"Computational Pattern Analysis in Physics and Cosmology"

Reinaldo R. Rosa[1],

Murilo S. Dantas, Gustavo Zaniboni[1] , Cristiane P. Camilo[1],
Ana Paula Andrade[2], André L. Ribeiro[2], German Gomero[3],
William S. Hipólito-Ricaldi[3] , Carlos Alexandre Wuenshe[4]
Haroldo F. C. Velho, Fernando M. Ramos[1],
Cesar A. Caretta[1], Nei Cardenuto[1]

[1]Núcleo para Simulação e Análise de Sistemas Complexos (NUSASC)
Laboratório Associado de Computação e Matemática Aplicada (LAC)
Instituto Nacional de Pesquisas Espaciais (INPE)
São José dos Campos – SP-Brasil

[2]UESC, [3]IFT/CBF, [4]DAS/INPE

Versão atualizada do trabalho apresentado no evento
"Conference on Computational Cosmology"
Abdus-Salam ICTP-Trieste, Maio de 2005

Classification of Physical Patterns (Representation Level):

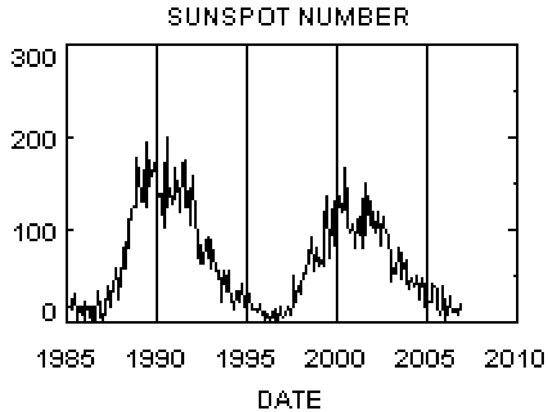
1st Order Patterns (Usual domains): time, space and frequency

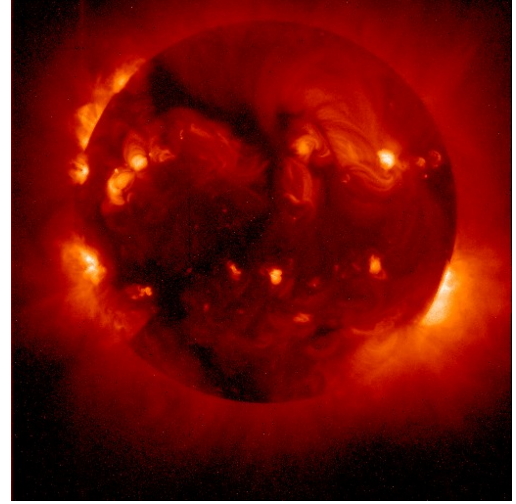
2nd Order Patterns: phase space

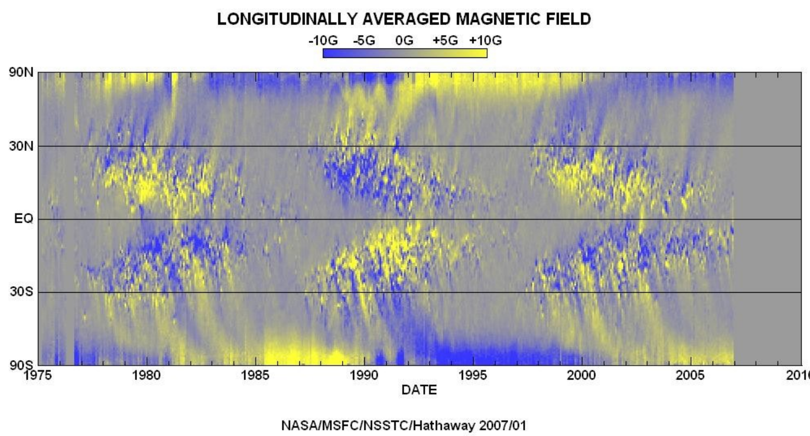
3rd Order (Statistical Parameters): Histogram and Power-law

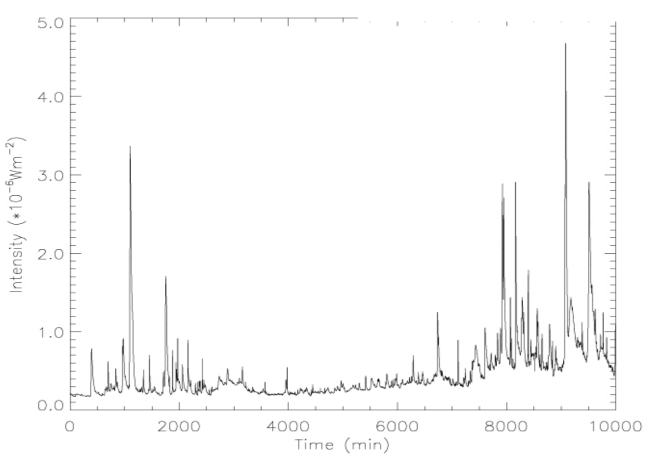
➔Quantifiers and Metrics ➔ Phenomelogical Characterization

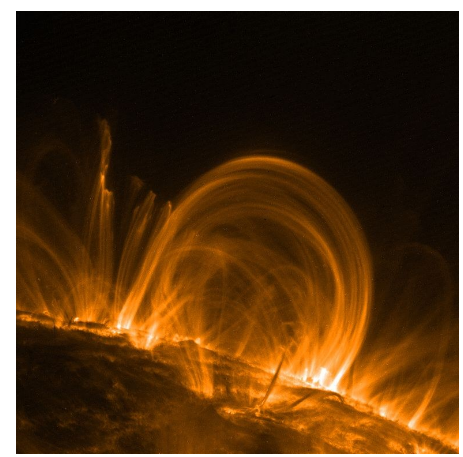
(mencionar noção de vetor de atributos e mineração de dados!)

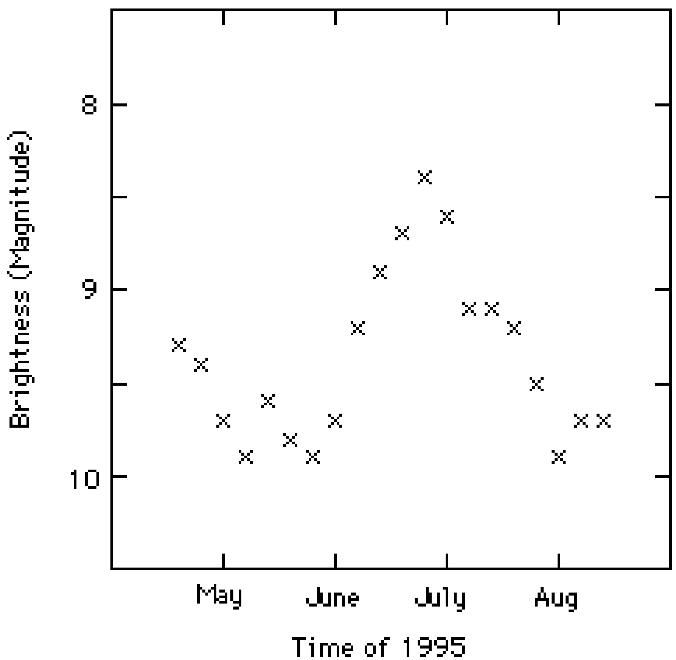

SUNSPOT NUMBER
300
200
100
0
1985
1990
1995
2000
2005
2010
DATE

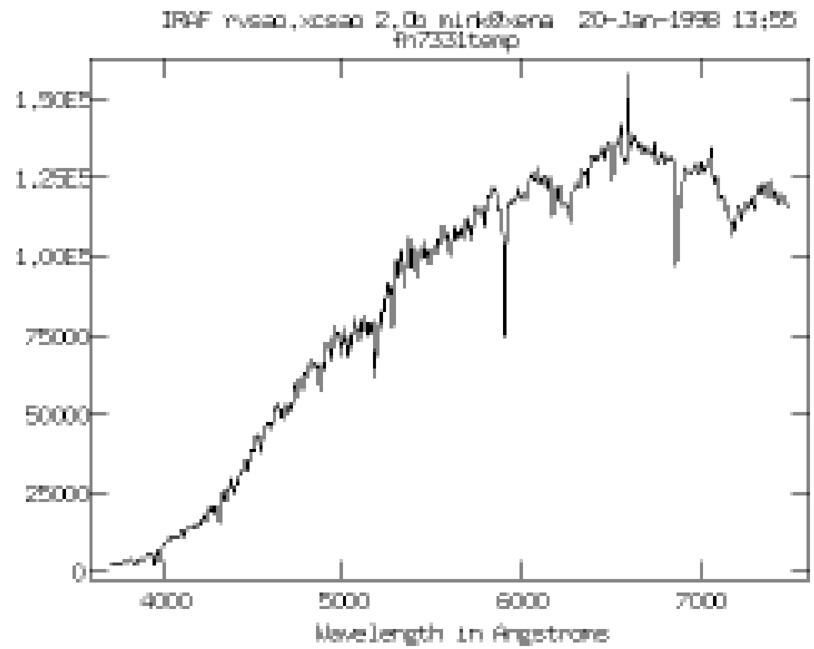


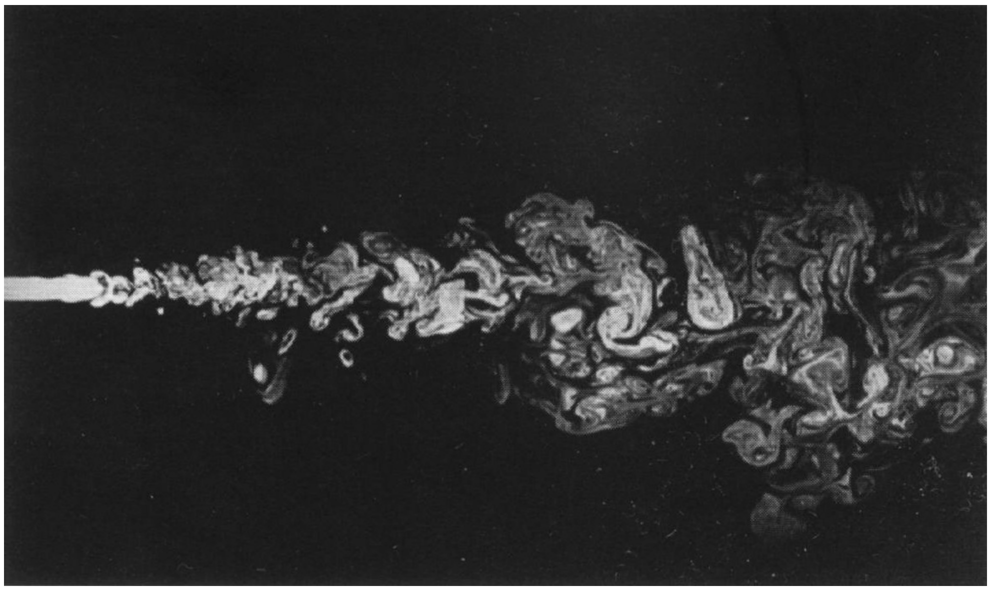
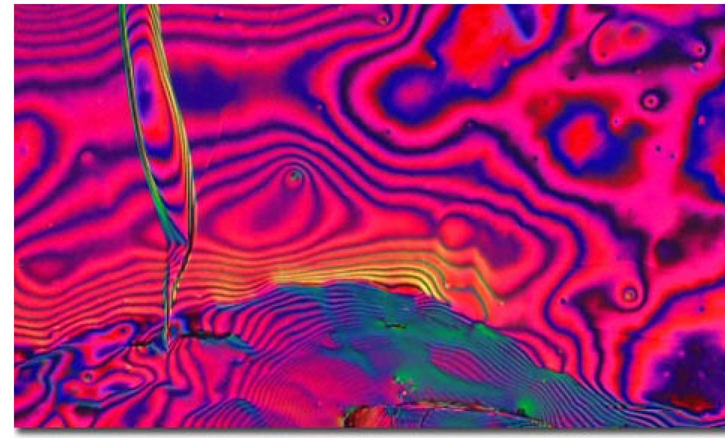
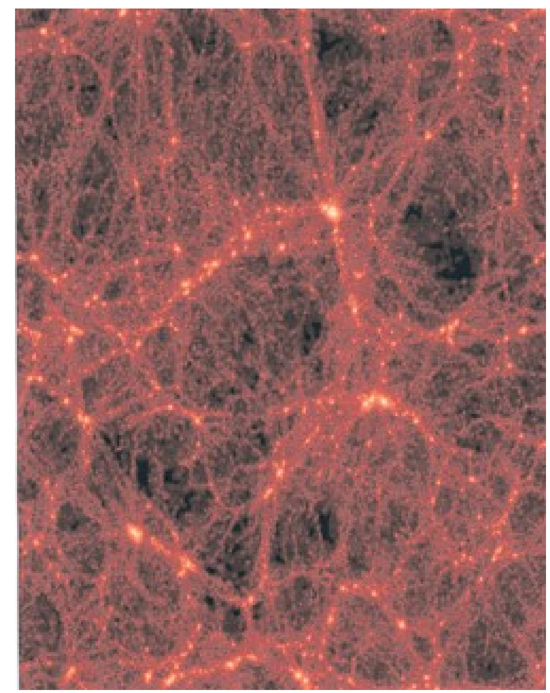

LONGITUDINALLY AVERAGED MAGNETIC FIELD
-10G -5G 0G +5G +10G
90N
30N
EQ
30S
90S
1975
1980
1985
1990
1995
2000
2005
2010
DATE
NASA/MSFC/NSSTC/Hathaway 2007/01


5.0
4.0
3.0
2.0
1.0
0.0
Intensity ($\times 10^{-6}$Wm^{-2})
0
2000
4000
6000
8000
10000
Time (min)




Brightness (Magnitude)
8
9
10
May
June
July
Aug
Time of 1995


1.50E5
1.25E5
1.00E5
75000
50000
25000
0
4000
5000
6000
7000
Wavelength in Angstroms

