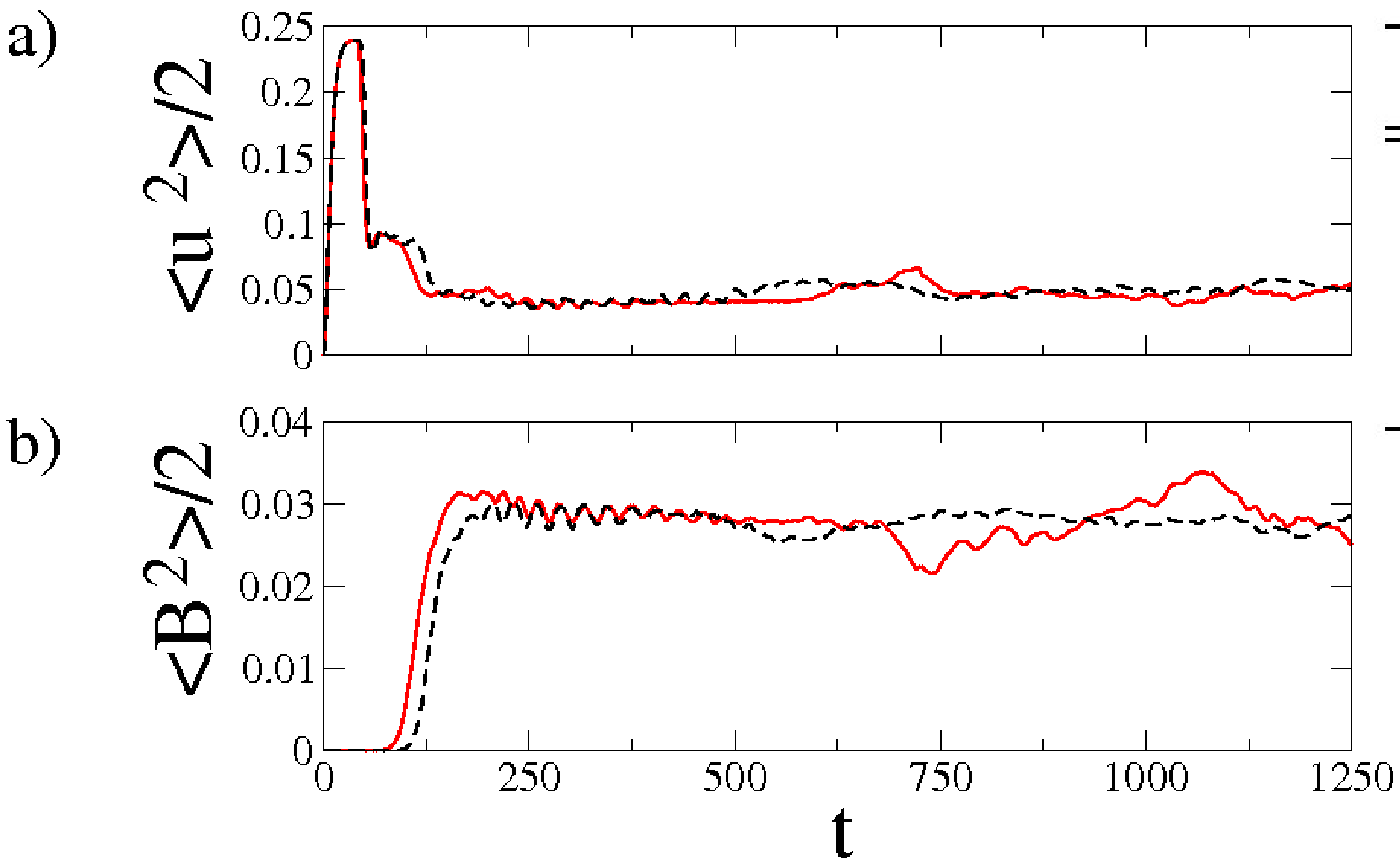
Max(S) = ln (*N*) ~ 3.42 (random state with $p_{k,t} = 1/N$ for any *k*)

