MINISTÉRIO DA CIÊNCIA E TECNOLOGIA
INSTITUTO NACIONAL DE PESQUISAS ESPACIAIS

Mathematics of Climate

Climate change and natural disasters

Haroldo Fraga de Campos Velho

E-mail: haroldo@lac.inpe.br

Web-page: www.lac.inpe.br/~haroldo

Climate change research

- The climate is going to another equilibrium point

- Some words in the agenda:

Climate change research

- The climate is going to another equilibrium point
  - What can we expect from this new point?
  - Is there fixed point? Or, is it a dynamic equilibrium?
  - This is part of the climate change research
- Some words in the agenda:

Climate change research

- The climate is going to another equilibrium point
  - What can we expect from this new point?
  - Is there fixed point? Or, is it a dynamic equilibrium?
  - This is part of the climate change research

- Some words in the agenda:
  - Mitigation
  - Adaptation
  - Diagnosis

Climate change research

- Scientific consensus
  - Earth climate is changing
  - Human society is a geophysical forcing (anthropogenic sources)
  - Colder winters
  - Hotter summers
  - Change statistics for extreme (severe) weather
    - Extreme events more intense
    - Extreme events more frequent

Climate change research

- Scientific consensus
  - Earth climate is changing
  - Human society is a geophysical forcing (anthropogenic sources)
  - Colder winters
  - Hotter summers
  - Change statistics for extreme (severe) weather
    - Extreme events more intense
    - Extreme events more frequent

  - But, … how can we know that?

Climate change research

- But, … how can we know that?
  - Several (a lot) evidences

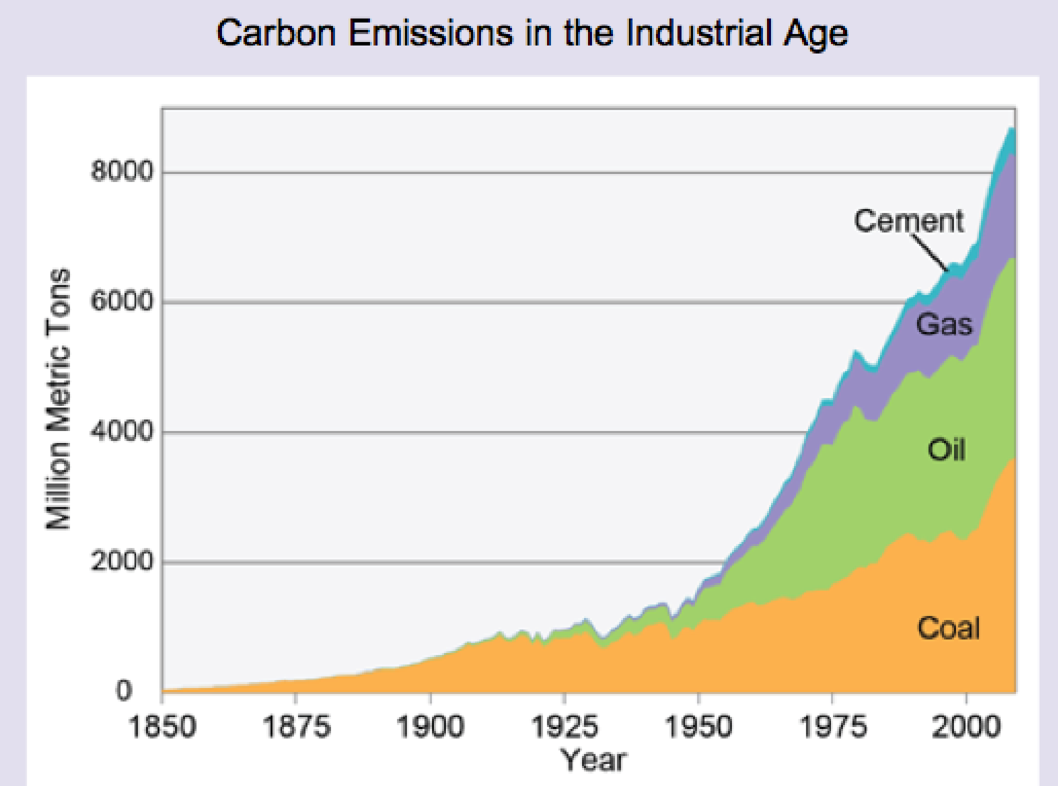


Figure 3. Global carbon emissions from burning coal, oil, and gas and producing cement (1850-2009). These emissions account for about 80% of the total emissions of carbon from human activities, with land-use changes (like cutting down forests) accounting for the other 20% in recent decades (Data from Boden et al. 2012[7]).

Climate change research

- But, … how can we know that?
  - Several (a lot) evidences

Muir Glacier – Alaska (USA): August 13, 1941 (left) / August 31, 2004 (right)

Climate change research

- Mathematical models
  - A scientific conquer is to use mathematical equations for doing predictions.
  - In geophysical fluid dynamics, meteorology has initiated the forecasting process by mathematical equations
  - However, we solve the equations **only approximately**!
  - Numerical methods need to be applied
  - Impact from model resolution
  - (More) Physical process need to be included

Climate change research

- Mathematical model (hard equations: non-linear)

Movement Equation *(momentum)*

$$\frac{du}{dt} - fv + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

$$\frac{dh}{dt} + g + \frac{1}{\rho}\frac{\partial p}{\partial z} = 0$$

$$\frac{dv}{dt} + fu + \frac{1}{\rho}\frac{\partial p}{\partial y} = 0$$

Continuity Equation *(mass)*

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho h) = 0$$

Thermodynamic equation *(energy)*

$$p = f(T)\rho \;\rangle \Rightarrow \frac{f(T)}{T} = \frac{g(\rho)}{\rho} \equiv R(cte)$$

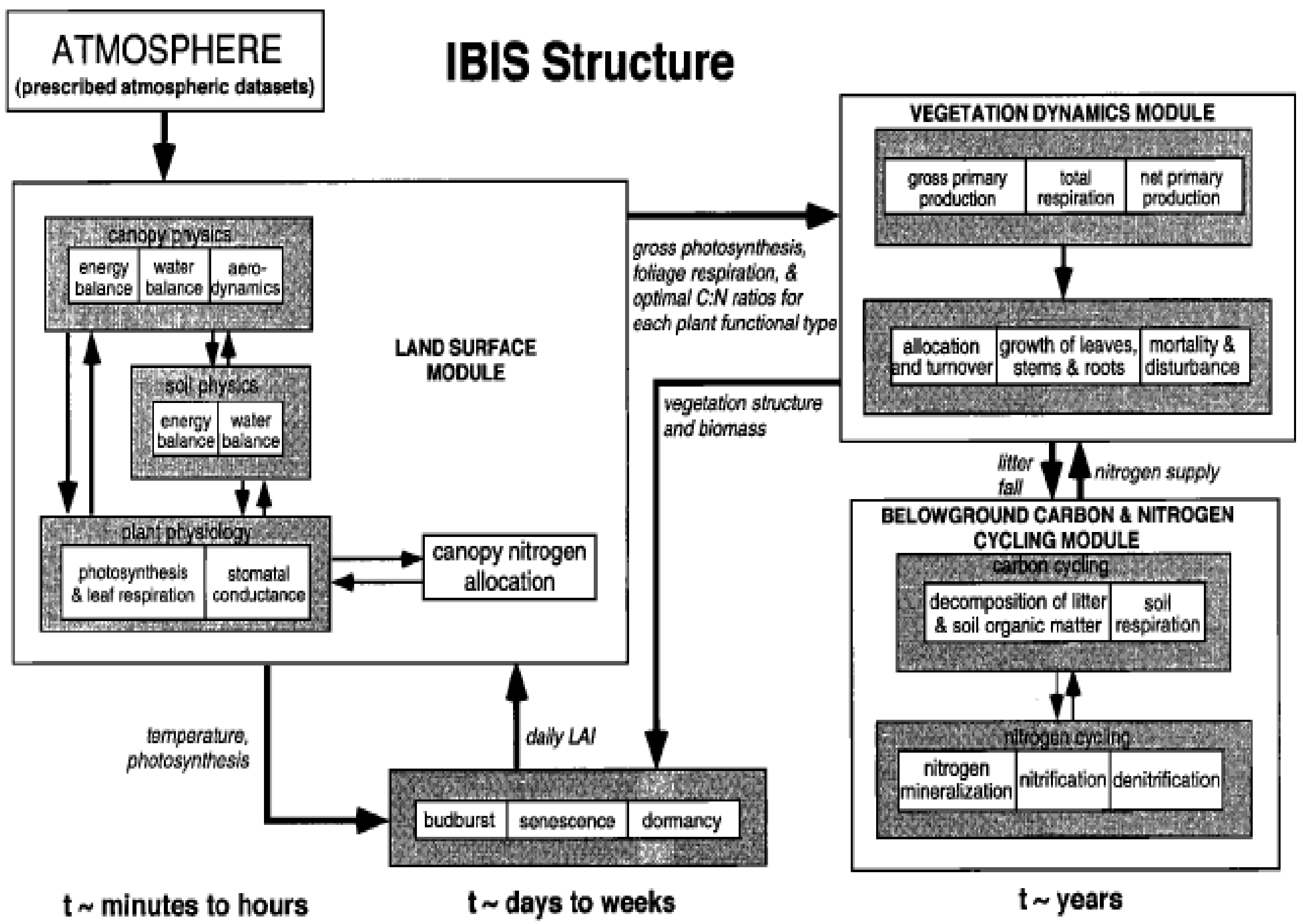
$$p = g(\rho)T$$

$$p = \rho R T$$

$$C_v \frac{dT}{dt} + p \frac{d(1/\rho)}{dt} = \frac{dq}{dt}$$

Climate change research

- Preparing the models: calibration



Climate change research

- Preparing the models: calibration (IBIS model)

No	Name	Description
1	*rhoveg_vis*	Leaf reflectance in the upper canopy - visible (dimensionless)
2	*rhoveg_NIR*	Leaf reflectance in the upper canopy - NIR (dimensionless)

Climate change research

- Preparing the models: calibration (IBIS model)

3	*tauveg_vis*	Leaf Tramitância in the upper canopy - visible (dimensionless)
4	*tauveg_NIR*	Leaf Tramitância in the upper canopy - NIR (dimensionless)
5	*chifuz*	Sheet guiding factor in the upper canopy (-1: vertical, 0: random, 1: horizontal)

Climate change research

- Preparing the models: calibration (IBIS model)

41	*alogl_cef*	Coefficient for calculation of the lower canopy roughness (dimensionless)
42	*alogu_coef*	Coefficient for calculation of the upper canopy roughness (dimensionless)
43	*avmuir_coef*	Coefficient for calculating the canopy emissivity (dimensionless)

Climate change research

- Preparing the models: calibration (IBIS model)

$$F_1(\mathbf{w}) = \sqrt{\frac{1}{N}\left[\sum_{j=1}^{9}\sum_{i=1}^{N}\left(Y_i^{\mathrm{Obs},(j)} - Y_i^{\mathrm{Mod},(j)}(\mathbf{w})\right)^2\right]} \quad (1)$$