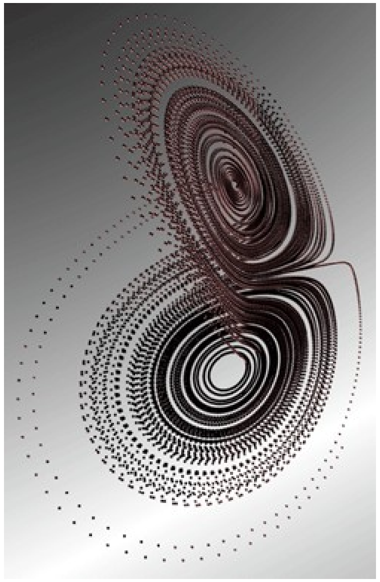




At one point, Edward Lorenz was looking for a way to model the action of the chaotic behavior of the gaseous system first mentioned above. Lorenz took a few "Navier-Stokes" equations, from the physics field of
fluid dynamics. He simplified them and got as a result the following three-dimensional system:

dx/dt = delta * (y - x)

dy/dt = r * x - y - x * z

dz/dt = x * y - b * z

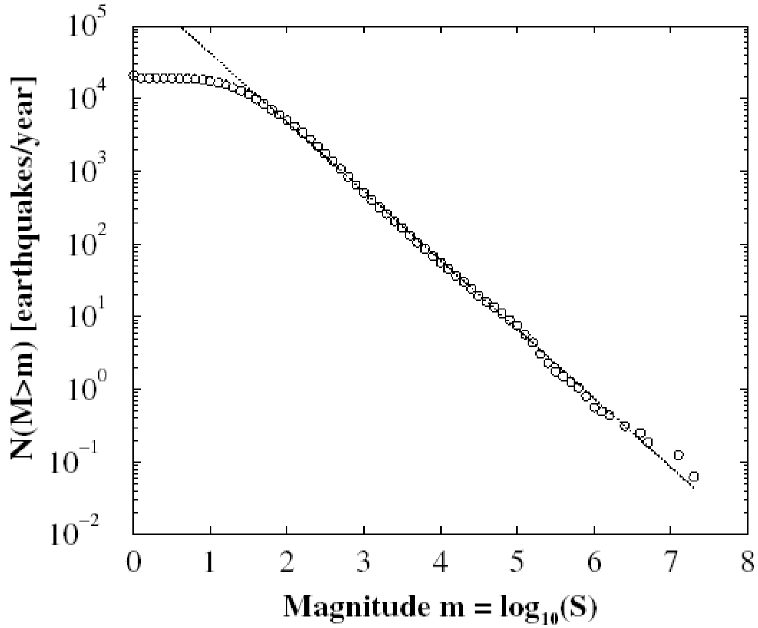


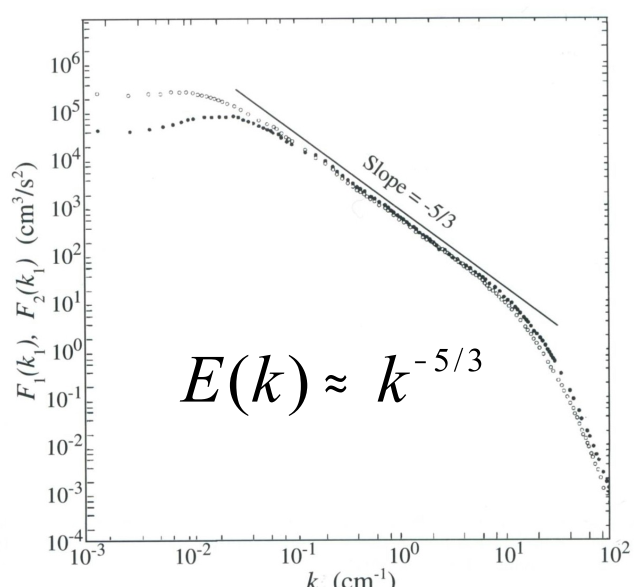
Here delta represents the "Prandtl number." This number, which one absolutely does not have to know the meaning of, is
the ratio of the fluid viscosity of a substance to its thermal conductivity (named after Ludwig Prandtl, a German physicist).
The value Lorenz used was 10.

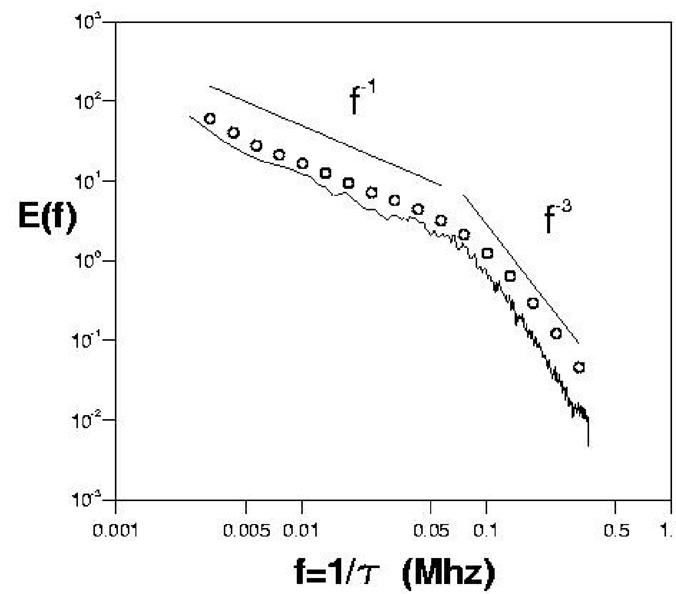
The variable r represents the difference in temperature between the top and bottom of the gaseous system. The value usually used in sample Lorenz attractors such as the one displayed here is 28.

The variable b is the width to height ratio of the box which is being used to hold the gas in the gaseous system.
Lorenz happened to choose 8/3, which is now the most common number used to draw the attractor.

The resultant x of the equation represents the rate of rotation of the cylinder, y represents the difference in temperature at opposite sides of the cylinder, and the variable z represents the deviation of the system from a linear, vertical graphed line representing temperature.

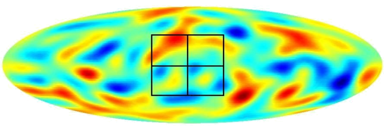

N(M>m) [earthquakes/year]
Magnitude m = log10(S)

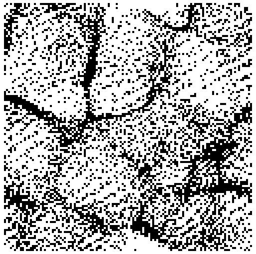

Slope = -5/3
$E(k) \approx k^{-5/3}$
$F_1(k_1)$, $F_2(k_1)$ (cm^3/s^2)
k_1 (cm^{-1})

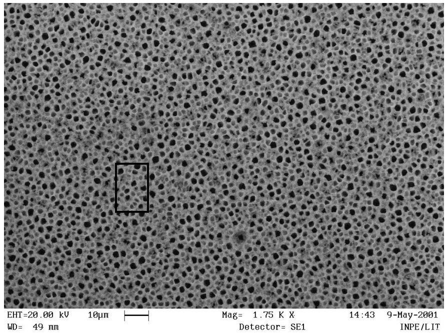

f^{-1}
f^{-3}
E(f)
$f=1/\tau$ (Mhz)

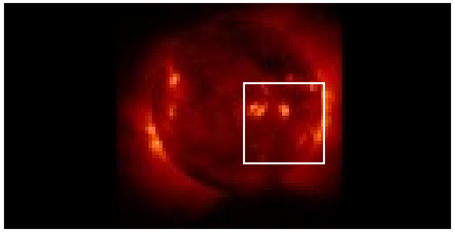
Computational Tools for Data Analysis:

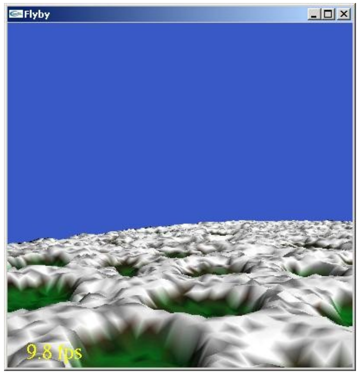
- **Pattern characterization for nanofabrication**
- **Pattern evolution in astrophysical plasmas**
- **Pattern classification of wave propagation in elastic manifolds (membrane vibration)**
- **Pattern recognition of temperature fluctuation and large scale structures in cosmology**

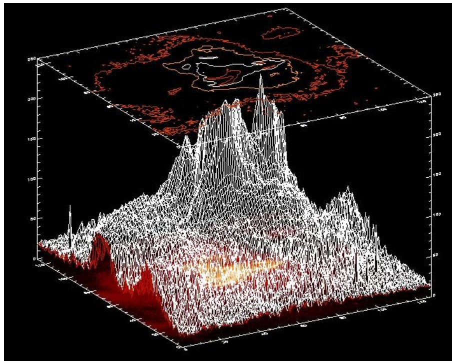


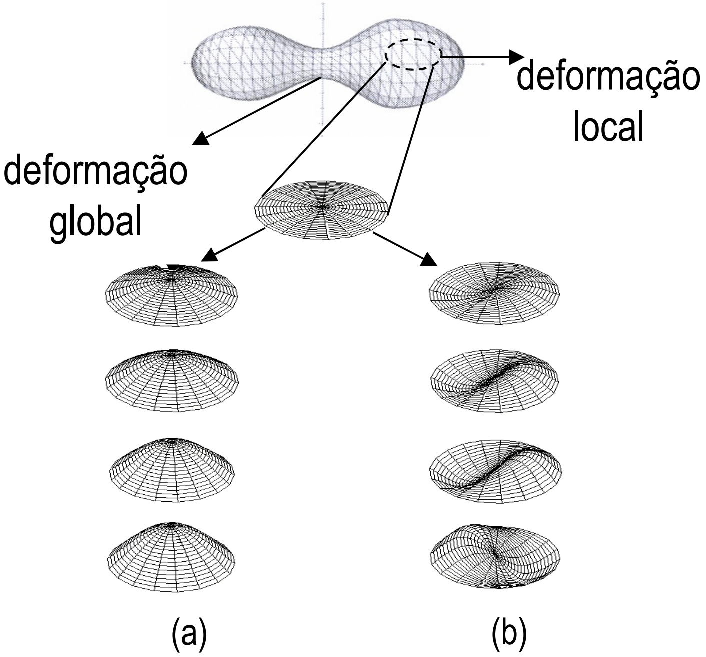




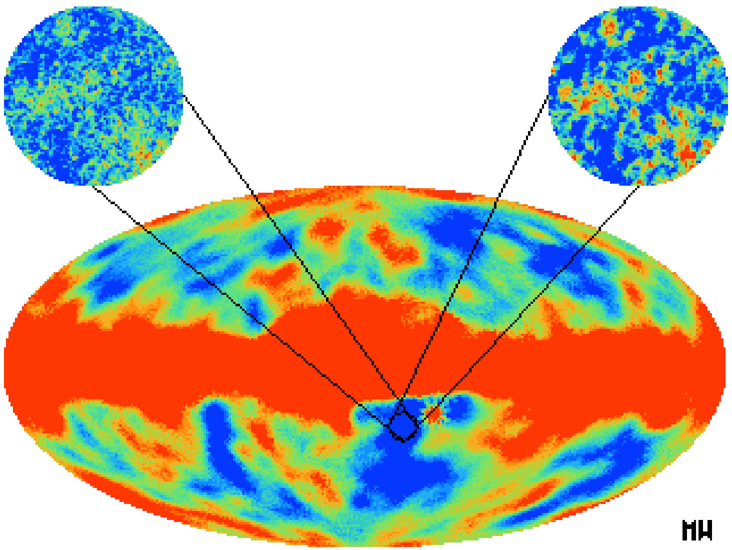


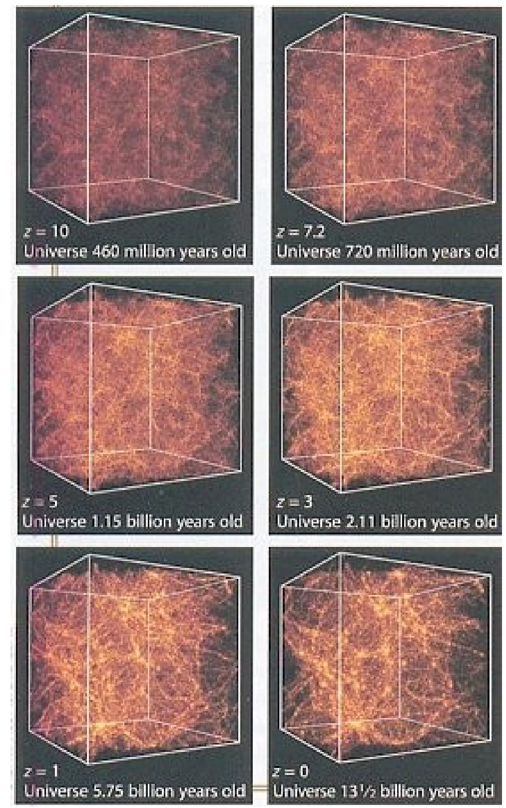


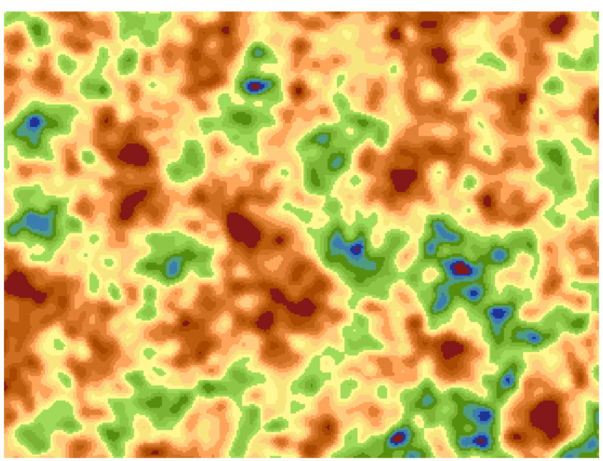


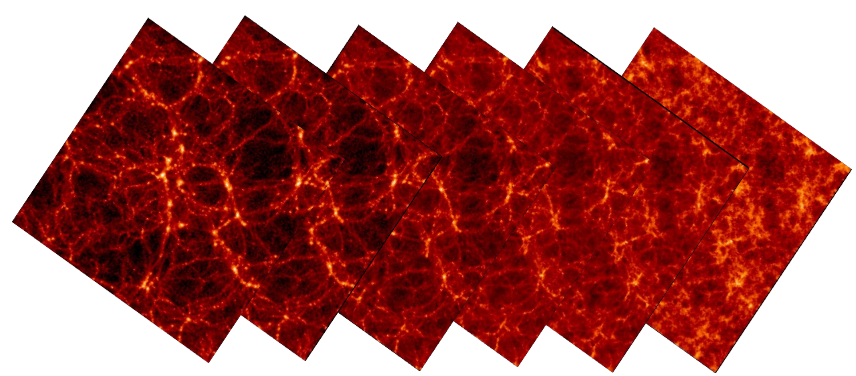


02 Projects: (1) Topological Signature ; (2) Turbulent-like model (poster)









Analysing Regular and Irregular Patterns

1. Searching for Regularities:
   2D FFT and 2D Wavelets Analysis

2. Searching for Irregularites:Gradient Pattern Analysis e MF

Advanced Techniques (2 & 3D)

- **Spatial Correlation Function and Statistical Exponents (universality classes)**

→ - **Aspect Ratio and Minkowski Functionals**

("integral geometry")