LAMINAR x TURBULENT MEAN-FIELD DYNAMO



NUMERICAL RESOLUTION

$$\eta = 0.02$$

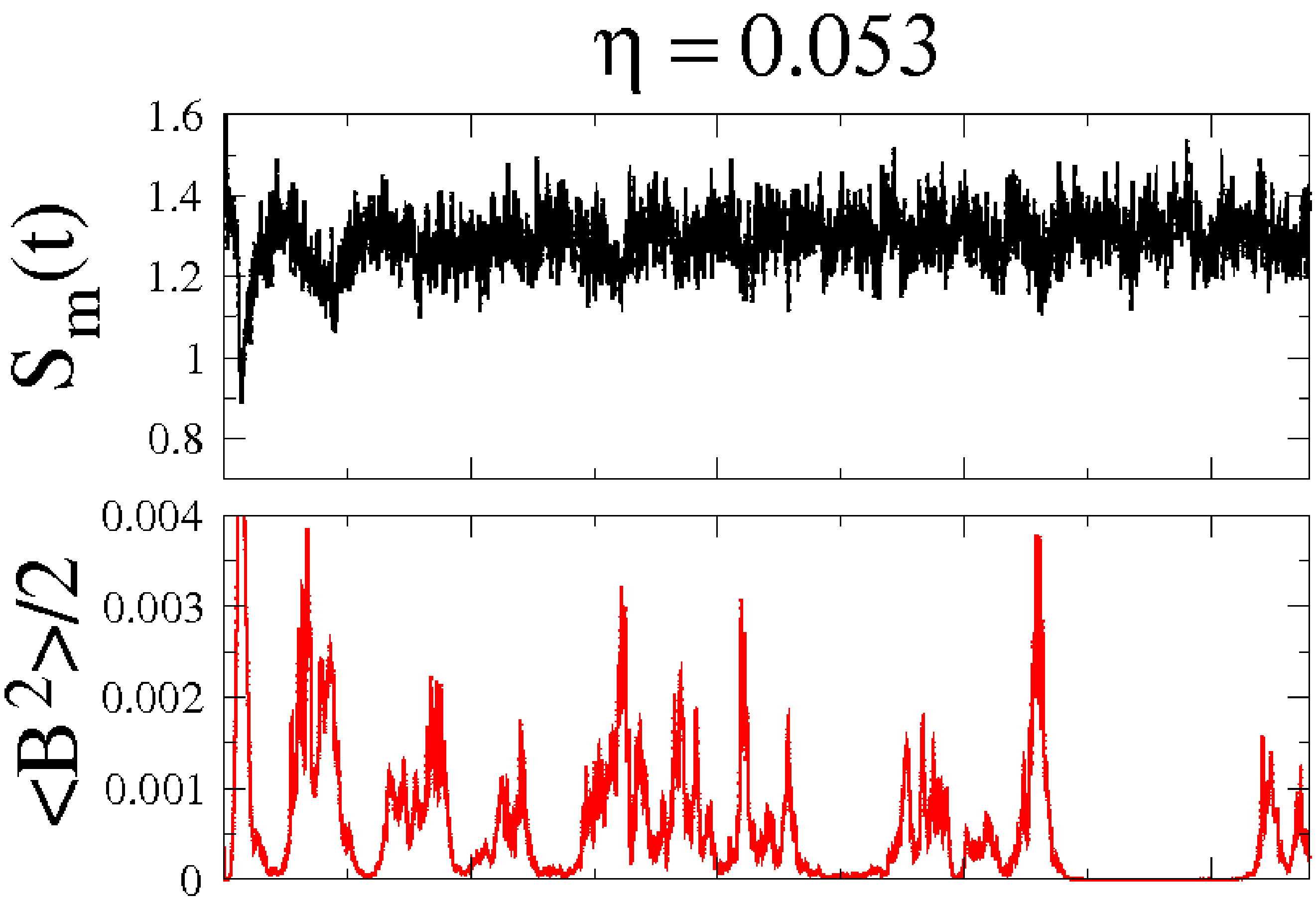


Res	64^3	128^3
$\langle S_k \rangle_t$	1.24	1.10
$\langle N_k \rangle_t$	4.77	4.96
$\langle S_m \rangle_t$	1.08	1.06
$\langle N_m \rangle_t$	2.85	2.93

64^3 (red) and 128^3 (black)

Source: Rempel et al., MNRAS, 400, 509-517 (2009)