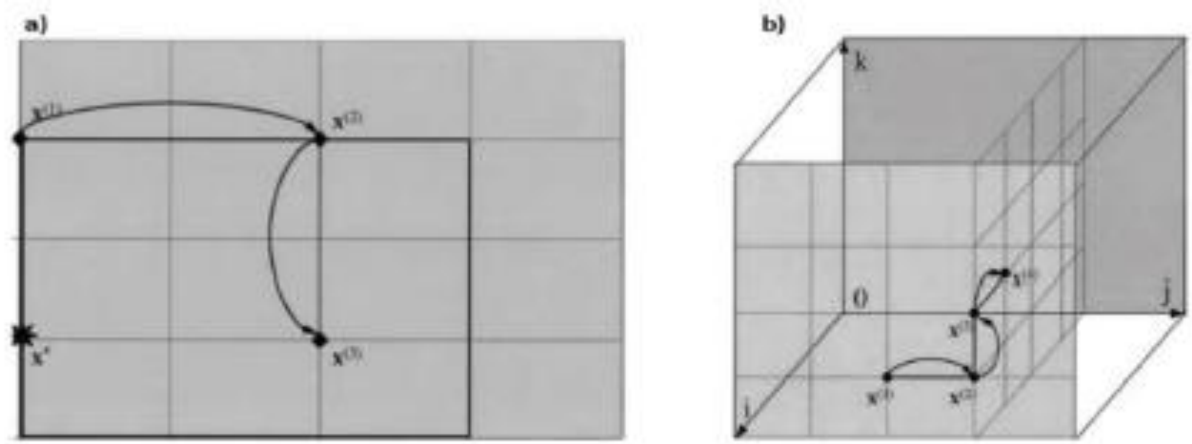
$$F_2(\mathbf{w}) = \sqrt{\frac{1}{N}\left[\sum_{j=1}^{9}\sum_{i=1}^{N}\left(\frac{1}{Y_i^{\mathrm{Obs},(j)}} - \frac{1}{Y_i^{\mathrm{Mod},(j)}(\mathbf{w})}\right)^2\right]} \quad (2)$$

Calibration: sensitivy analysis (2)

- Morris' method
  - Sensitivity analysis

$$d_k(\mathbf{w}) = \frac{Y(w_1,\ldots,w_k+\Delta,w_{k+1},\ldots,w_p)-Y(\mathbf{w})}{\Delta} \quad (3)$$

  - Trajectories in the search space (2D (a) / and 3D (b))



Numerical results [1]

- Identifying faster to slower processes (Figure 1)

(a) L1: radiative flows: PAR_0 and f_{APAR} (3 parameters);
(b) L2: surface radiation balance: R_n (3 parameters);
(c) L3 - Turbulence: u_* (3 parameters);
(d) L4: turbulent flows: NEE, HE, LE (16 parameters);
(e) L5: Carbon Allocation: LAI (6 parameters).

Numerical results (2)

- Model parameter estimation

Parameter	Exact	Estimated
rhoveg–vis	0.0872	0.0778
tauveg–vis	0.0498	0.0498
chifuz	–0.2249	–0.1643
rhoveg–NIR	0.2966	0.3468
tauveg–NIR	0.2038	0.2235
avmuir–coef	370.9740	366.4872
dispu–coef	0.9779	0.9895
alogl–coef	3.9289	4.4682
alogu–coef	7.2151	6.4447
vmax–pft	0.0001	0.0001
coefmub	7.5494	7.6680
chs	33448.5189	22679.8452
beta2	0.7838	0.8081
funca–coef	5477.4147	6678.2708
funcbcoef	5900.2127	5306.0478
root–coef	0.8429	1.0703
rwood–coef	0.1563	0.0857
tempvm–coef	3961.8800	3502.3284
stressf–coef	–5.3121	–5.1114
clitls–coef	1.4686	2.1727
clitrs–coef	4.5124	4.9226
clitws–coef	1.0452	1.0452
csoislon–coef	0.2502	0.3429
csoislop–coef	6.3736	6.6974
kfactor	1.4744	1.3810
rgrowth–coef	0.2965	0.2738
tauleaf	0.5808	0.7208
specla	31.2950	32.9062
aleaf	0.2654	0.1856
aroot	0.3269	0.1870
awood	0.6280	0.4773

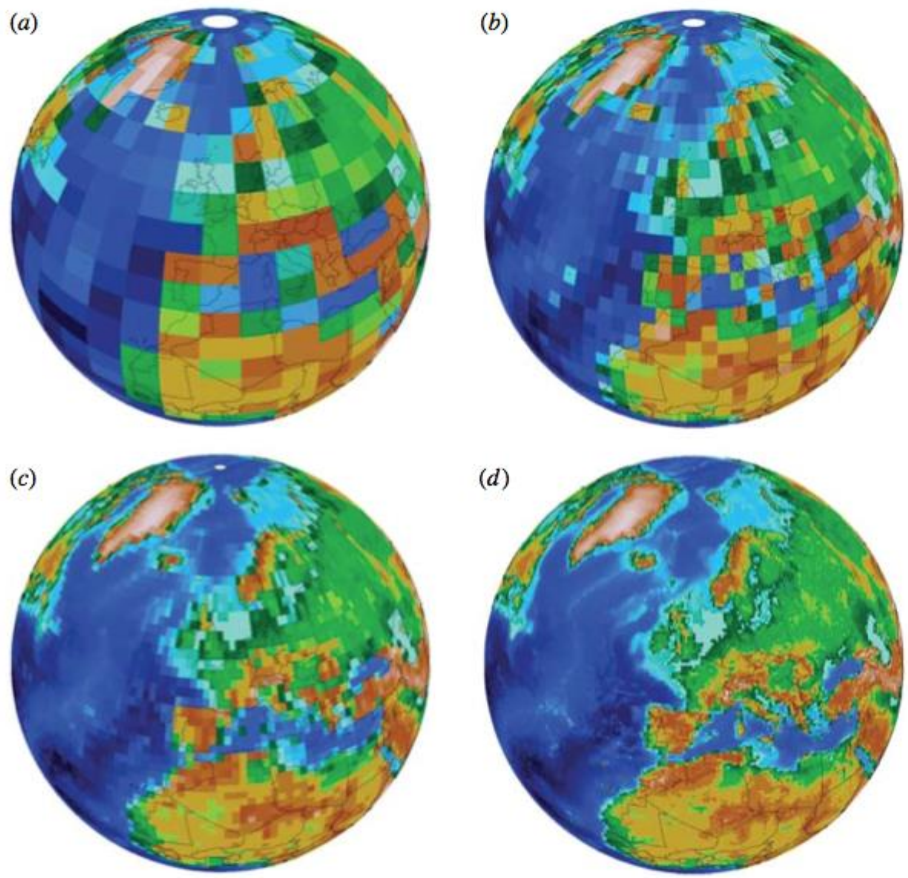
Climate change research

- Preparing the models: calibration
  - Each module should have its calibration process
  - Calibration: multi-objective optimization
  - Calibration: sentitivy analysis is essential

- Calibration: depends on prediction time-scale
  - Sentitivy analysis shows more relevant parameters
  - Parameters relevance change with prediction time-scale
    - Nowcasting
    - Forecasting
    - Climate: seasonal, year, decadal, century(ies)

Climate change research

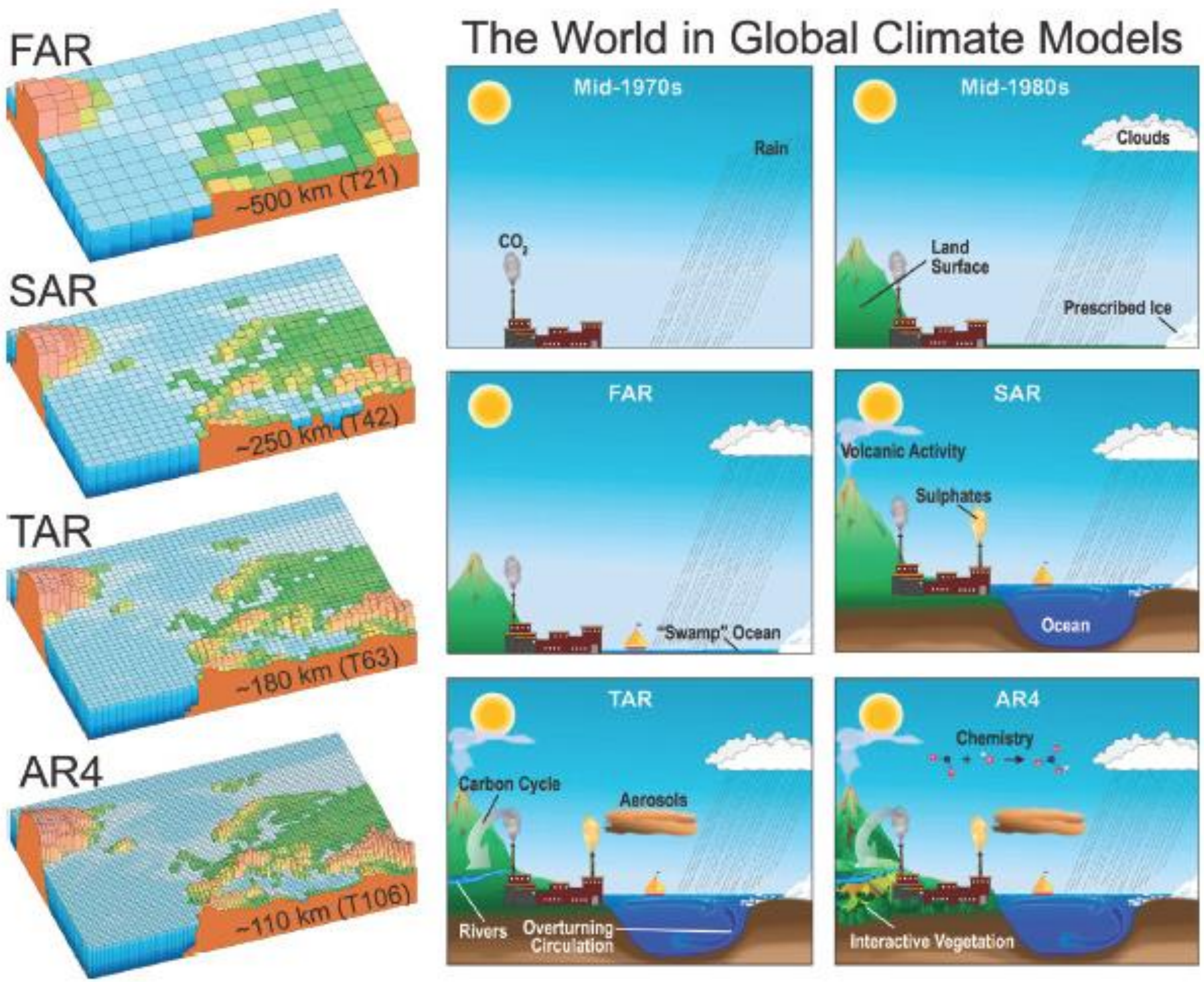
- Mathematical models (hard equations: resolution impact)