- **Geometric Dimensions**

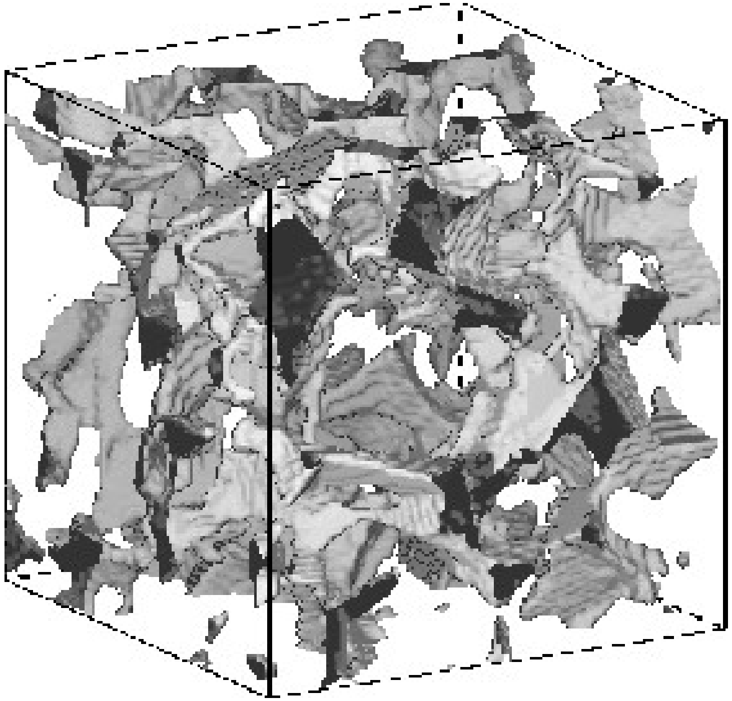
- **Disorder Function**

→ - **Gradient Pattern Analysis**

("convex geometry")

- **Structural Entropy**

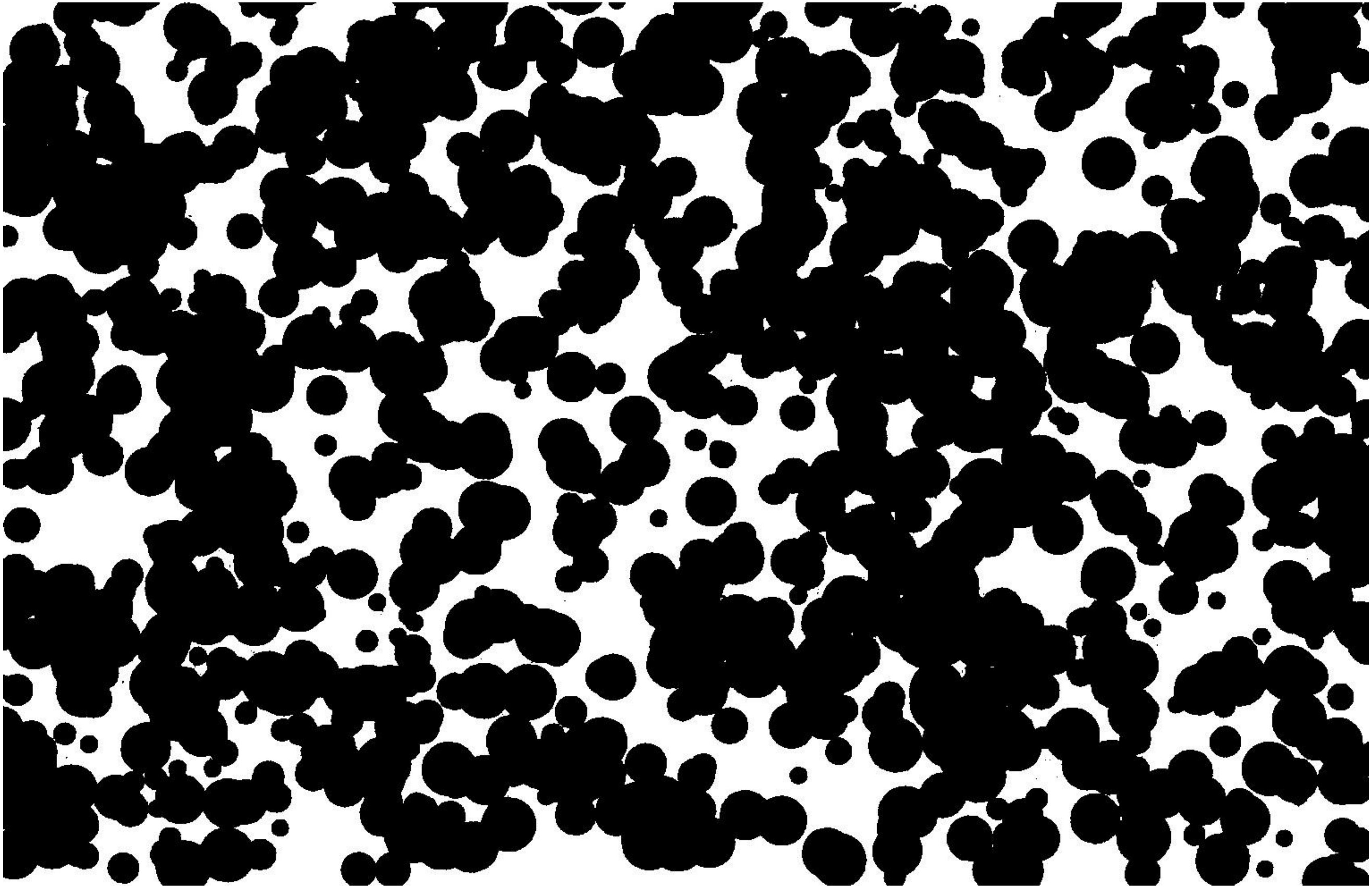
Integral Geometry in Physics: the shape of matter



3D-structure information in real space

Minkowski functionals

Boolean model: overlapping spheres



Minkowski functionals of black region ? From Prof. Mecke

Gaussian random fields: Minkowski functions

Minkowski functionals : $f_\nu(\rho) \sim \lambda^\nu H_{\nu-1} [\rho(x)]$

with the Hermite functions $H_{\nu-1}(x) = (-\partial/\partial x)^{\nu-1} (1/2\pi)\, e^{-1/2x}$

in two dimensions

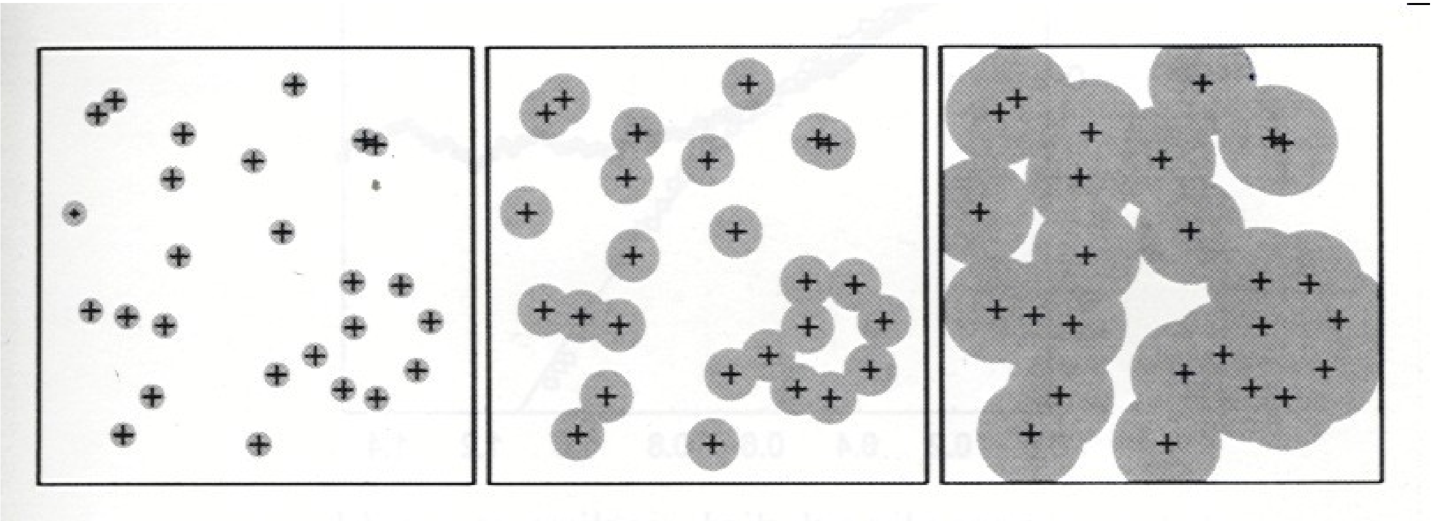
area $$F(\rho) = 1 - \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{\rho} dx\, e^{-\frac{1}{2}(x-\rho_0)^2/\sigma^2}$$

boundary $$U(\rho) = \sqrt{\frac{\pi}{2}}\lambda e^{-\frac{1}{2}(\rho-\rho_0)^2/\sigma^2}$$

curvature $$\chi(\rho) = \frac{1}{\sqrt{2\pi}}\lambda^2 \frac{\rho-\rho_0}{\sigma} e^{-\frac{1}{2}(\rho-\rho_0)^2/\sigma^2}$$

Surface (2D) into Volume (3D):

F(ρ) *U*(ρ) χ(ρ)

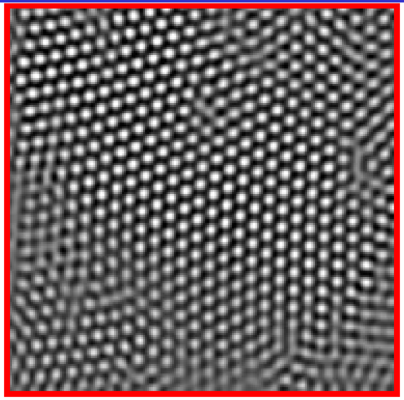


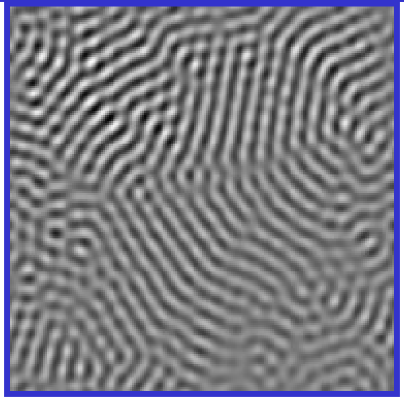
F(ρ) = *superfície de cobertura*

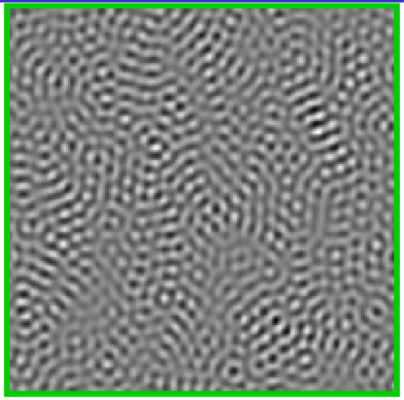
U (ρ)= *contorno*

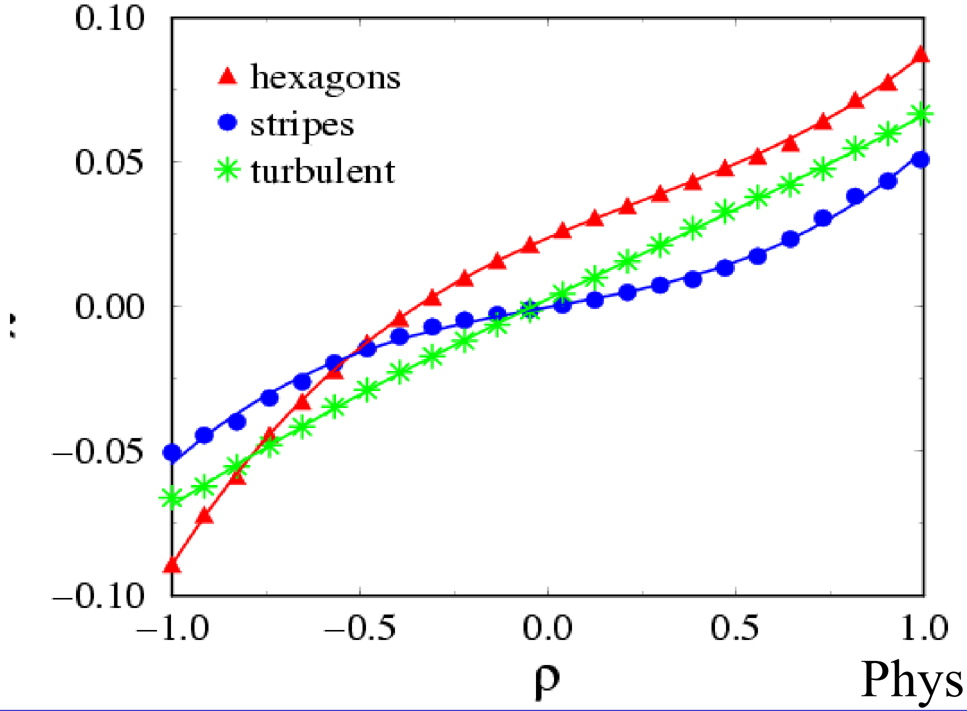
χ(ρ) = *característica de Euler (conectividade)*

Turing patterns: Euler characteristic





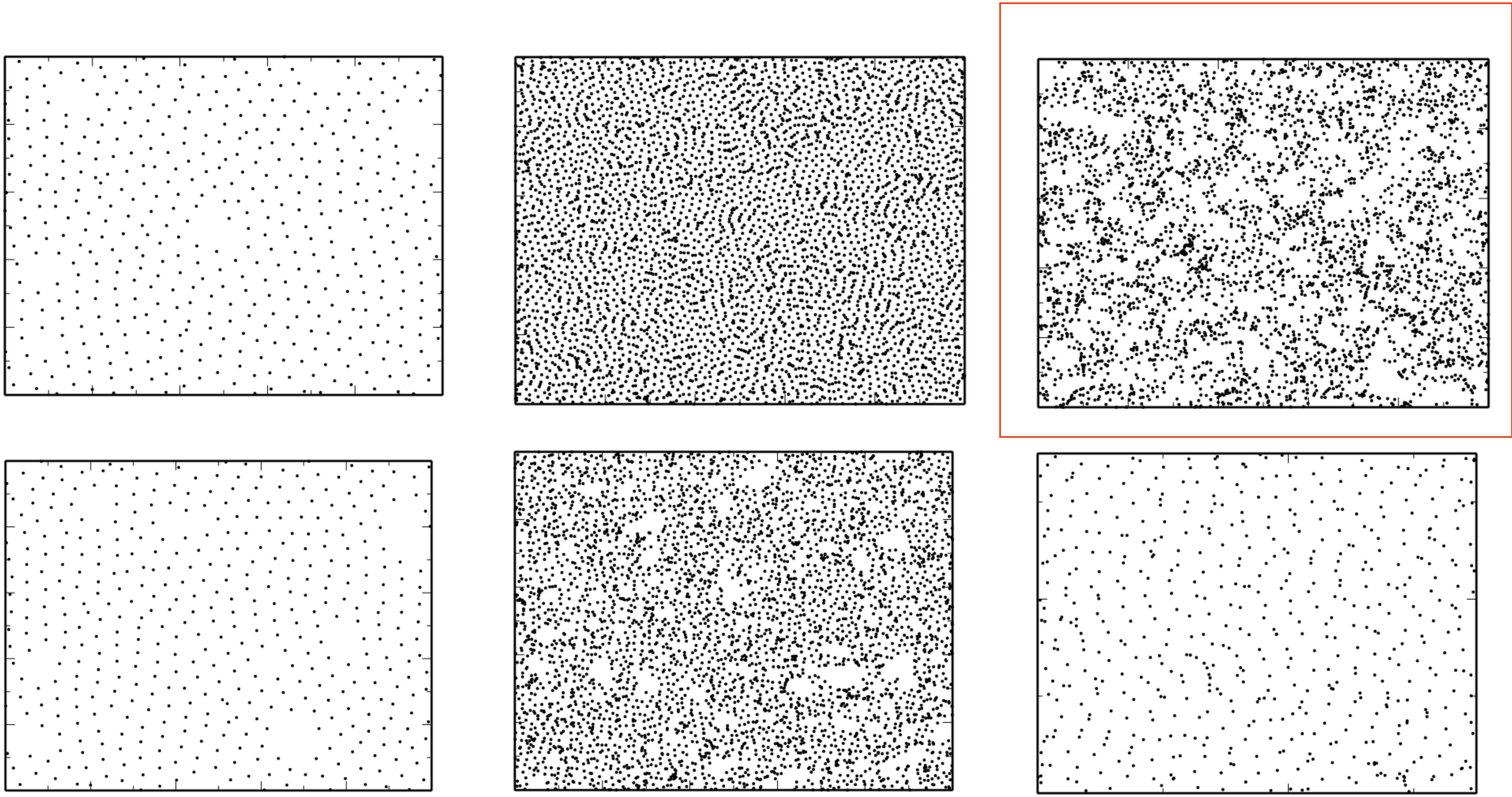




Phys. Rev. E 53,4794 (1996)