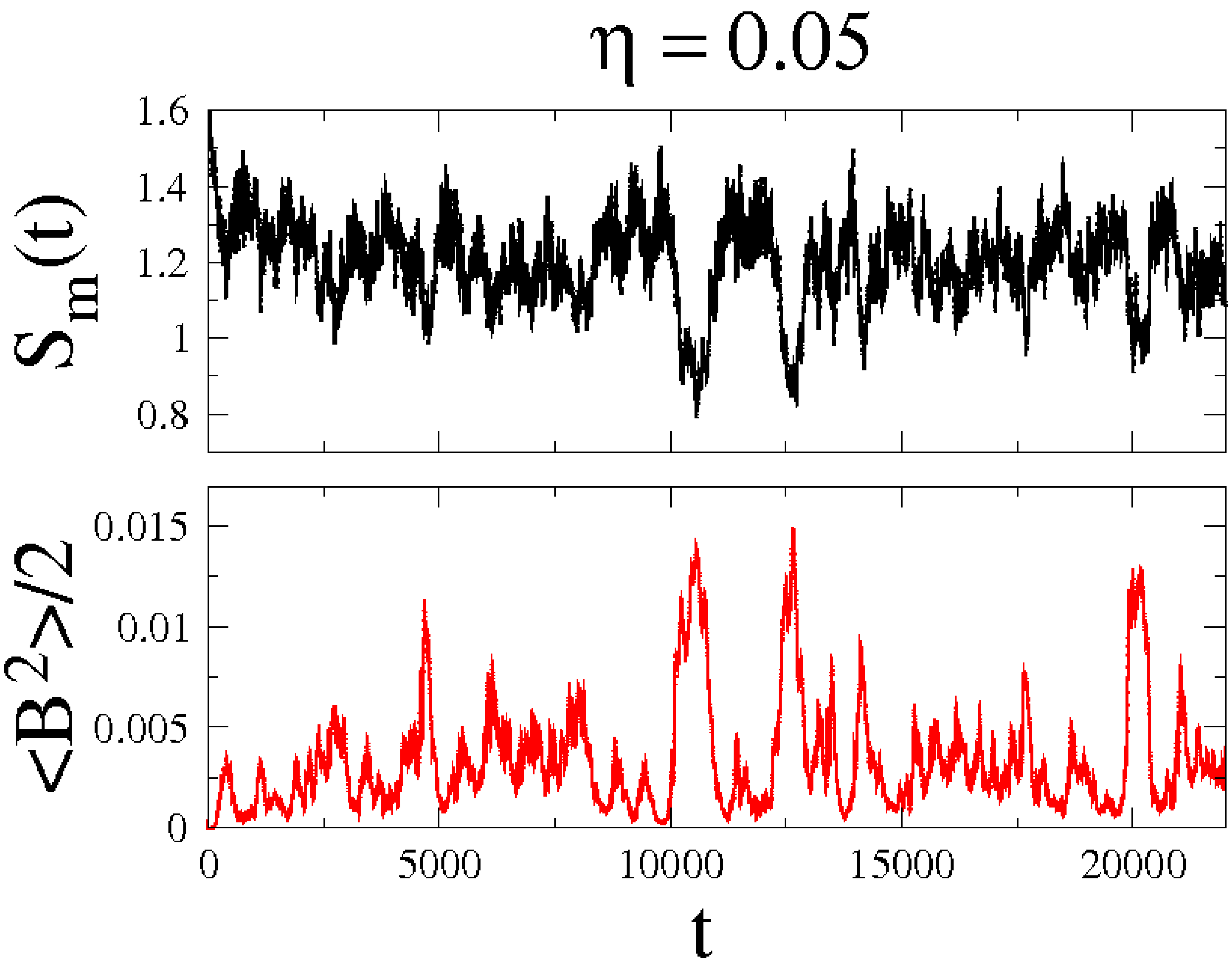
ON-OFF INTERMITTENCY



Magnetic energy (red) and entropy (black) time series

Source: Rempel et al., MNRAS, 400, 509-517 (2009)

COHERENCE-INCOHERENCE INTERMITTENCY



Magnetic energy (red) and entropy (black) time series

Source: Rempel et al., MNRAS, 400, 509-517 (2009)

CONCLUSIONS

Main results:

- Measure of spatial complexity in an intermittent dynamo;
- Characterization of coherent/incoherent intermittency;
- "Bifurcation diagram" for ABC-flow dynamo with inverse-cascade.

Open questions:

- How general is the coherent/incoherent intermittency mechanism?
- Is it present in turbulence with rotation and shear?
- What are the causes?