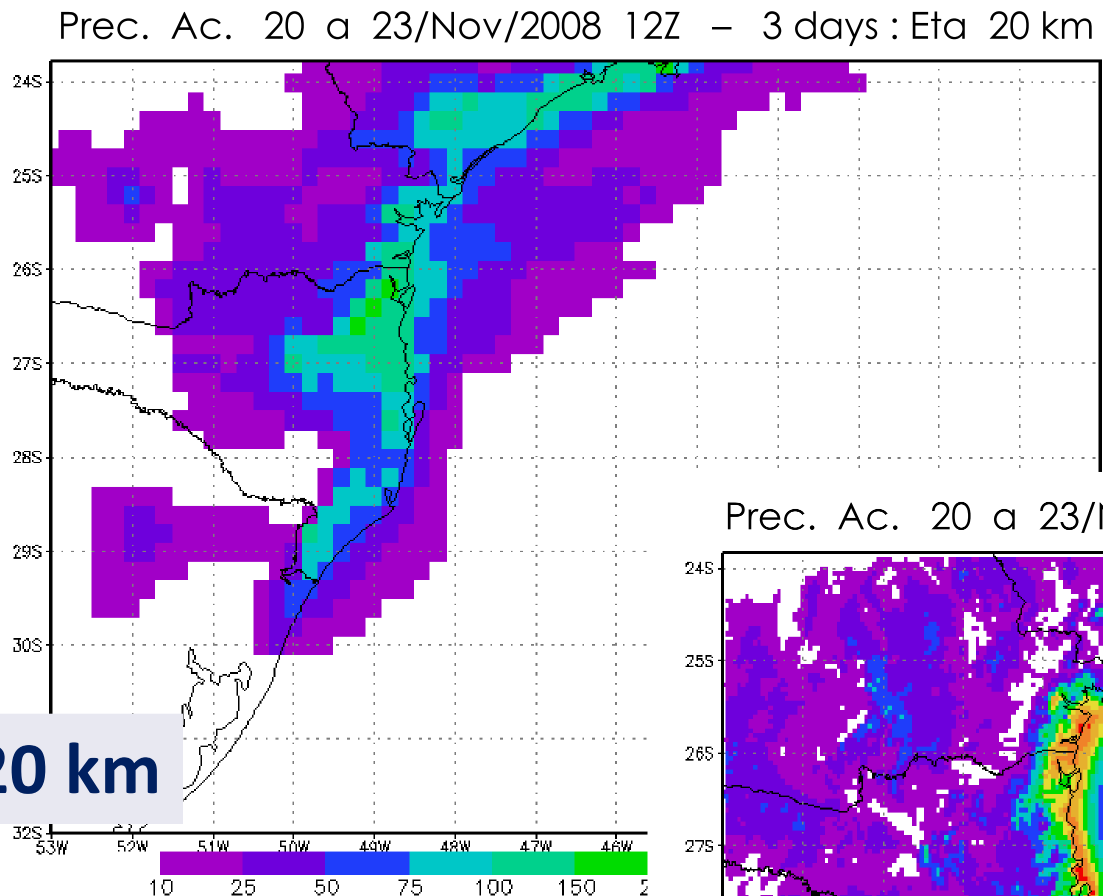


(a) 500 km (b) 300 km (c) 150 km (d) 75 km

Climate change research

- Mathematical models (hard equations: resolution impact)