Minkowski functions are cubic polynoms in threshold ρ (From Prof. Mecke)

At this moment: simulating complex fluid configurations

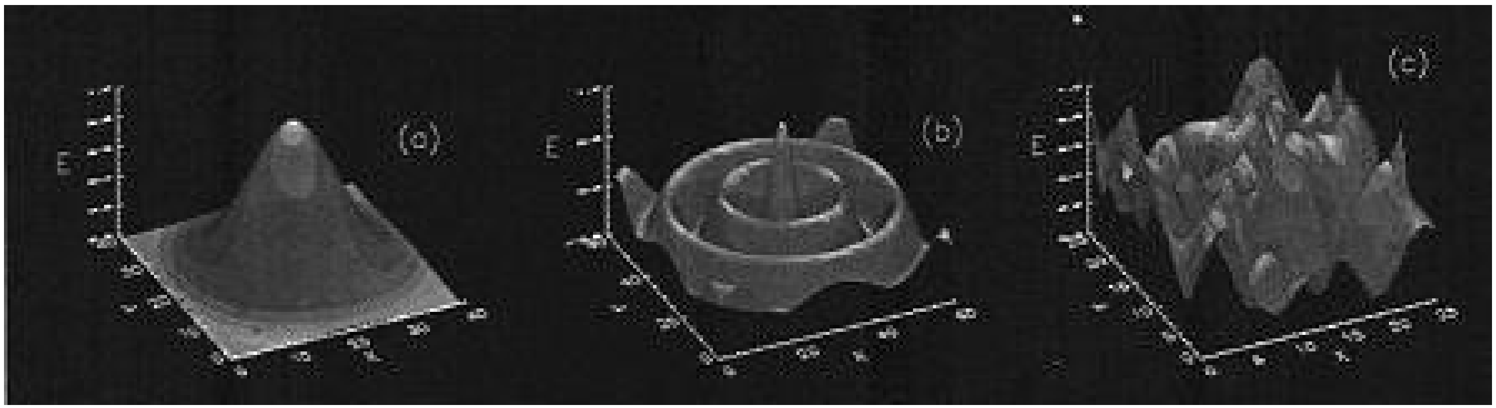


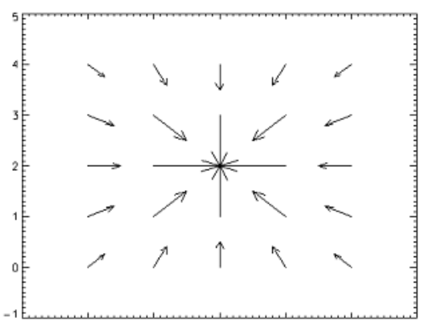
What about asymmetries?

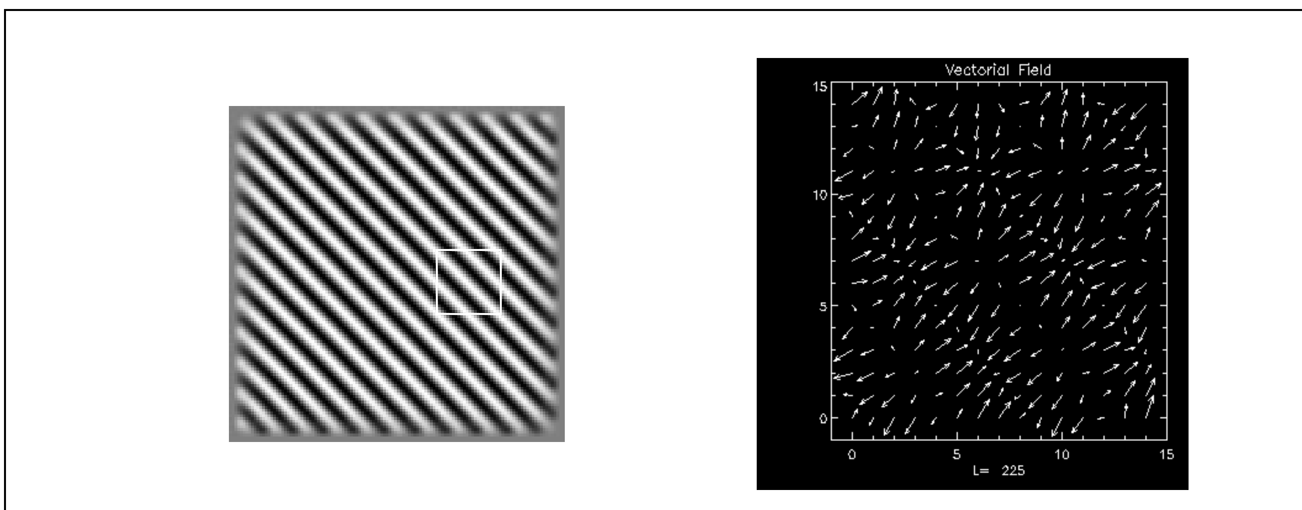
The Notion of Local & Global Fluctuations

4.56	4.67	4.59	5.01	5.03	6.98	6.98	6.87	5.67	6.51	5.00
4.00	4.04	4.09	4.34	4.56	4.34	4.52	5.56	5.78	4.56	5.67
3.88	3.45	3.56	6.45	6.34	6.89	5.09	5.56	5.77	5.67	5.68
3.34	3.56	4.54	4.00	4.04	5.08	5.20	6.34	5.34	4.56	4.59
5.56	4.56	4.59	5.11	5.23	5.67	5.23	6.02	8.79	8.55	9.00
7.89	8.09	7.29	8.08	7.34	6.78	6.79	9.08	0.08	0.06	0.89
9.02	9.08	9.08	9.04	9.01	8.79	8.88	8.88	0.09	0.06	1.24
9.08	9.06	9.78	9.89	9.56	9.05	9.08	9.06	1.23	1.34	0.09
4.56	4.67	4.59	5.01	5.03	6.98	6.98	6.87	1.22	1.45	0.09
4.00	4.04	4.09	4.34	4.56	4.34	4.52	5.56	1.34	1.56	1.01
3.68	3.45	3.16	6.45	5.34	6.19	5.09	5.56	1.56	1.56	1.55
5.14	3.56	3.14	6.00	4.04	5.08	7.22	9.34	4.56	4.01	4.45
5.56	4.56	4.59	2.11	5.23	5.67	9.23	9.00	5.67	5.01	6.09
5.98	4.03	3.09	3.01	7.32	6.78	5.69	9.01	8.00	7.78	6.00

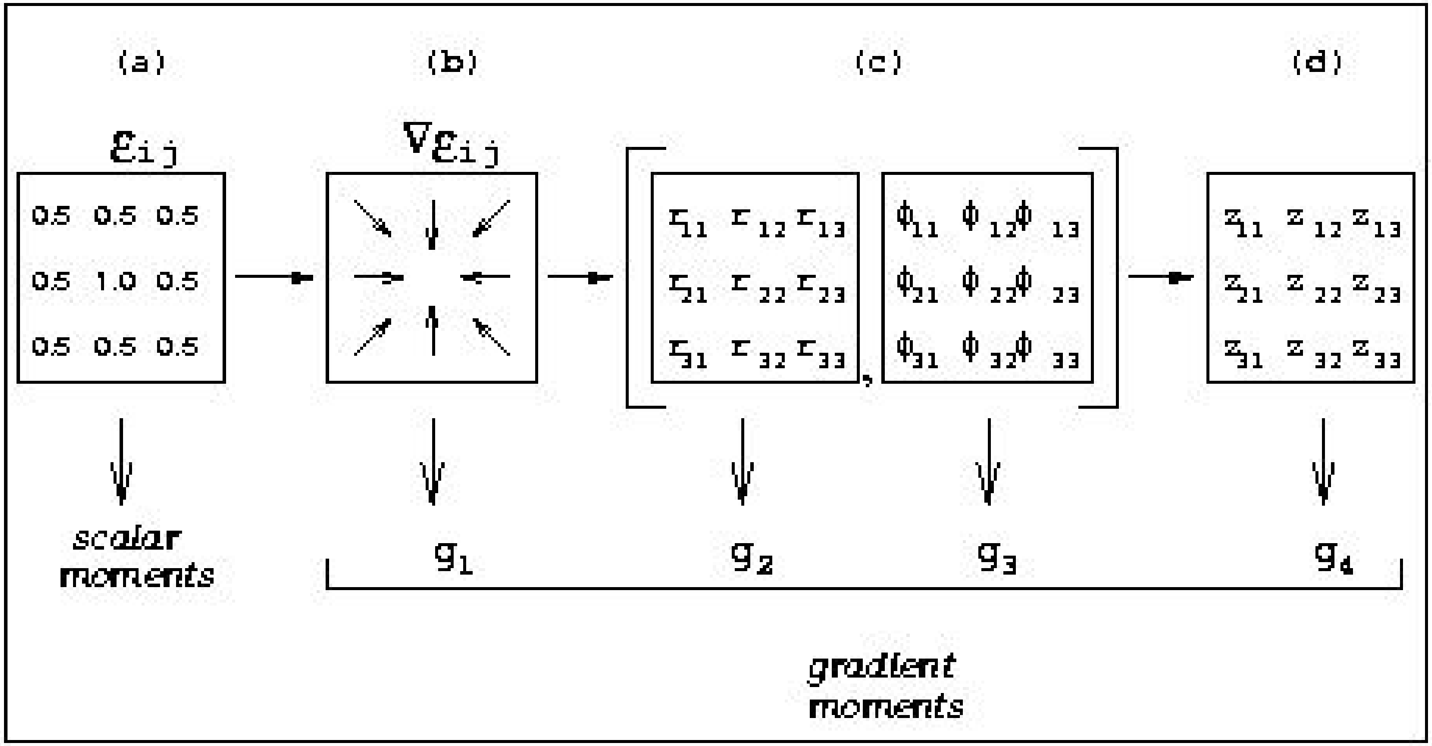
Symmetry Breaking in the Gradient Field of the Amplitudes ➔ GPA







Gradient Pattern Analysis



G_t $= \nabla[E(x,y,t]_t$ is represented by 4 n.s. **gradients *moments***:

$g_1^{(t)} \equiv h_1\,(\{(r_{1,1}, \phi_{11}) \dots , (r_{ij}, \phi_{ij}) \dots , (r_{kk}\, \phi_{kk})\}_t)$
$g_2^{(t)} \equiv h_2\,(\{(r_{11}), \dots , (r_{ij}), \dots , (r_{kk})\}_t)$
$g_3^{(t)} \equiv h_3\,(\{(\phi_{11}), \dots , (\phi_{ij}), \dots , (\phi_{kk})\}_t)$
$g_4^{(t)} \equiv h_4\,(\{(z_{11}) \dots , (z_{ij}) \dots , (z_{kk})\}_t)$

As {r} and {φ} are compact groups, spatially distributed, they can be geometrically constructed as *Haar-like* measures ==> rotational and amplitude translation invariant

How to compute g_1, g_2, g_3 e g_4 ?

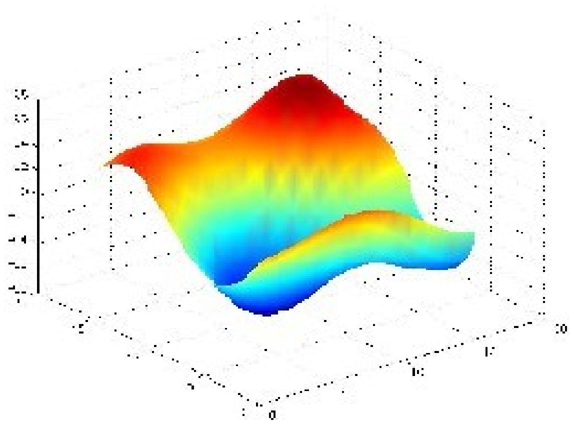
$g_1 \equiv (C - V_A)/V_A$, $C > V_A$

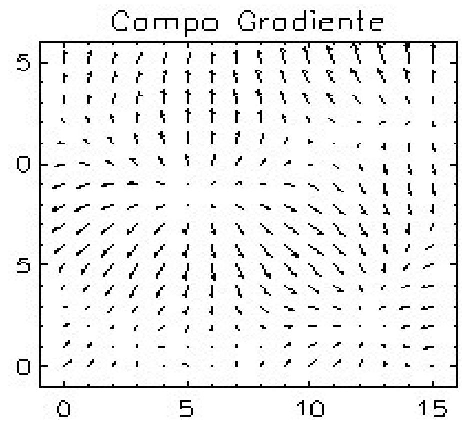
V_A = amount of asymmetric vectors, $(v1 + v2 \neq 0)$

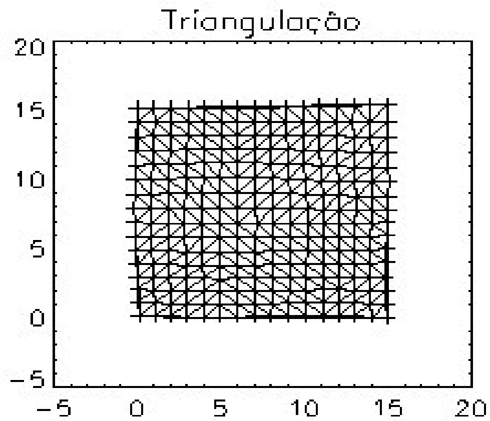
C = amount of geometric correlation lines (Delaunay triangulation $T_D(C,V_A)$)

(*Asymmetric Amplitude Fragmentation* - AAF, Rosa et al.,

Int. J. Mod. Phys. C, 10(1)(1999):147.)







$$g_2 \equiv \sum (r_{ij} - r_{mn})^2 / N \; ;$$

$$g_3 \equiv \sum (\Phi_{ij} - \Phi_{mn})^2 / N$$

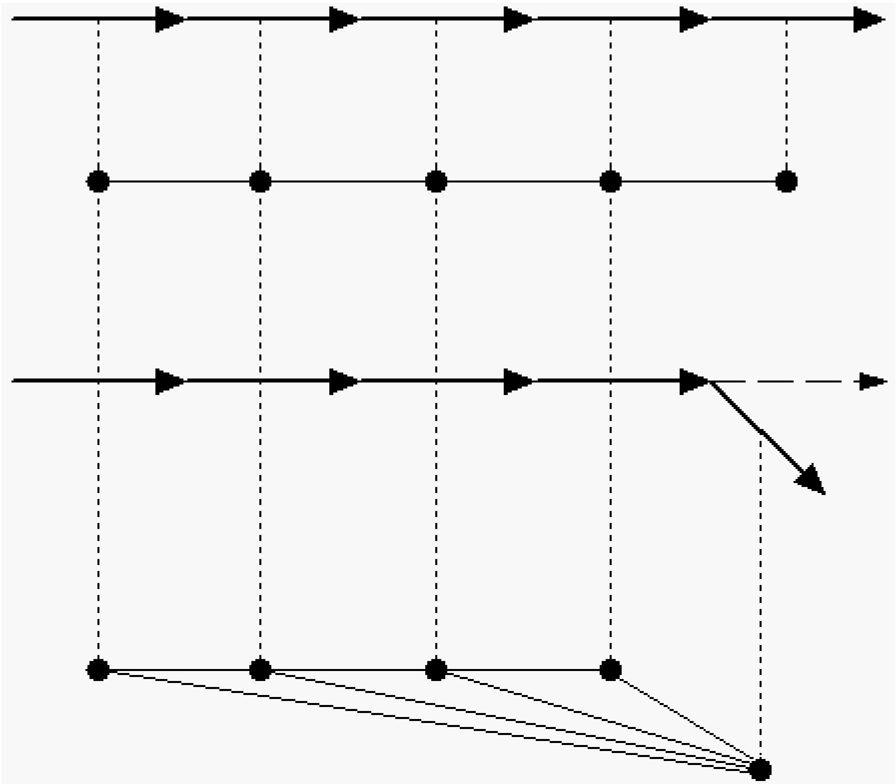
$$g_4: \quad S_z = - \sum z_{i,j}/z \ln (z_{i,j}/z) = \mathrm{Re}(S_z)\, e^{iSz}$$

$$|g_4| = \mathrm{Re}(S_z) = S(|z|) \text{ and } \Phi(g_4) = \mathrm{Im}(S_z)$$

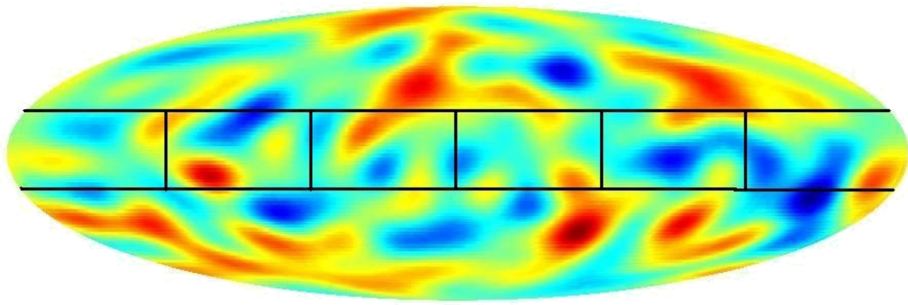
Thus, $|g_4|$ and $\Phi(g_4)$ are invariant measurements of norm and phase of the gradient entropy

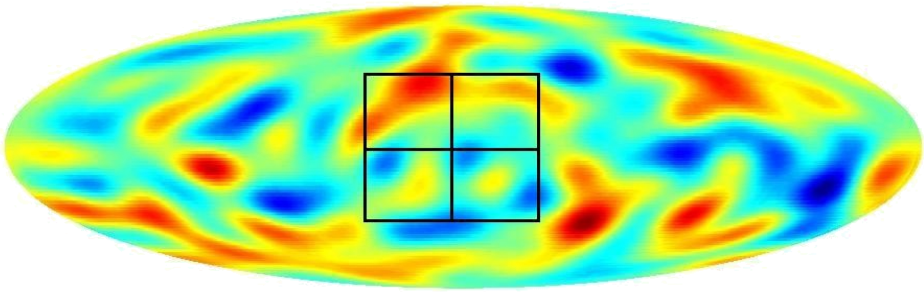
(*Complex Entropic Form* (CEF) by Ramos et al. Physica A283(2000):171.)

GPA: Rosa et al. Braz. J. of Physics, 2003



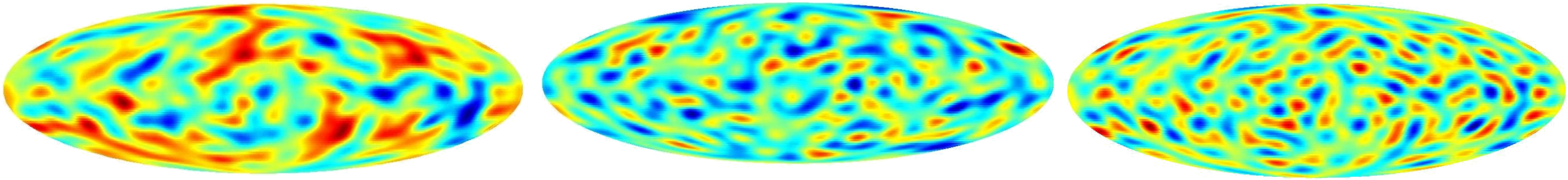
GPA of Temperature Fluctuations on a Harisson-Zeldovich Sphere





Modelling Irregular Amplitude Fluctuations on a Spherical Surface

$$\frac{\delta A}{A}(\theta,\varphi) = \frac{A(\theta,\varphi) - A_0}{A_0}$$

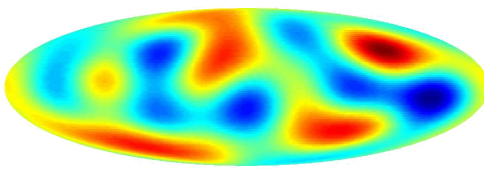
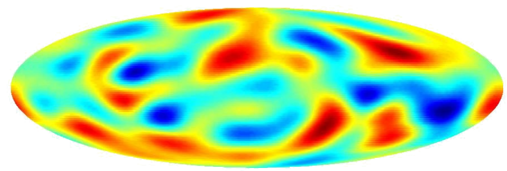
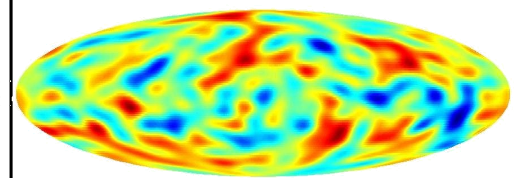
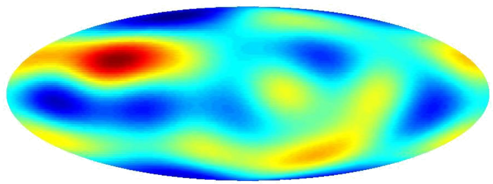
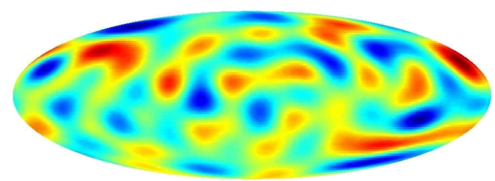
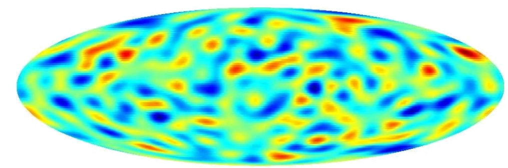
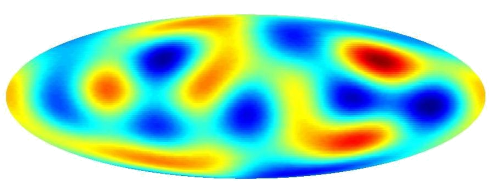
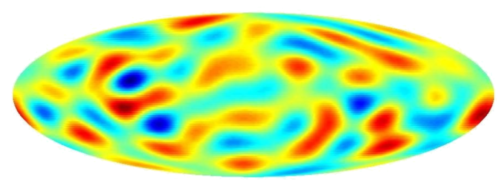
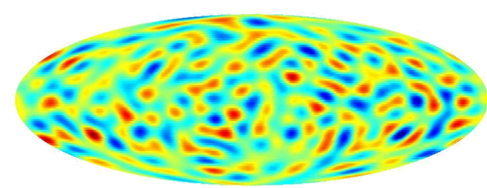


$$\frac{\delta T}{T}(\theta,\varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta,\varphi)$$

- Plane wave decomposition in spherical harmonics:

$$e^{i\vec{k}.\vec{r}} = 4\pi \sum_{lm} i^l jl(kr)\, Y^*_{lm}(\theta_{\vec{k}},\varphi_{\vec{k}})\, Y_{lm}(\theta,\varphi)$$

Irregular Fluctuations:

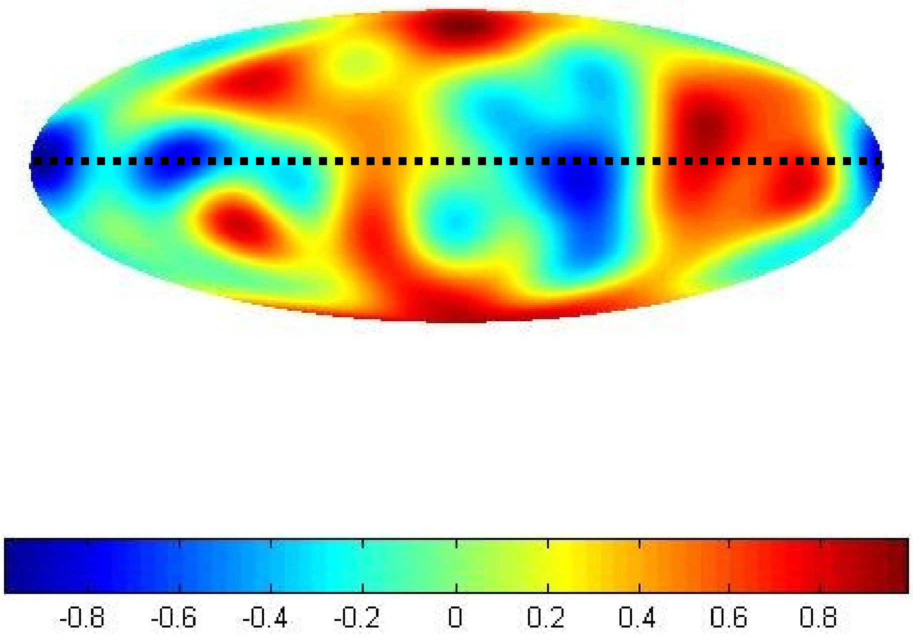
	$L_{max}=5$	$L_{max}=10$	$L_{max}=20$
$C_l=1/l(l+1)$			
$C_l=1$			
$C_l=l(l+1)$			

Asymmetric Aspect Ratio (AAR)

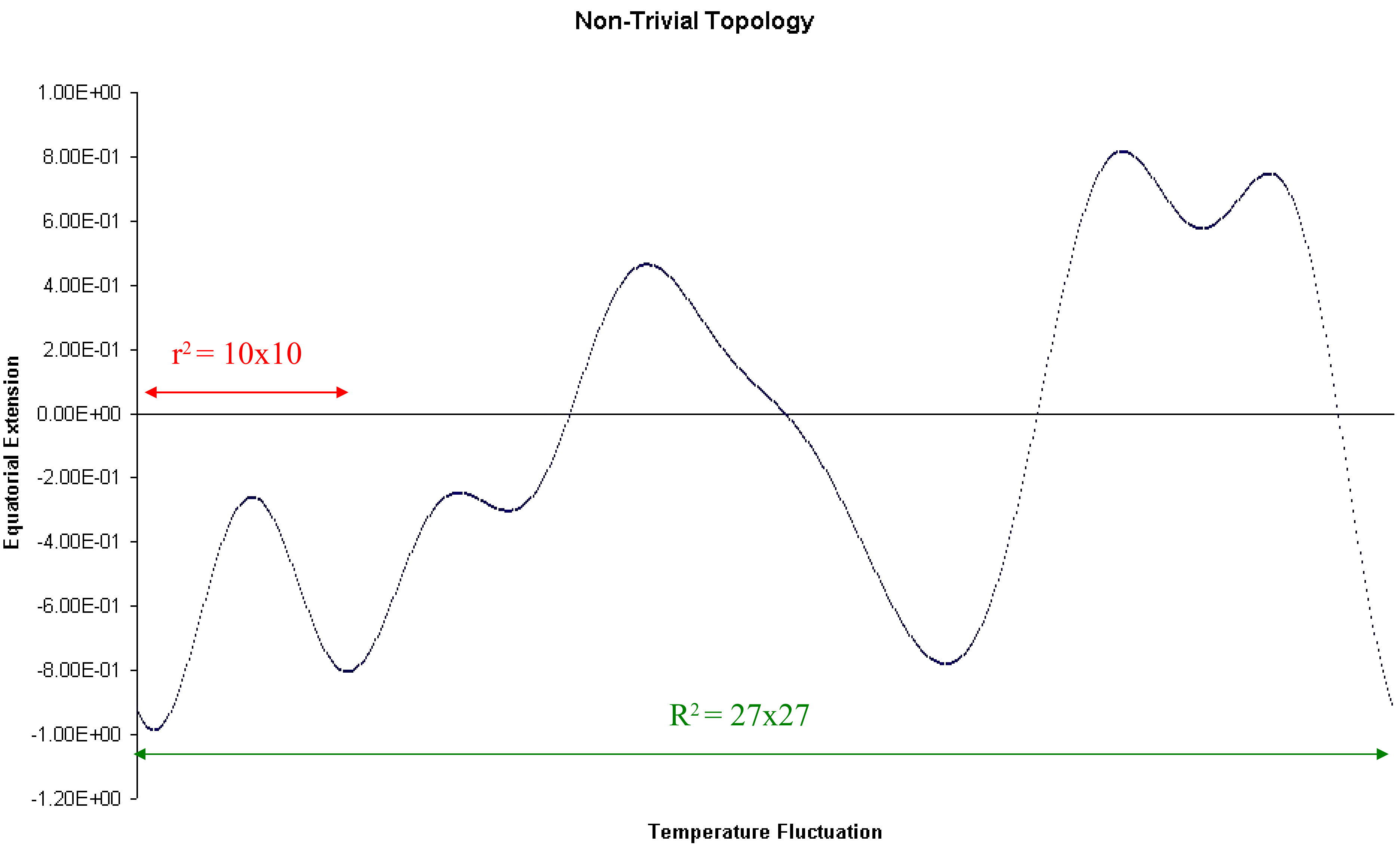
- In our approach we take from each CMB map a relation between the maximum variation of asymmetries for two characteristic scales: the global scale *R* where the major matrix (*R* x *R*) is obtained and the first local scale *r* for that the values of g_a are maximally different when different random seeds are considered in the simulation.