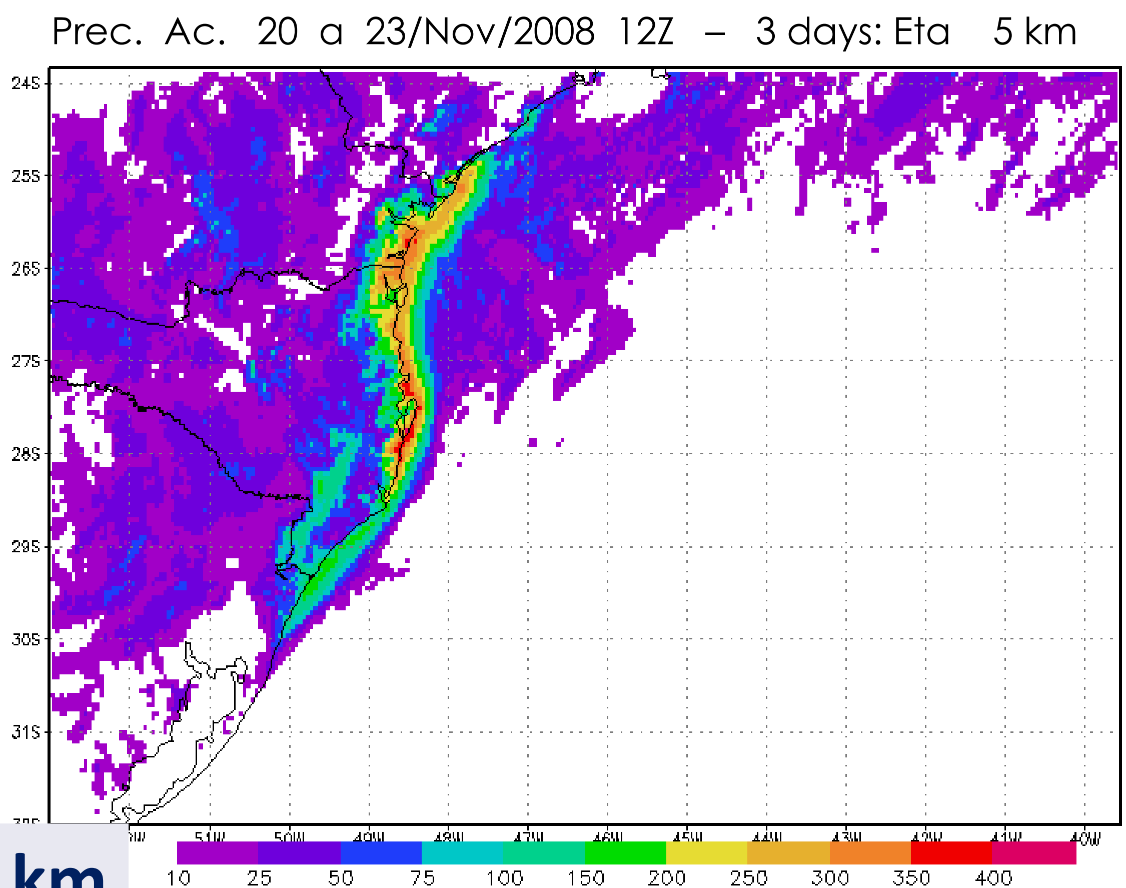




20 km

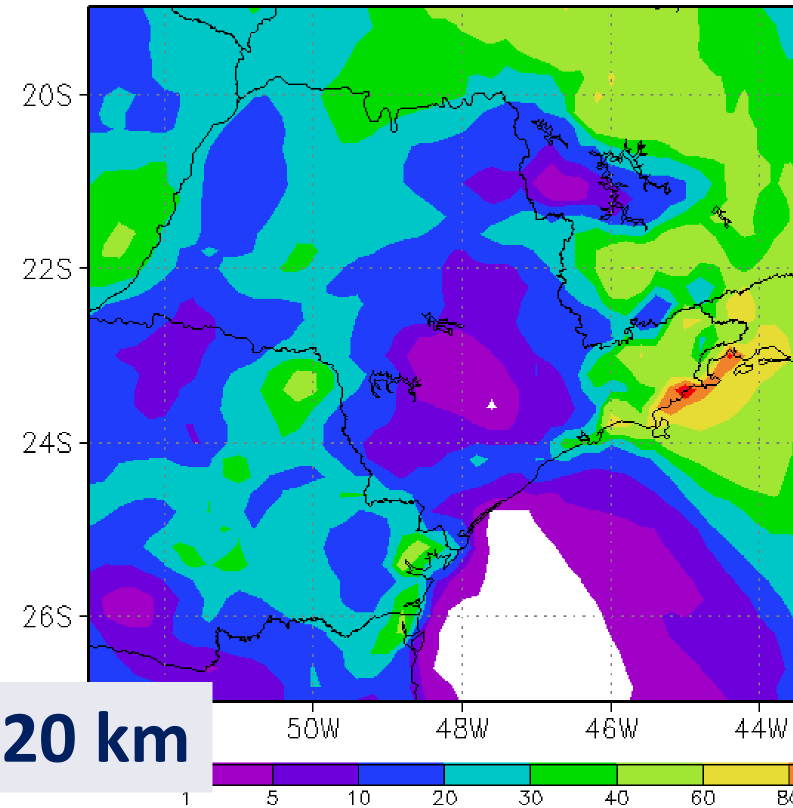
Santa Catarina (BR) November/2008

135 deaths / 78,000 homeless



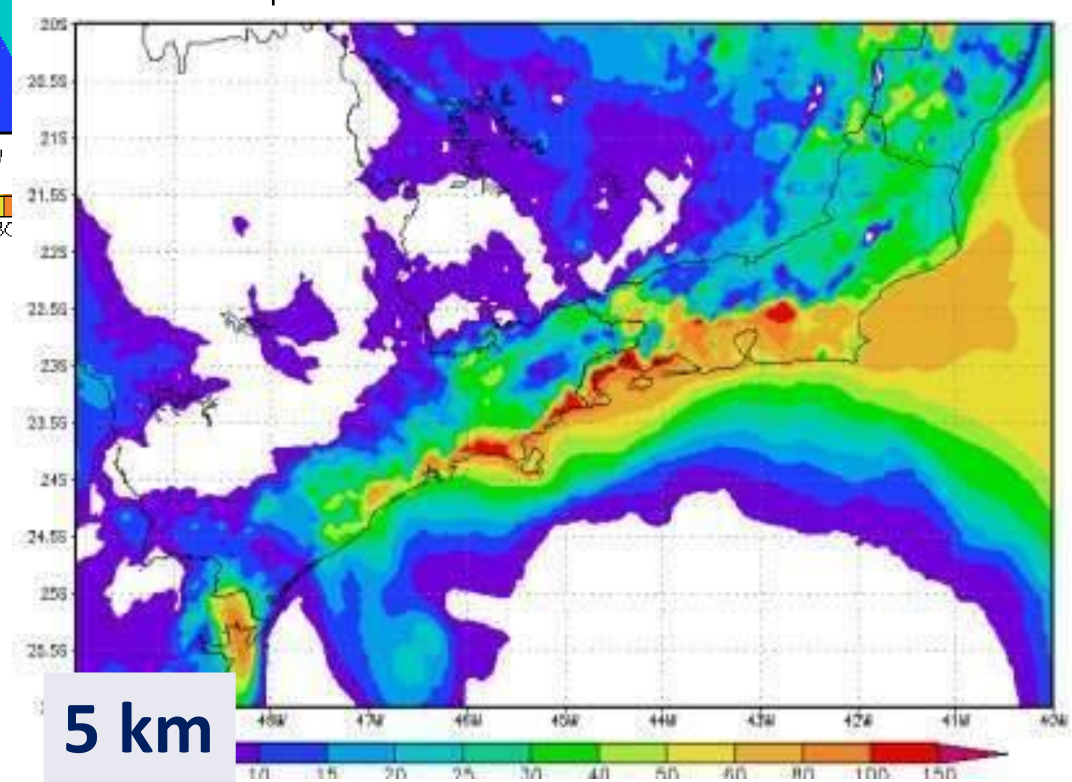
5 km

Precipitation Total (mm) – Eta 20 km



20 km

Precipitation for24 horas – Eta 5 km

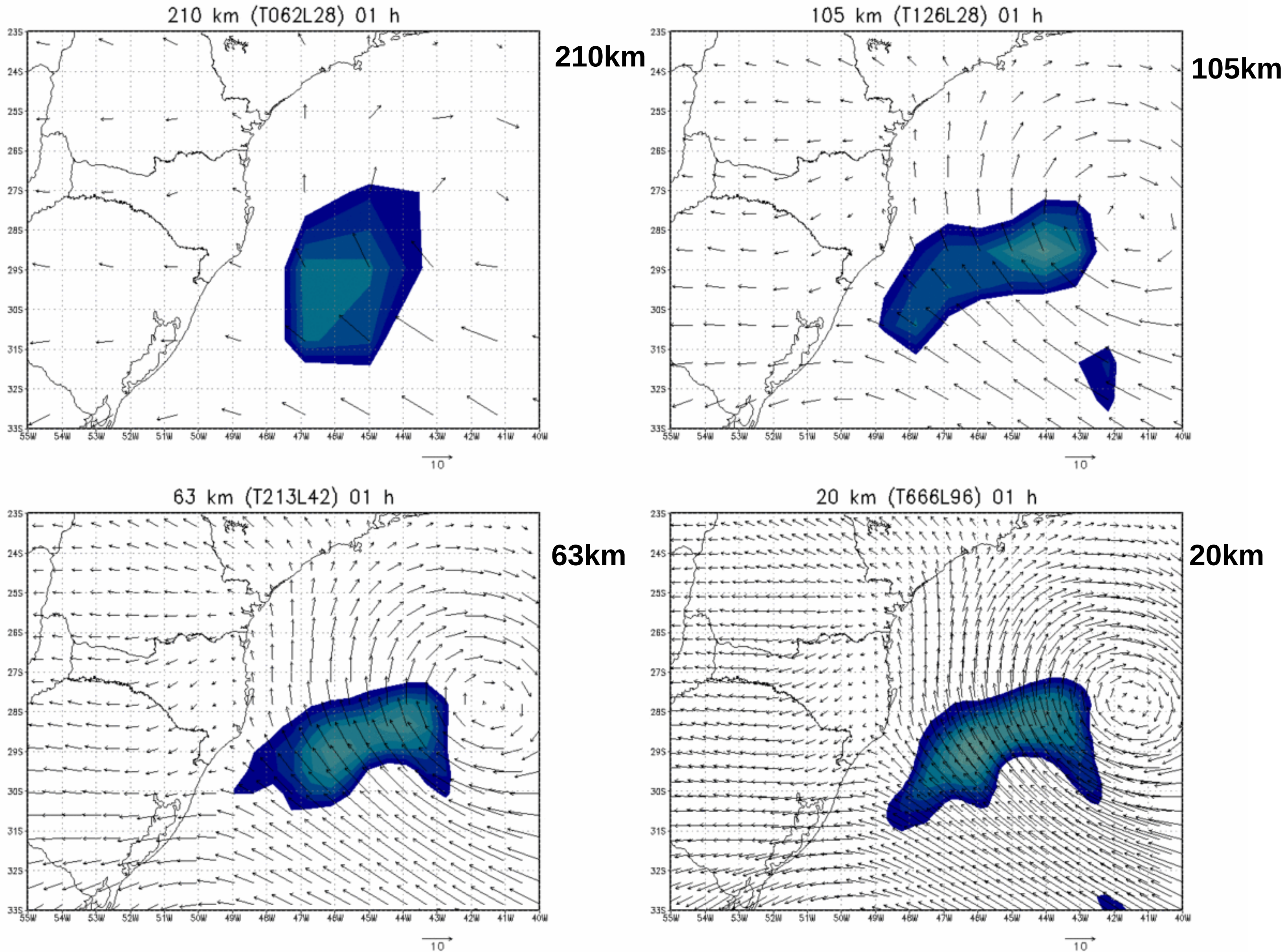


5 km

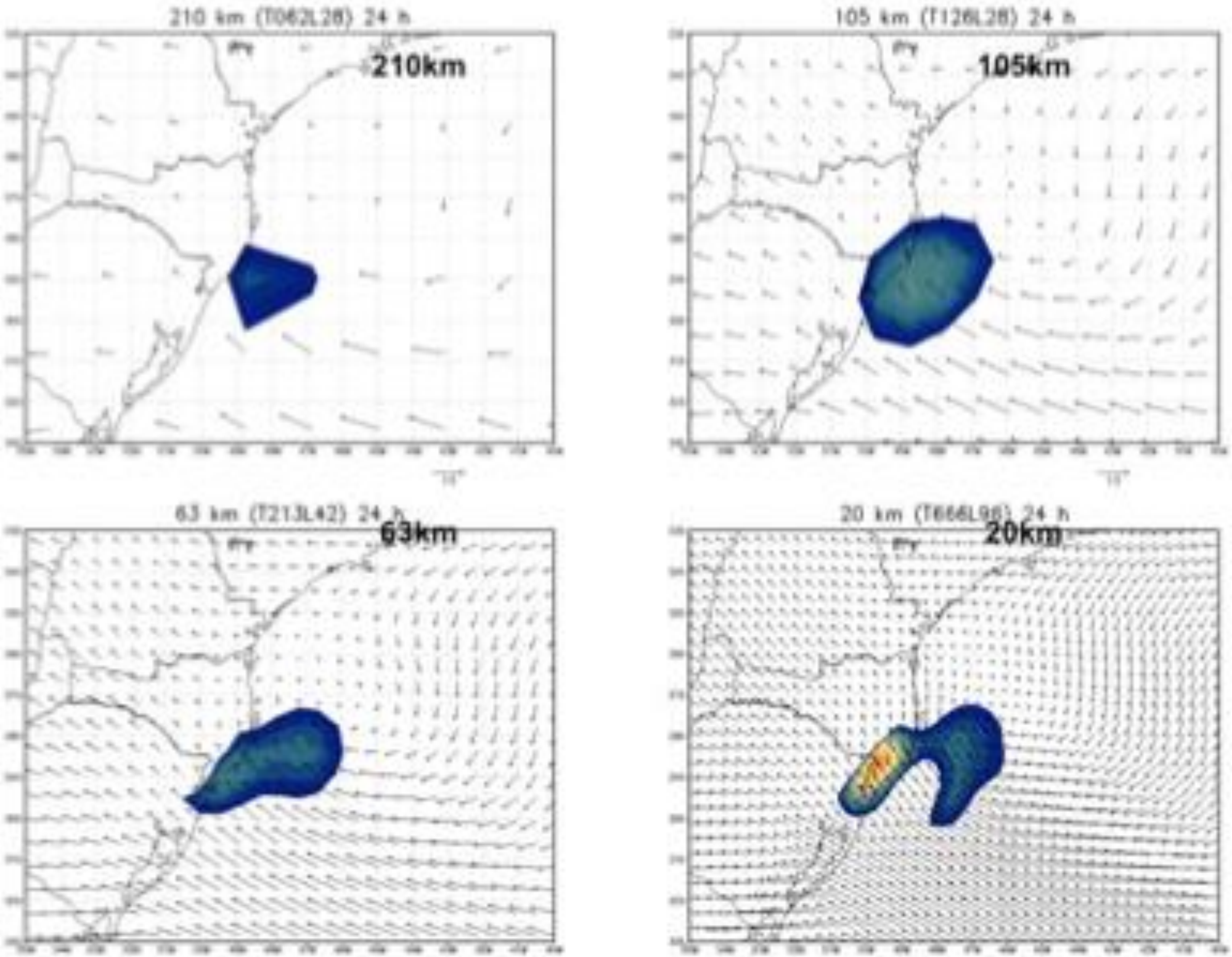
Rio and Angra dos Reis, 31/Dec/2009 (prediction for 24 h)

Catarina Hurricane: images from space

Catarina Hurricane: differents resolutions (Global model)

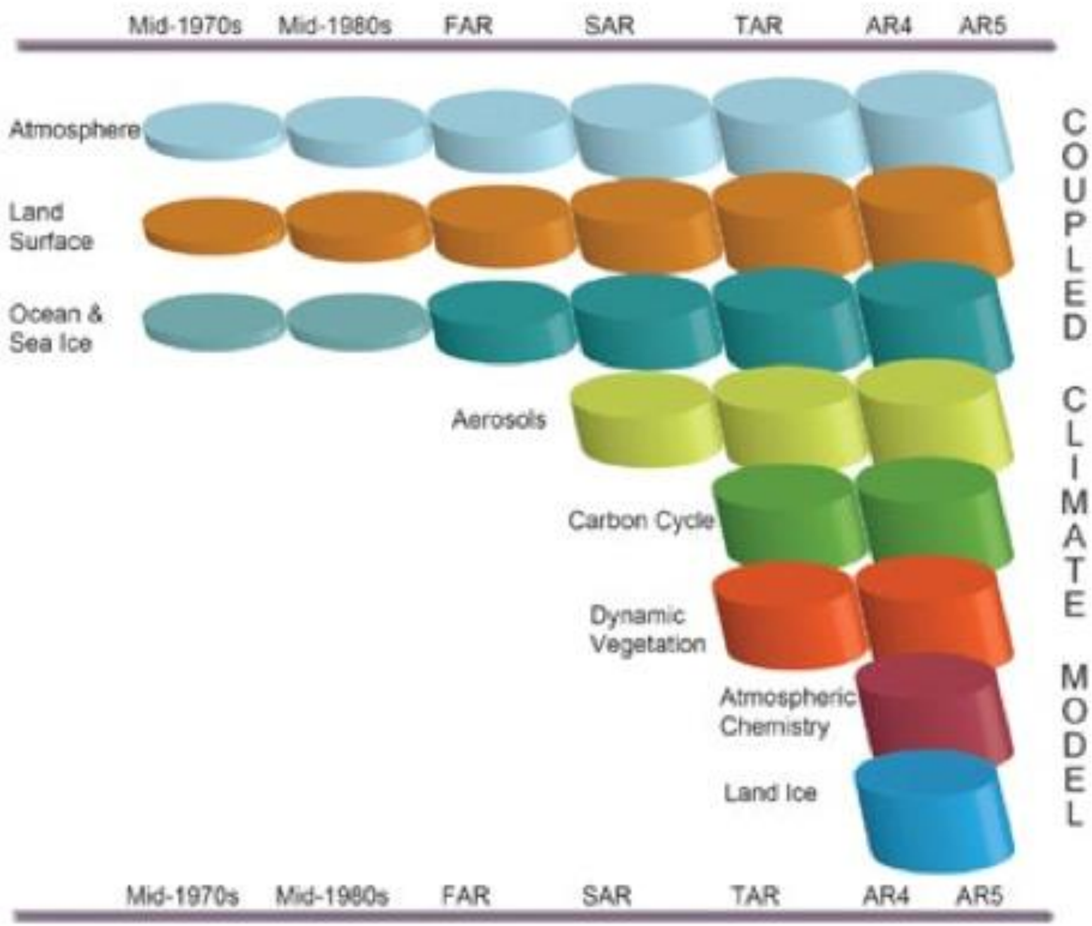


Catarina Hurricane: differents resolutions (Global model)



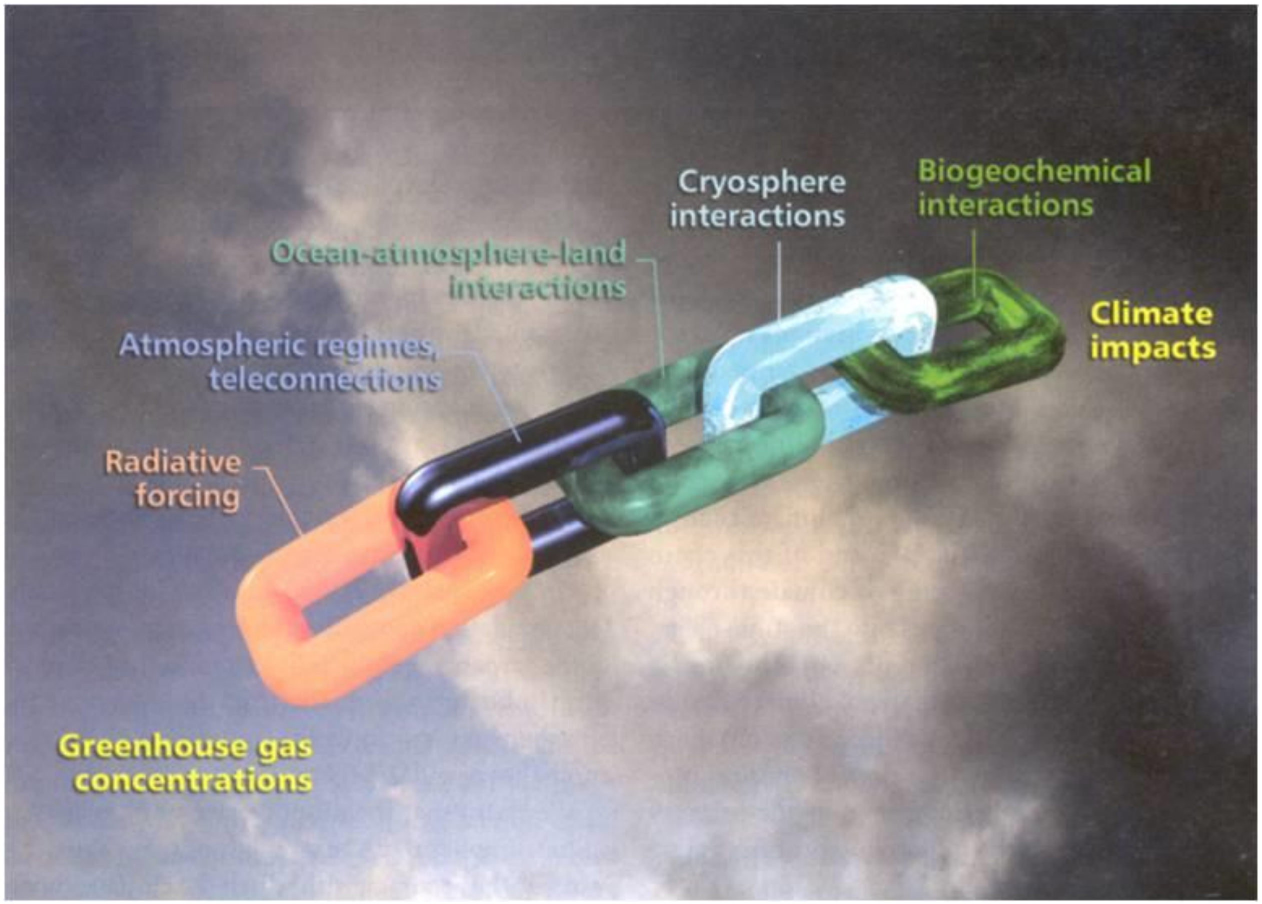
Climate change research

- Mathematical models (more physical process)



Climate change research

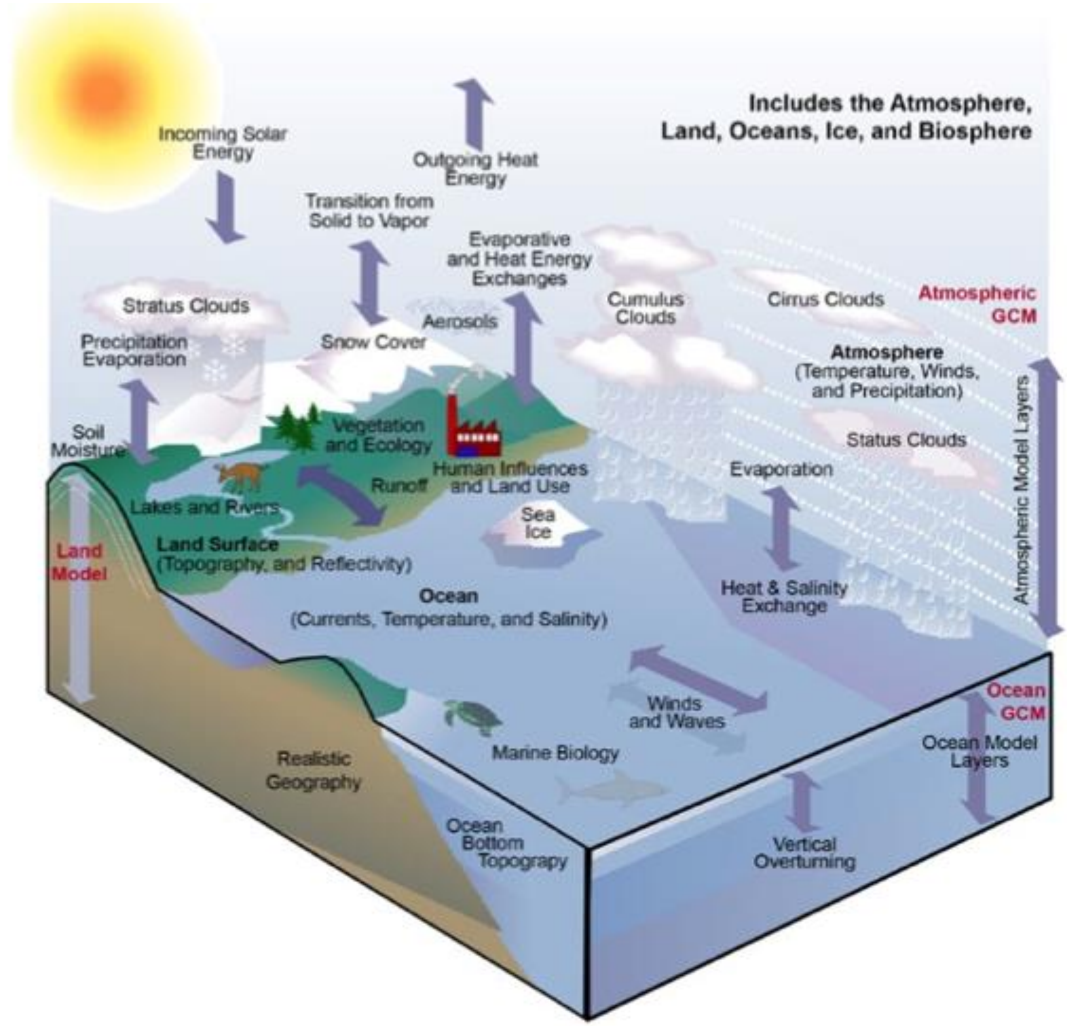
- Mathematical models (more physical process)



Bull. American Met. Soc. – 2008

Climate change research

- Mathematical models (more physical process)



Climate change research

- Mathematical models
  - Hard (*difficult* and stiff) equations
  - Finer resolution
  - More physical process involved
  - More data to be processed

  - This means …
  - **Computer power**

Climate change research

- Solving the models: computers and computing
  - Computers
  - Computing

Climate change research

Multi-processing machine with distributed memory

CPTEC-INPE: Cray XE6

1280 nodes

30720 cores

Climate change research

Hybrid computing: CPU multi-core + Cell

IBM Cell project: CPU + co-processor (project was finished)

+

Climate change research

Hybrid computing: CPU multi-core + FPGA

SGI RASC: CPU + FPGA
(project was finished)

RASC: Reconfigurable Application-Specific Computing

Climate change research

Hybrid computing: CPU multi-core + GP-GPU

Top 500: **1°** **Year 2009 – Tianhe-1**

Climate change research

Hybrid computing: CPU multi-core + CPU many-core

Top 500: **1°** **Milkway-2 (Tianhe-2):** CPU + Xeon Phi

+

Climate change research

Hybrid computing: CPU multi-core + (MIC+FPGA)

+

Climate change research

- Mathematical model (computing initial condition)

Movement Equation *(momentum)*

$$\frac{du}{dt} - fv + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \qquad \frac{dh}{dt} + g + \frac{1}{\rho}\frac{\partial p}{\partial z} = 0 \qquad \frac{dv}{dt} + fu + \frac{1}{\rho}\frac{\partial p}{\partial y} = 0$$