- Therefore, when we compare two different *Asymmetric Aspect Ratio* (AAR) where each one is coming from a different topology (usually, a nontrivial one -NT- in comparison to the trivial standard flat topology-T), we can to conjecture the existence of a parameter **γ** whose values, when different than unity, can distinguish (or classify) different classes of nontrivial topologies:

$$\frac{\Delta g^{a}_{T,R}}{\Delta g^{a}_{T,r}} \approx \gamma \left(\frac{\Delta g^{a}_{NT,R}}{\Delta g^{a}_{NT,r}} \right)$$

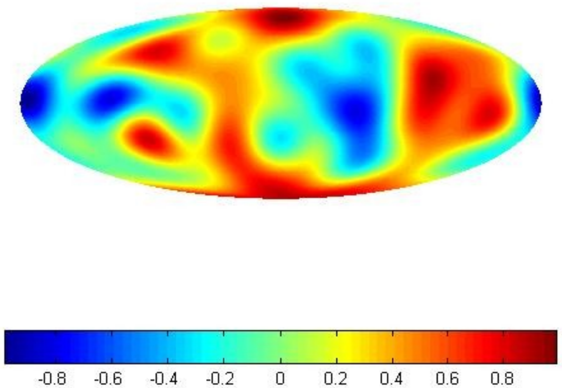


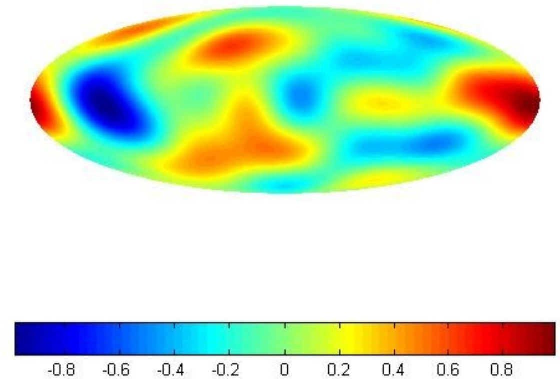
The equatorial fluctuation pixel profiles, at $f_\theta = 0$, as a function of the longitude sampled in 768 pixels (R^2 =27x27) (CT1)

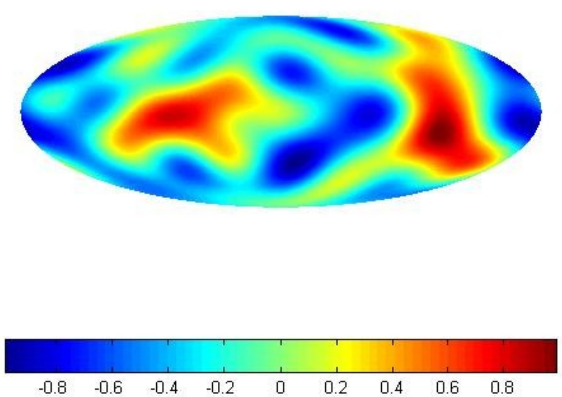


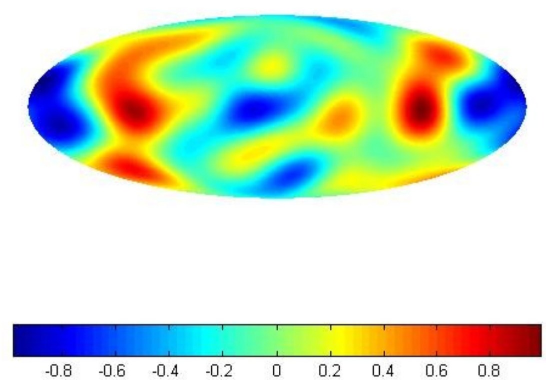
Non-Trivial Topology
1.00E+00
8.00E-01
6.00E-01
4.00E-01
2.00E-01
0.00E+00
-2.00E-01
-4.00E-01
-6.00E-01
-8.00E-01
-1.00E+00
-1.20E+00
Equatorial Extension
Temperature Fluctuation
r2 = 10x10
R2 = 27x27

TC

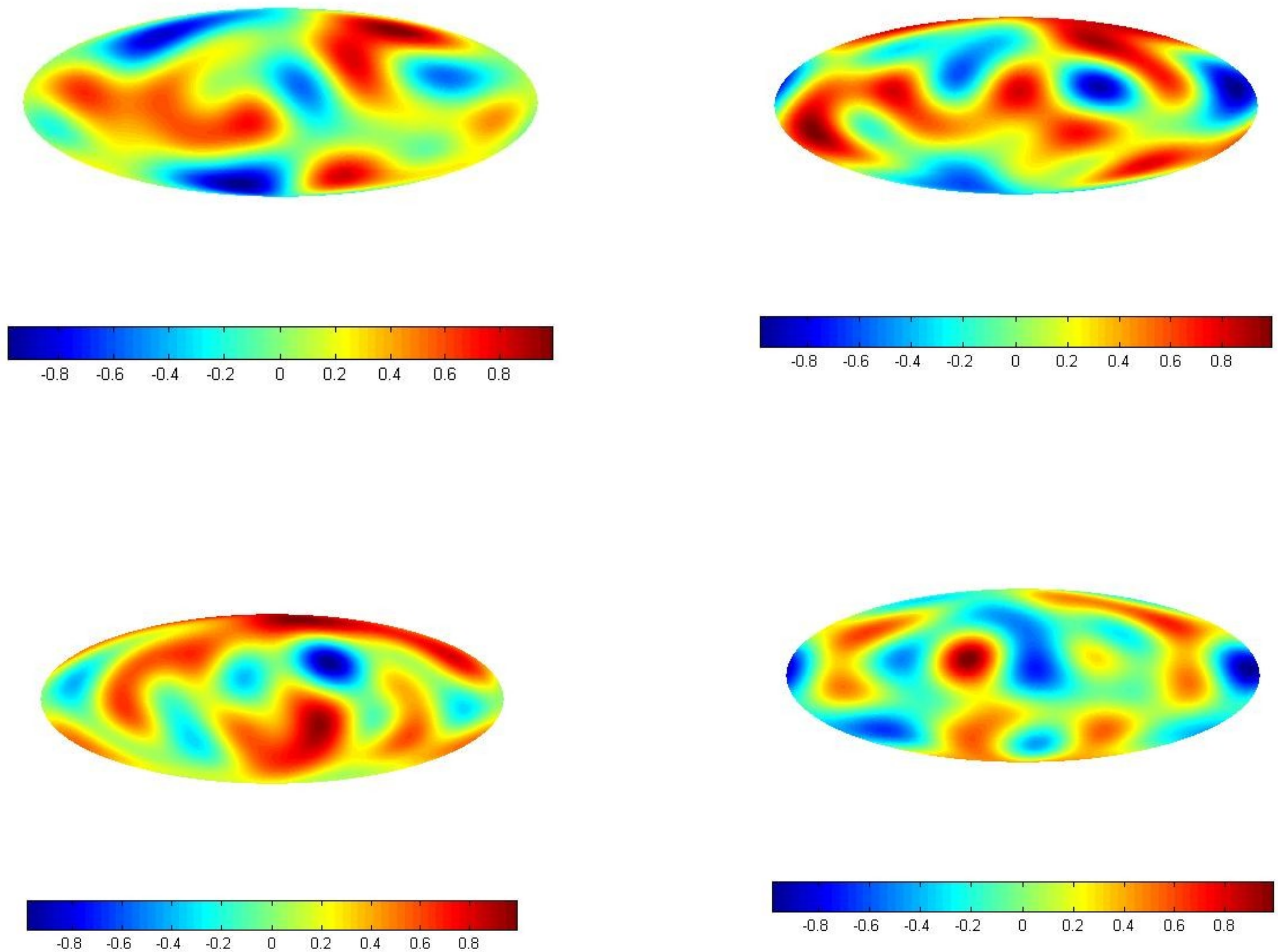

-0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8

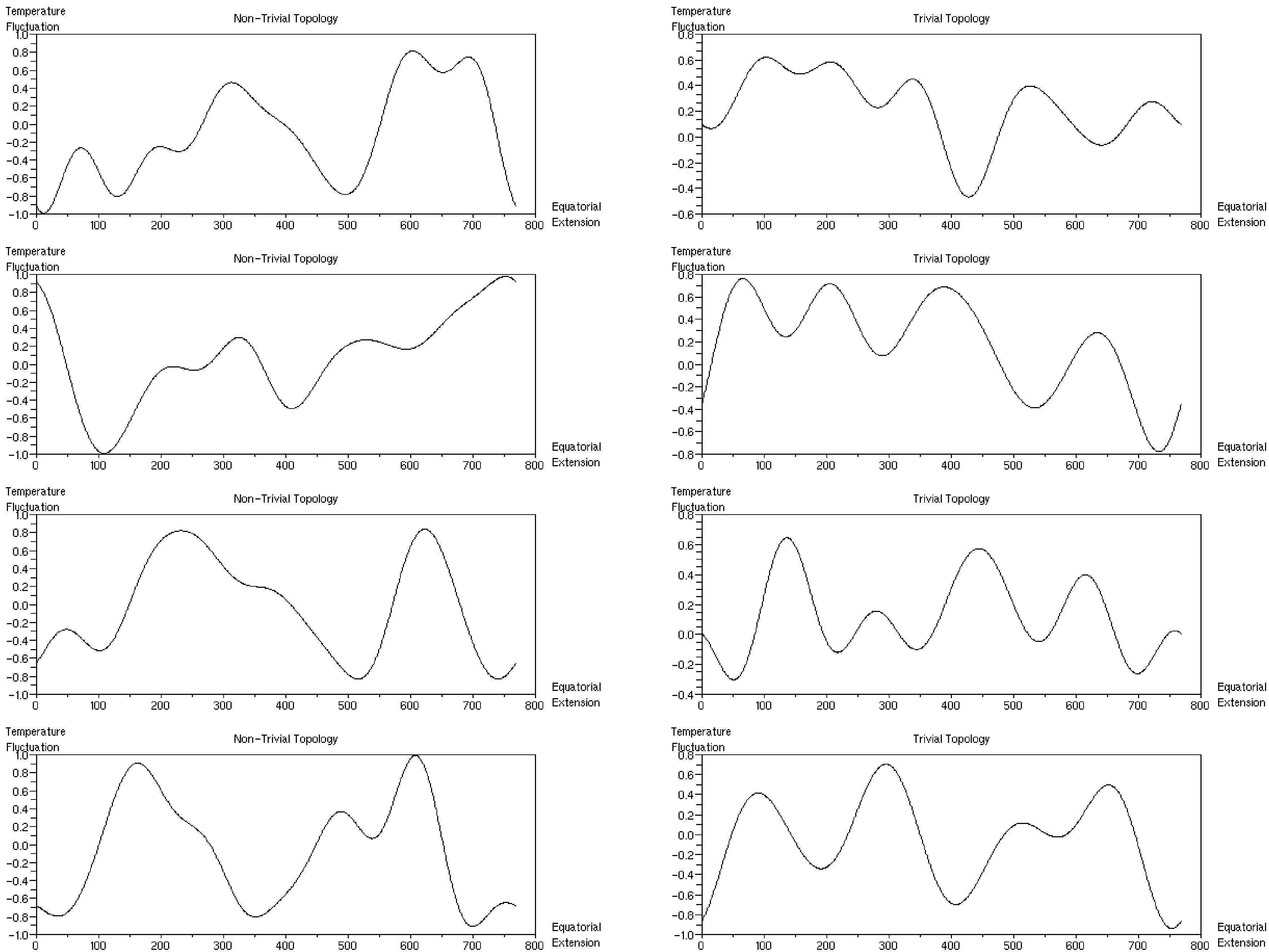

-0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8


-0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8


-0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8

TT


-0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8
-0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8
-0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8
-0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8


Temperature Fluctuation
Non-Trivial Topology
Equatorial Extension
Temperature Fluctuation
Trivial Topology
Equatorial Extension
Temperature Fluctuation
Non-Trivial Topology
Equatorial Extension
Temperature Fluctuation
Trivial Topology
Equatorial Extension
Temperature Fluctuation
Non-Trivial Topology
Equatorial Extension
Temperature Fluctuation
Trivial Topology
Equatorial Extension
Temperature Fluctuation
Non-Trivial Topology
Equatorial Extension
Temperature Fluctuation
Trivial Topology
Equatorial Extension

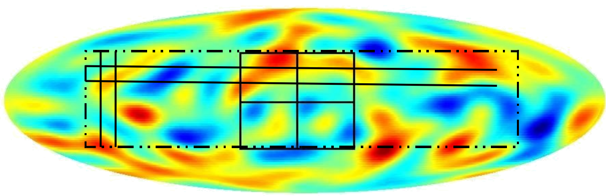
Resultados e Conclusões Preliminares

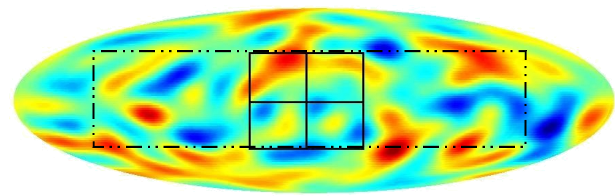
TC		TT	
g_a (729)	g_a (100)	g_a (729)	g_a (100)
1.96	1.79	1.94	1.80
1.96	1.81	1.95	1.81
1.98	1.87	1.92	1.80
1.95	1.74	1.99	1.87

$$\frac{\Delta g^{a}_{C,27}}{\Delta g^{a}_{C,10}} \approx 4\left(\frac{\Delta g^{a}_{T,27}}{\Delta g^{a}_{T,10}}\right)$$

Trivial

Torus





Gradient Classes Cn(ΛCDM)

Gradient Classes Cn(Torus)