Continuity Equation *(mass)*

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho h) = 0$$

Thermodynamic equation *(energy)*

$$p = f(T)\rho \ \rangle \Rightarrow \frac{f(T)}{T} = \frac{g(\rho)}{\rho} \equiv R(cte) \qquad p = g(\rho)T$$

$$p = \rho R T$$

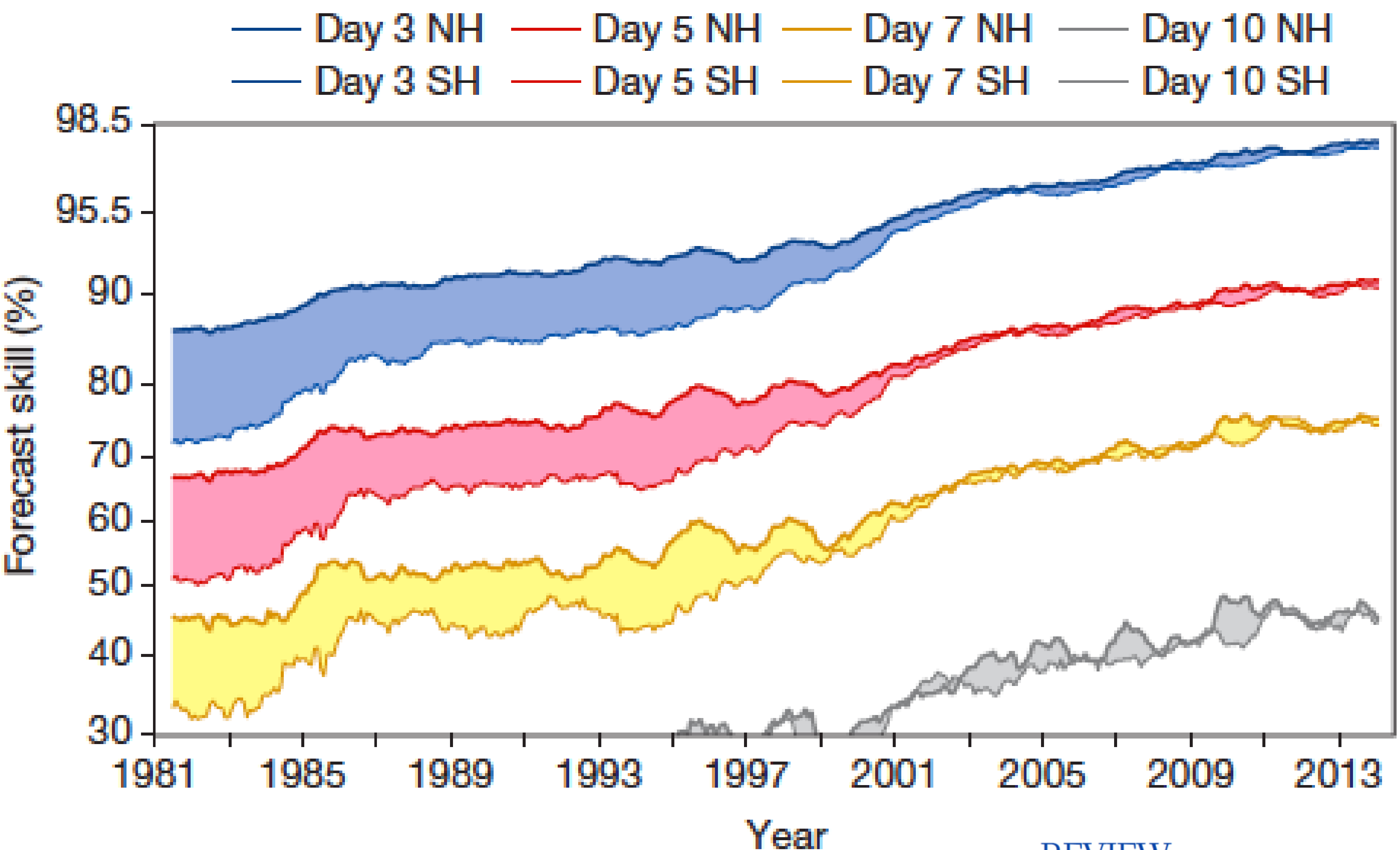
$$C_v \frac{dT}{dt} + p \frac{d(1/\rho)}{dt} = \frac{dq}{dt}$$

Observations × weather predictions

Advanced parameterization for the water cycle has been considered in the modern models. Better numerical weather predictions have been obtained using efficient data assimilation, employing all information available (observational data from satellites, radars, GPS, etc). This promotes a feedback mechanism, enhancing our understanding on the atmospheric water cycle itself.

Forecasts Scores

ECMWF



REVIEW

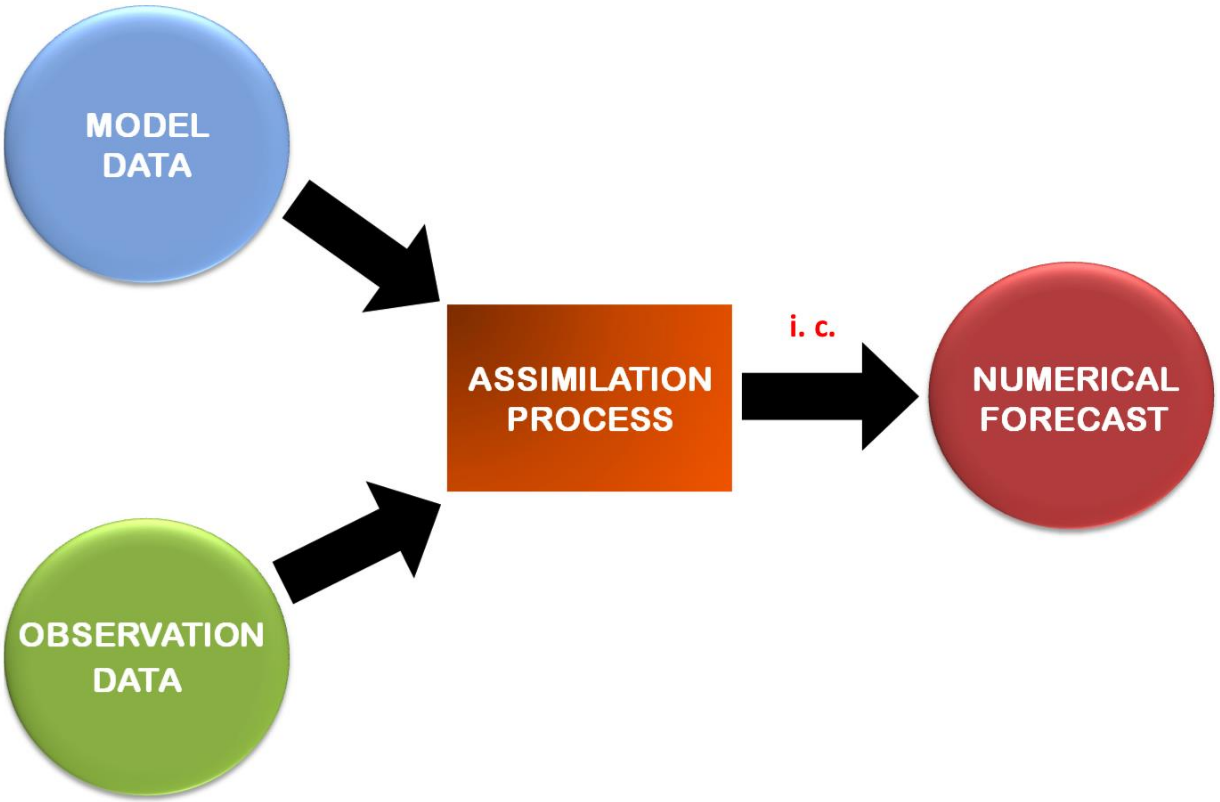
doi:10.1038/nature14956

The quiet revolution of numerical weather prediction

Peter Bauer[1], Alan Thorpe[1] & Gilbert Brunet[2]

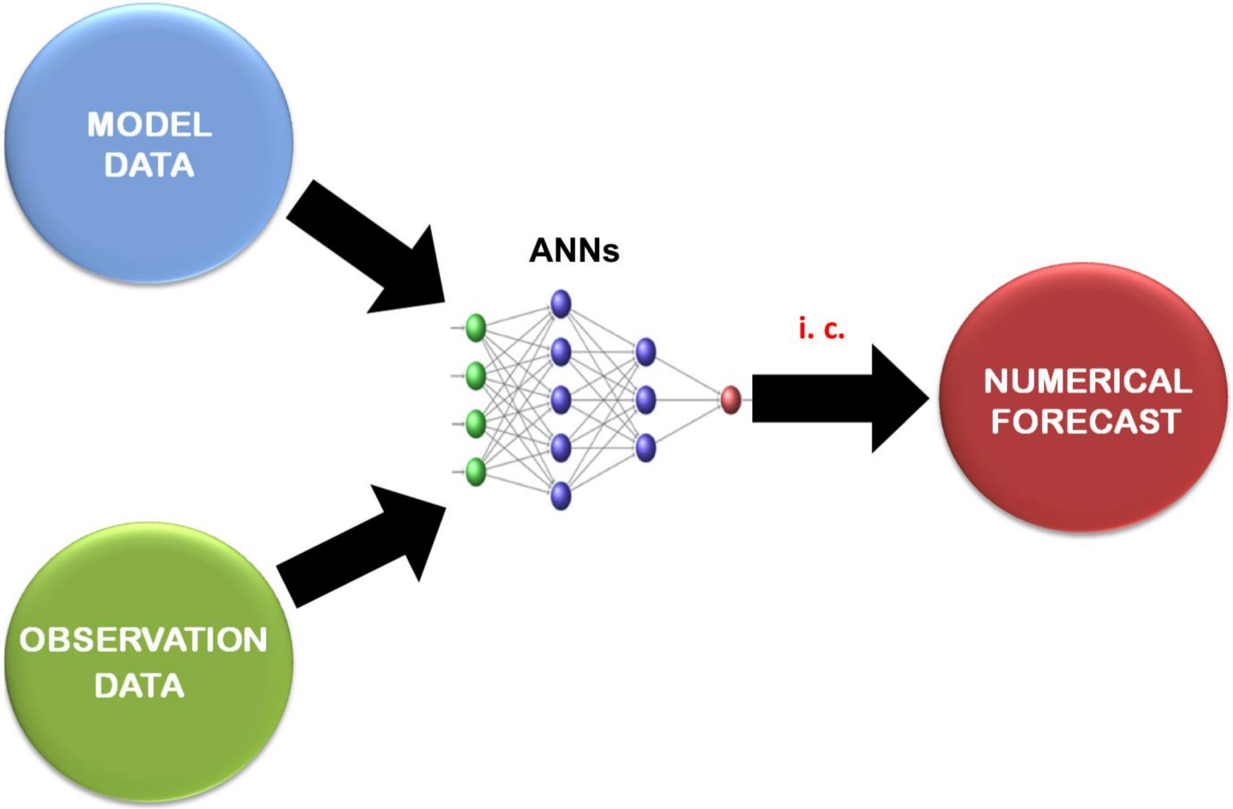
PART 2 – Applications: Data assimilation

- Data assimilation: data fusion



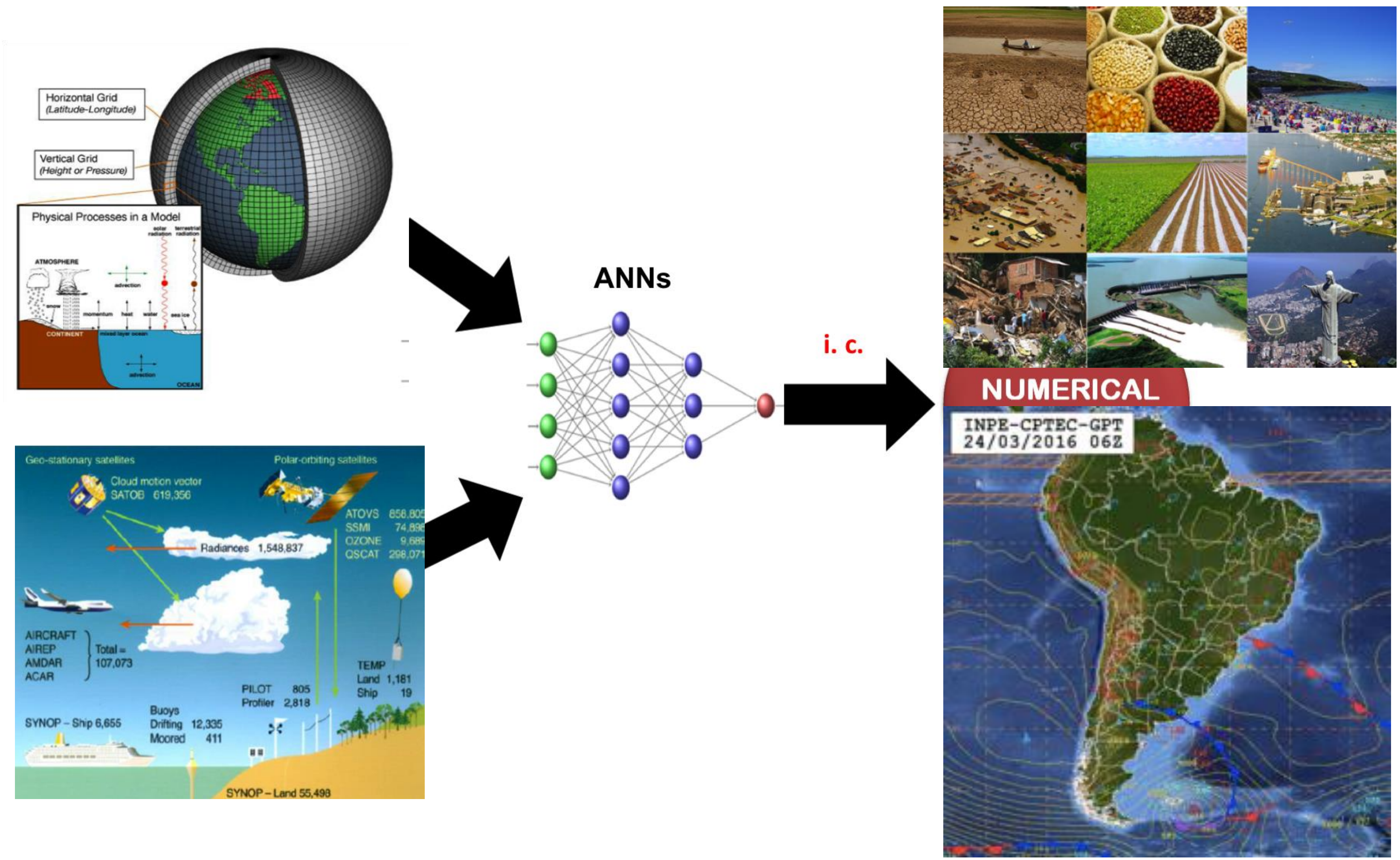
PART 2 – Applications: Data assimilation

- Data assimilation: data fusion

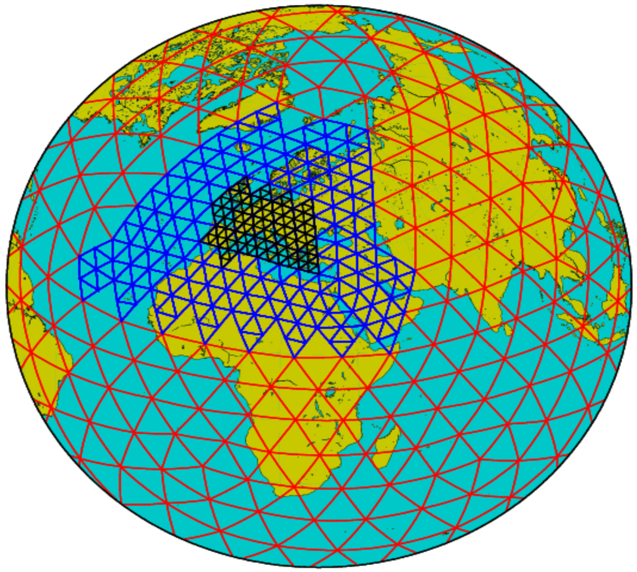


PART 2 – Applications: Data assimilation

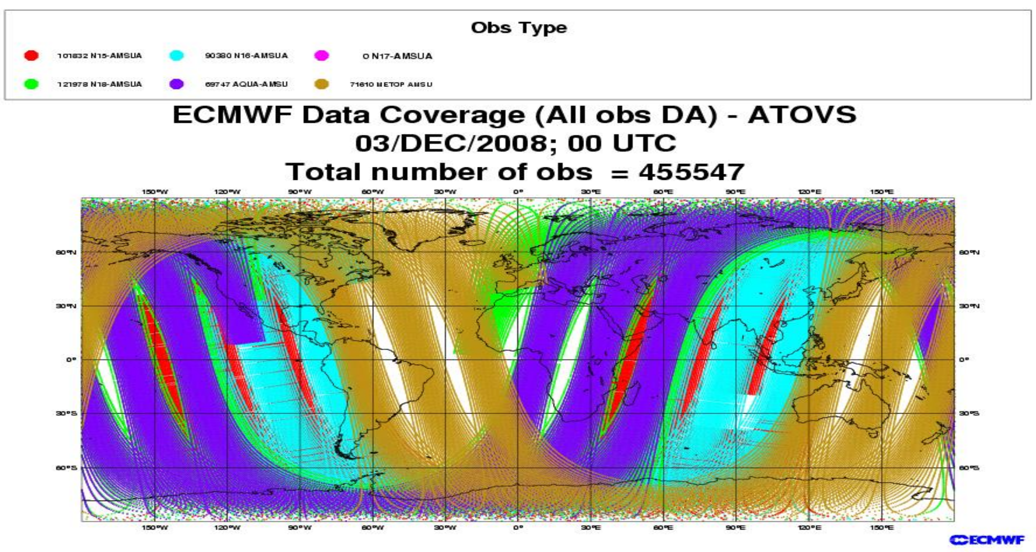
- Data assimilation: data fusion



Impact with the exponential growth for the available data



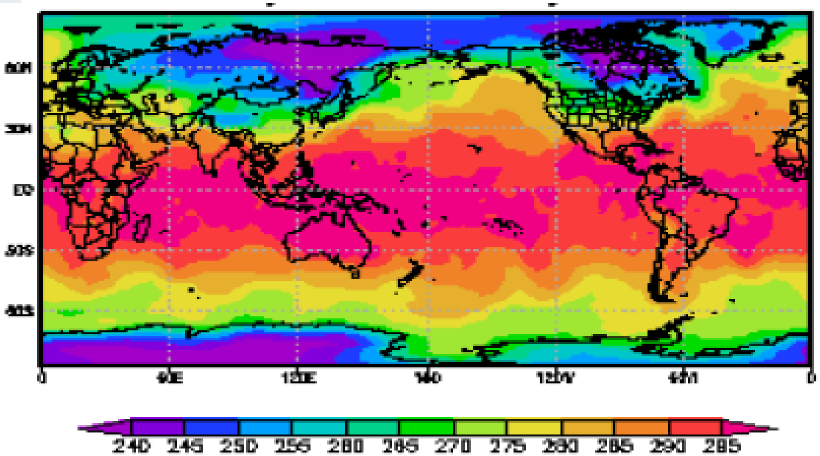
Numerical models with very high resolution

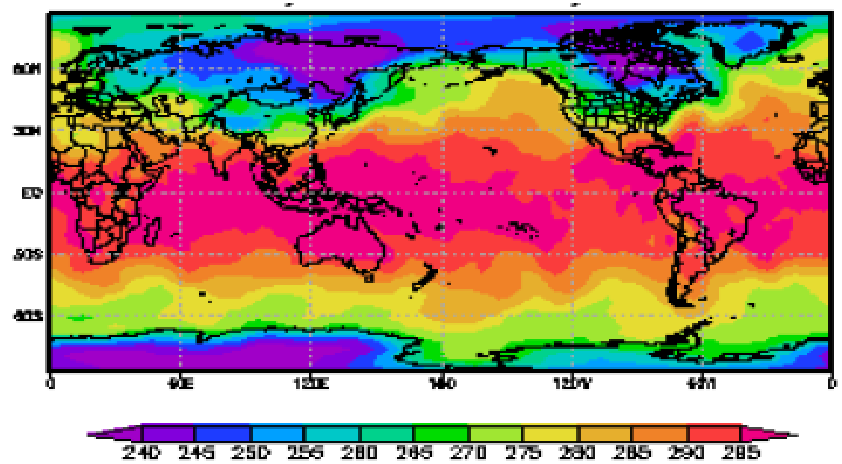


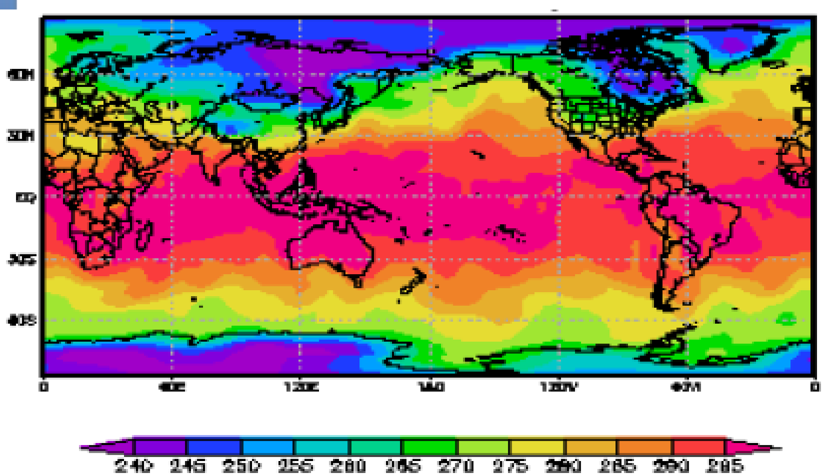
Number of observation are increasing: different satellites with thousands of bands, sensor cost decreasing.