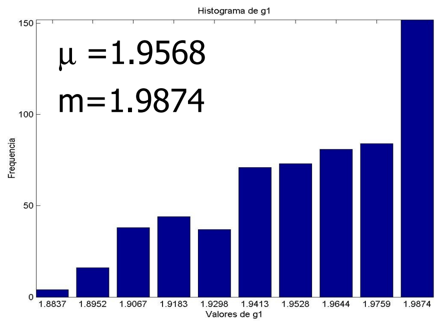
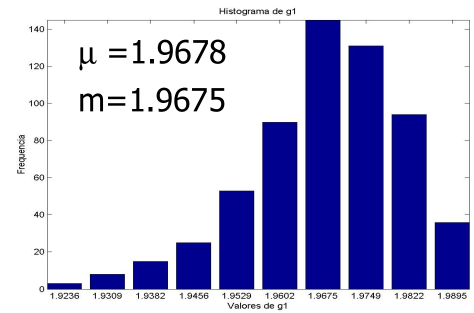
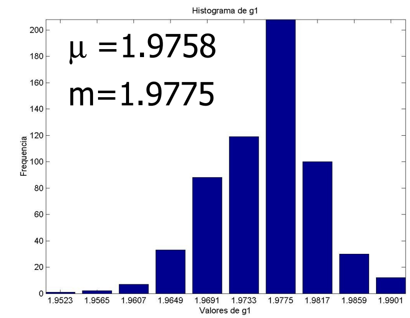
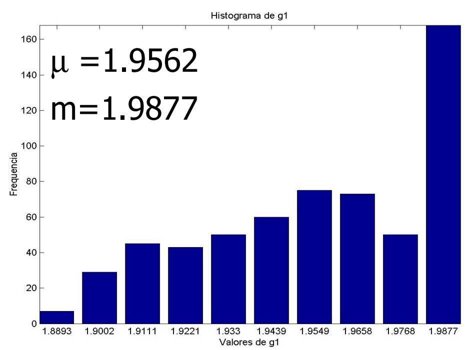
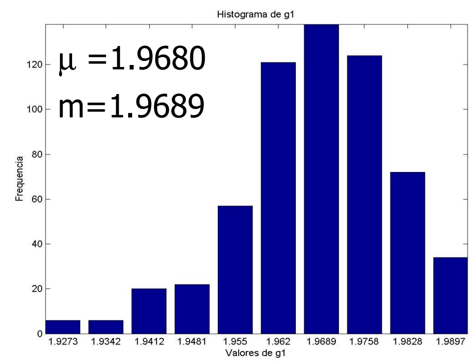
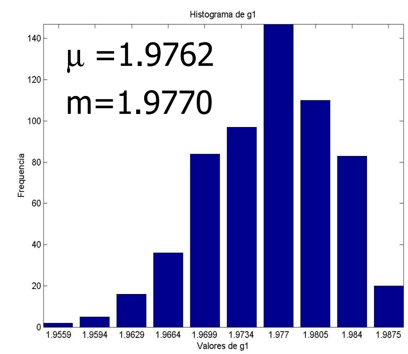
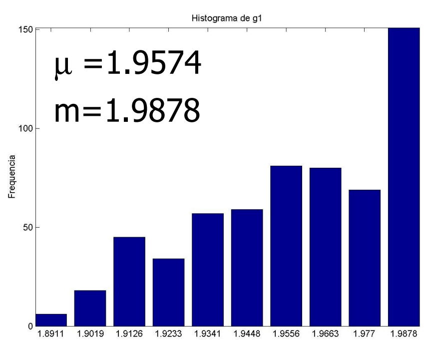
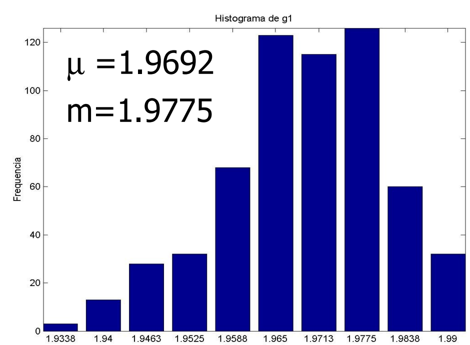
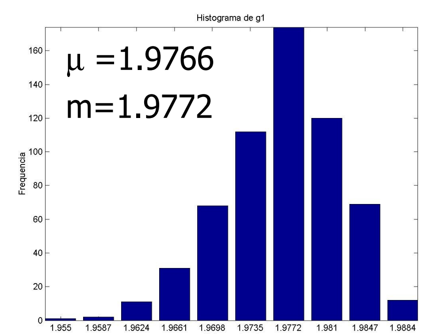
≈ ?

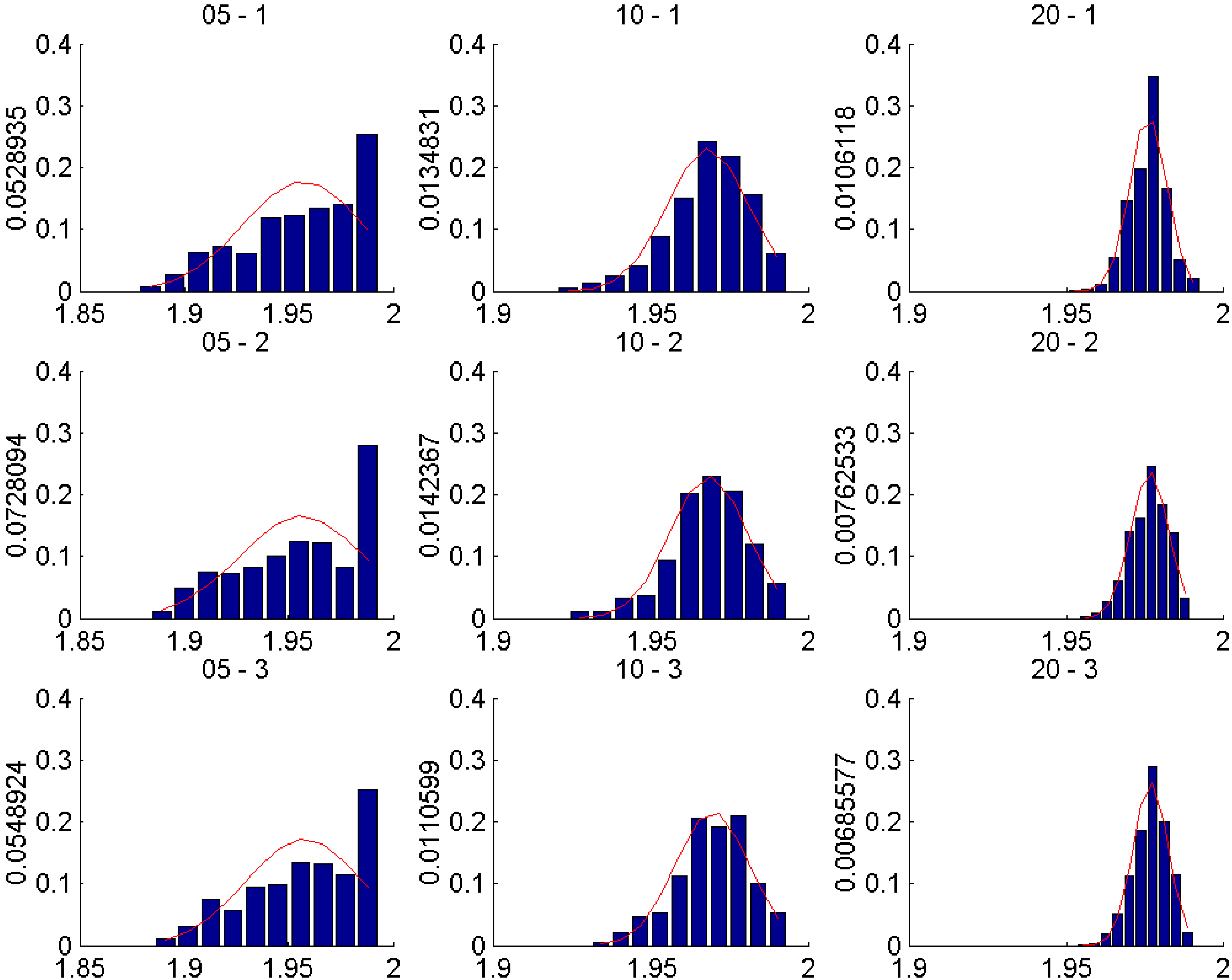
≈?

≈?

REAL DATA ==> Gradient Classes Cn(ex. WMAP)

Results from the GPA of 900 simulations:

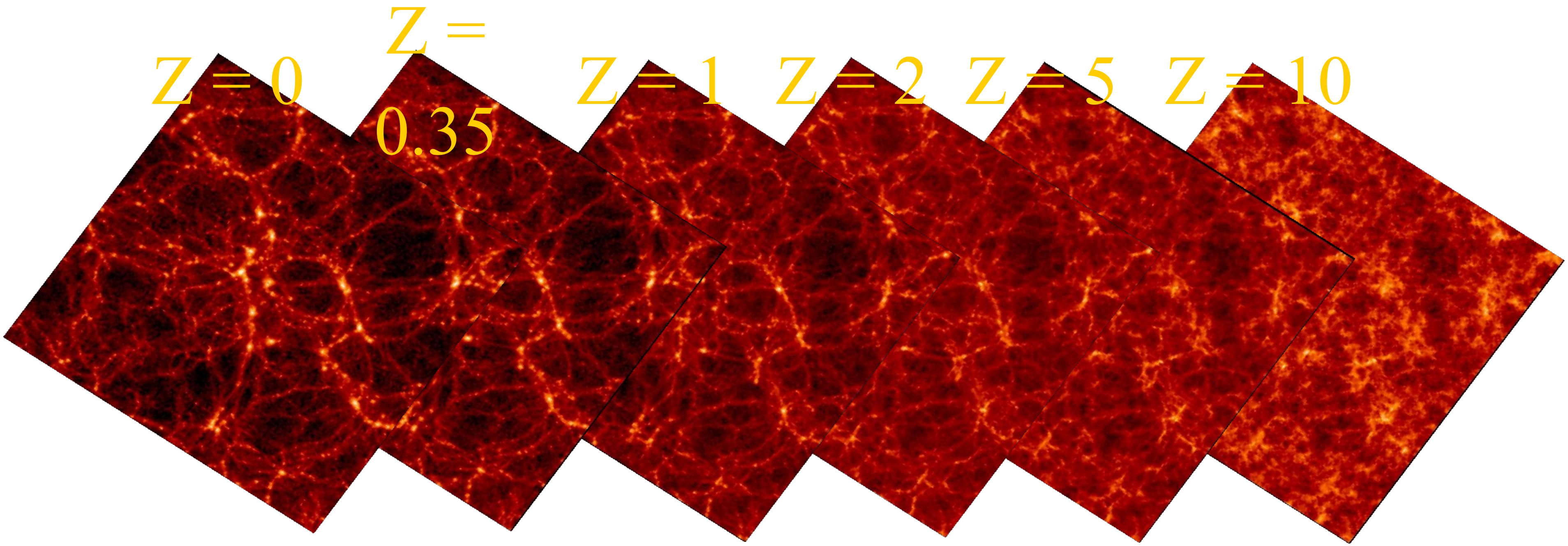
	$L_{max}=5$	$L_{max}=10$	$L_{max}=20$
$C_l=1/l(l+1)$			
$C_l=1$			
$C_l=l(l+1)$			

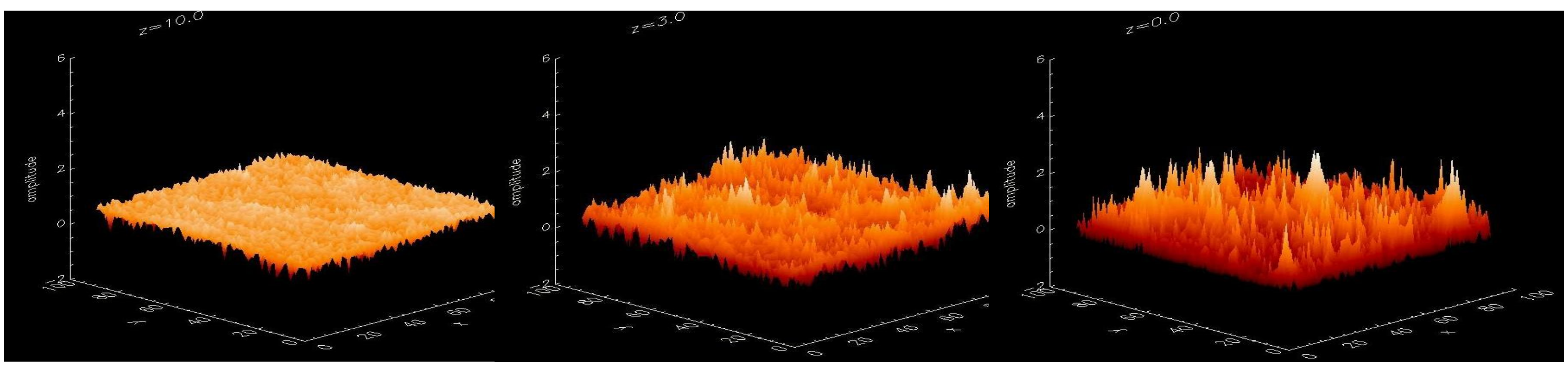

05 - 1
0.0528935
10 - 1
0.0134831
20 - 1
0.0106118
05 - 2
0.0728094
10 - 2
0.0142367
20 - 2
0.00762533
05 - 3
0.0548924
10 - 3
0.0110599
20 - 3
0.00685577

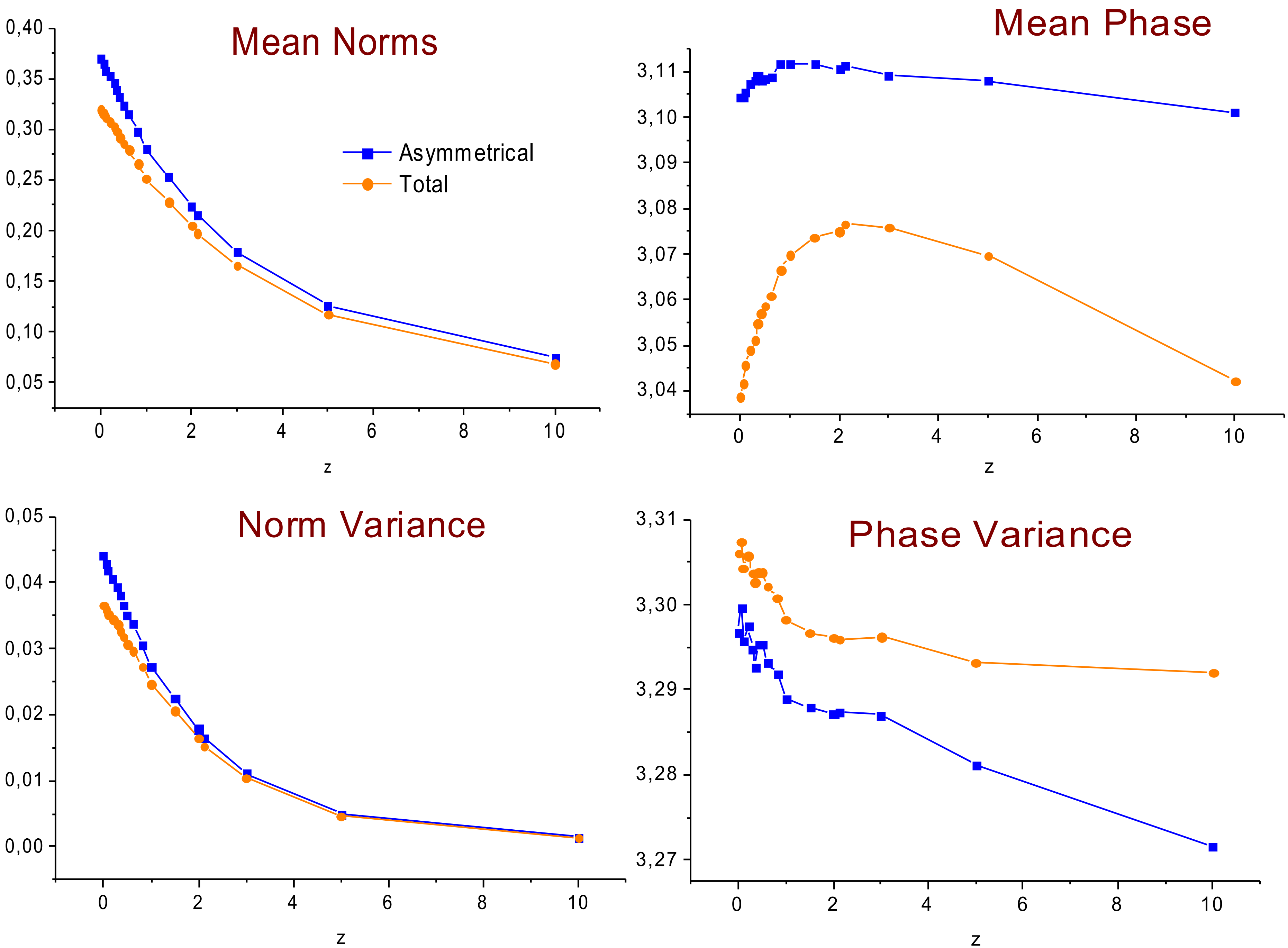
Resultados

	Lmax=5	Lmax=10	Lmax=20
C1	0.053	0.014	0.011
C2	0.073	0.014	0.008
C3	0.056	0.011	0.007

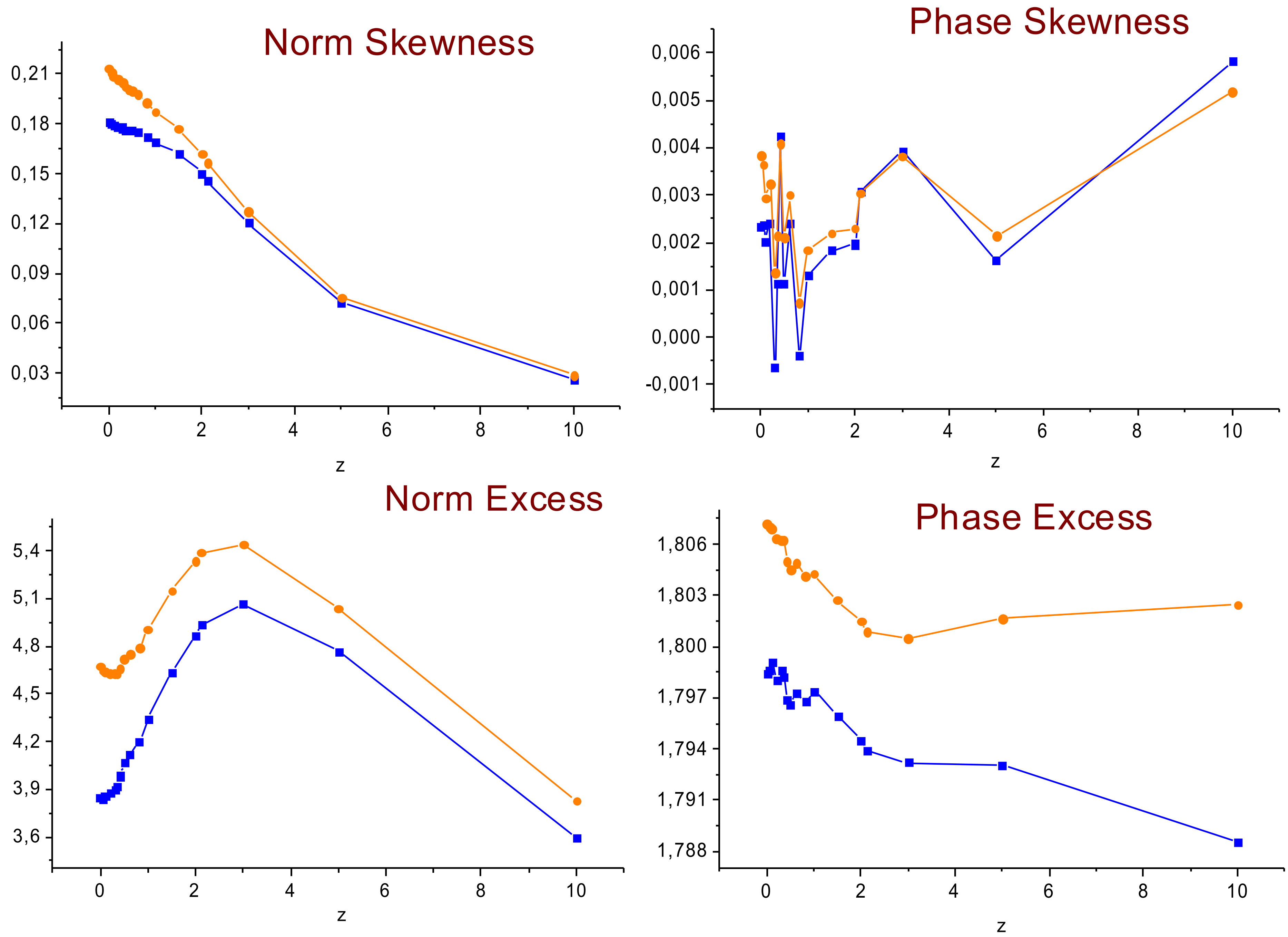
GRADIENT PATTERN ANALISYS IN STRUCTURE FORMATION



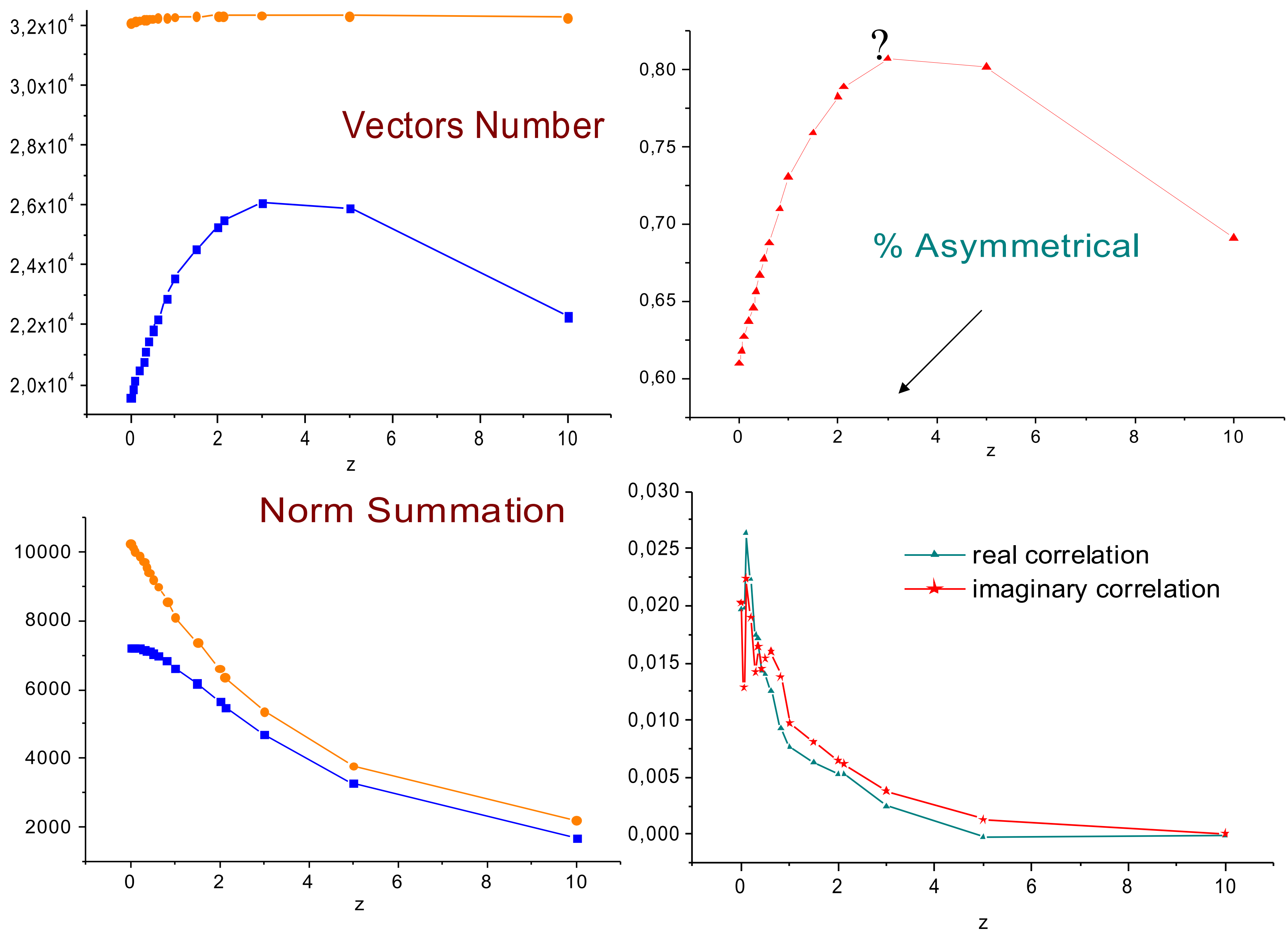




Mean Norms
Asymmetrical
Total
0,40
0,35
0,30
0,25
0,20
0,15
0,10
0,05
0
2
4
6
8
10
z
Mean Phase
3,11
3,10
3,09
3,08
3,07
3,06
3,05
3,04
0
2
4
6
8
10
z
Norm Variance
0,05
0,04
0,03
0,02
0,01
0,00
0
2
4
6
8
10
z
Phase Variance
3,31
3,30
3,29
3,28
3,27
0
2
4
6
8
10
z

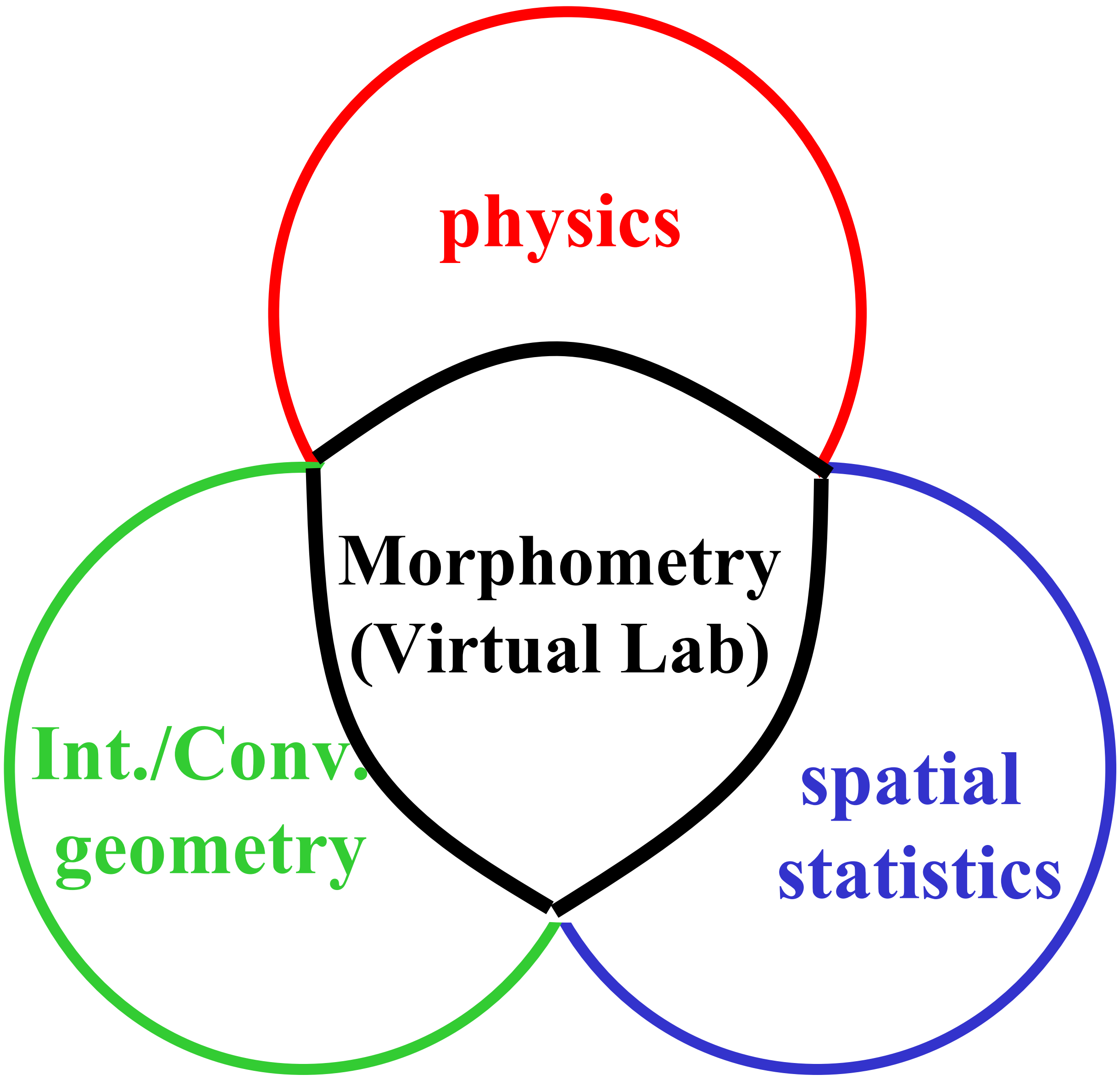


Norm Skewness
0,21
0,18
0,15
0,12
0,09
0,06
0,03
0
2
4
6
8
10
z
Phase Skewness
0,006
0,005
0,004
0,003
0,002
0,001
0,000
-0,001
0
2
4
6
8
10
z
Norm Excess
5,4
5,1
4,8
4,5
4,2
3,9
3,6
0
2
4
6
8
10
z
Phase Excess
1,806
1,803
1,800
1,797
1,794
1,791
1,788
0
2
4
6
8
10
z



Vectors Number
3,2x10^4
3,0x10^4
2,8x10^4
2,6x10^4
2,4x10^4
2,2x10^4
2,0x10^4
0
2
4
6
8
10
z
?
0,80
0,75
0,70
0,65
0,60
% Asymmetrical
z
Norm Summation
10000
8000
6000
4000
2000
z
0,030
0,025
0,020
0,015
0,010
0,005
0,000
real correlation
imaginary correlation
z

Computational Morphometry for Cosmology:



→ BRAVO! (php 5 & php-GK)

PHP is a widely-used general-purpose scripting language that is especially suited for Web development and can be embedded into HTML (http://www.php.net/)

Pattern Analysis and Simulation projects in LAC-INPE based on *php* environment:

- 2D and 3D Analysis (GPA & MF) (ex. MHD from ZEUS)
- Dynamical 3D Analysis (a more complex usual domain)
- Solving PDEs (Burgers & KPZ)
- Analytical flyby