Data assimilation: an essential issue

- Temperature: ANN assimilation experiment







LETKF

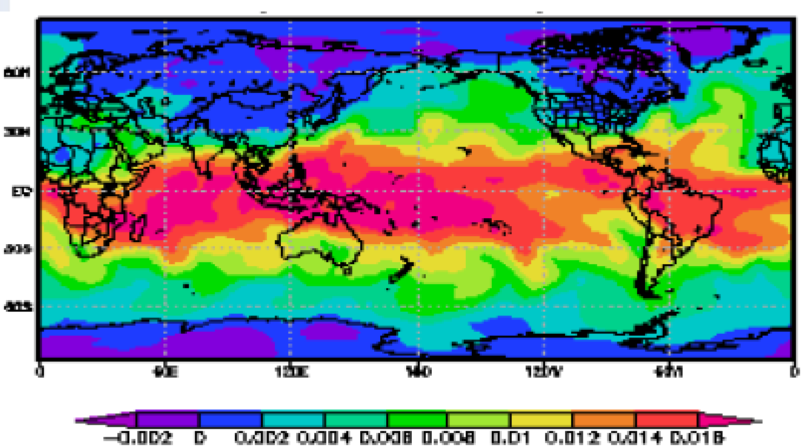
neural network

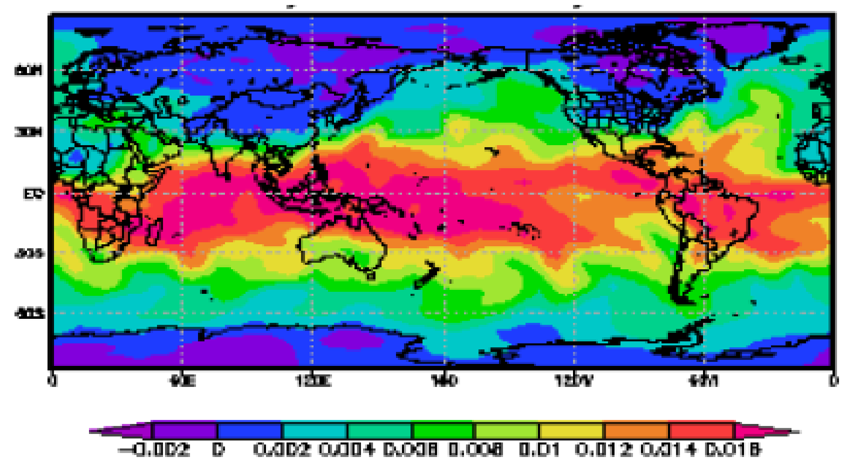
True

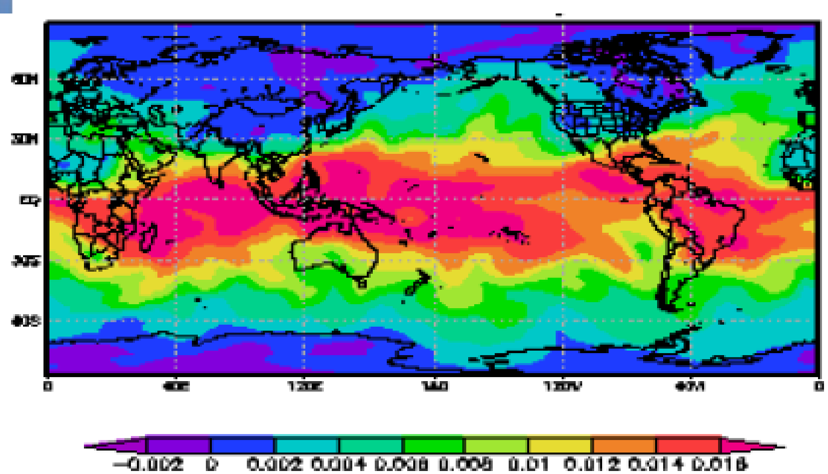
Results from Rosangela Cintra PhD thesis (2011)

Data assimilation: an essential issue

- Moisture: 1 month assimilation experiment







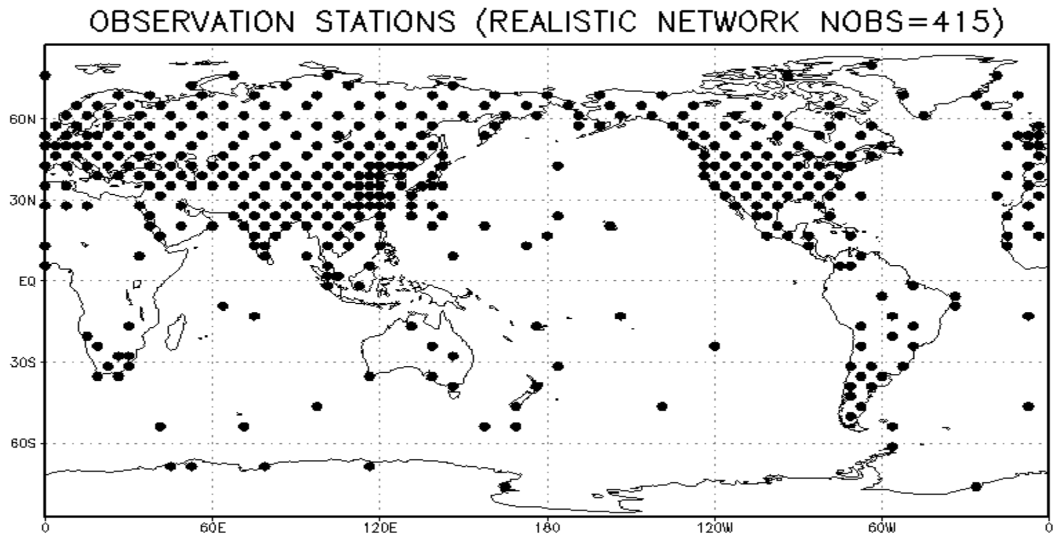
LETKF

neural network

True

Results from Rosangela Cintra PhD thesis (2011)

Numerical experiment: LEKF and ANN



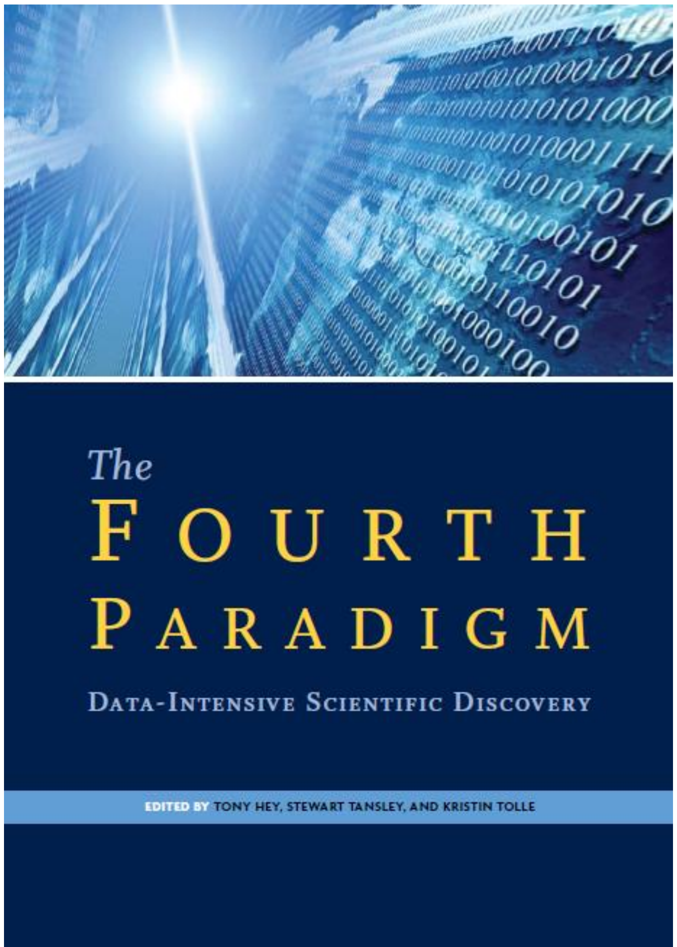
Execution time	
LETKF method	RNA method
04:20:39	00:02:53
hours : minutes : seconds	

Atmospheric general model circulation (spectral model): **3D SPEEDY** (**S**implified **P**arameterizations primitiv**E** **E**quation **DY**namics)

Gaussian grid: 96 x 48 (horizontal) x 7 levels (vertical) = T30L7
Total grid points: 32,256   Total variables in the model: 133,632
Observations: (00, 06, 12, 18 UTC) – radiosonders "OMM stations"
Observations: 12035 (00 and 12 UTC) = 415 x 4 x 7 + 415
Observations: 2075 (06 and 18 UTC) = 415 x 5 (only surface)

Climate change research

- Mathematical models (post-processing)



The
FOURTH
PARADIGM

DATA-INTENSIVE SCIENTIFIC DISCOVERY

EDITED BY
TONY HEY, STEWART TANSLEY,
AND KRISTIN TOLLE

Data science – Data mining example

Association rules: database from Agriculture Eng. (Unicamp)

Association rules: database from Agriculture Eng. (Unicamp)

It was used DMII-CBA (Classification Based on Association) - software developed by School of Computing, National University of Singapore

http://www.comp.nus.edu.sg./~dm2/p_overview.html

Temperatura [^{0}C]	Nomenclatura	Sensação do animal
<20	T1	frio
20-23	T2	frio
23-25	T3	frio
25-27	T4	conforto
27-30	T5	conforto
30-32	T6	calor
>32	T7	calor

Níveis de ruído [dba]	Nomenclatura	Classificação
<60	RU1	ambiente tranqüilo
60-70	RU2	barulho
70-80	RU3	muito barulho
80-85	RU4	nível preocupante
>85	RU5	nível prejudicial

Umidade [%]	Nomenclatura	Sensação do animal
<60	UR1	desconforto
60-70	UR2	conforto
70-80	UR3	conforto
>80	UR4	desconforto

0	UR3,T4,RU5
1	UR2,T3,RU5
2	UR2,T4,RU5
3	UR2,T5,RU5
4	UR1,T5,RU5
5	UR1,T5,RU5
6	UR1,T5,RU5

```
Rule 28:
        UR1 = Y
        ->      RU5 = Y
    (42.857% 100.00% 3 3 42.857%)
```

Example of rule created by DMII-CBA software

BraVO@INPE

2. Decision tree for astronomical data classification

Classification

Star/galaxy

It is not easy task

See the figure:

(a) Easy

BraVO@INPE

2. Decision tree for astronomical data classification

Classification

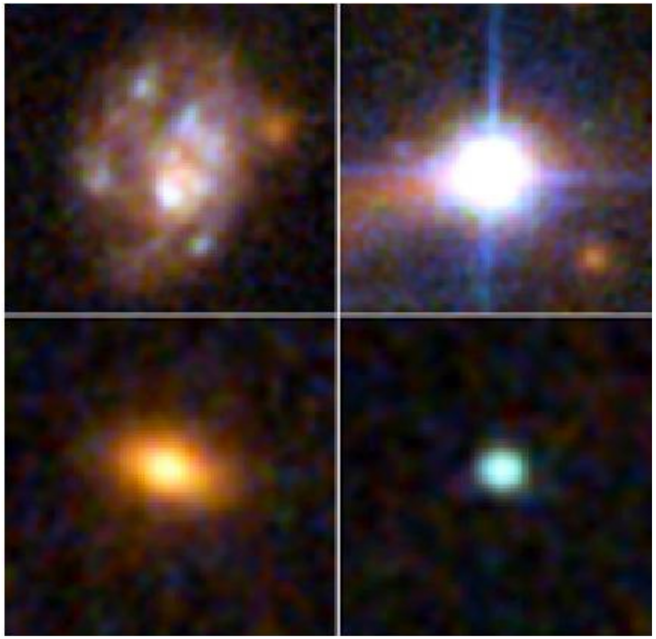
Star/galaxy

It is not easy task

See the figure:

(a) Easy

(b) More complicated



BraVO@INPE

2. Decision tree for astronomical data classification

Classification

Star/galaxy

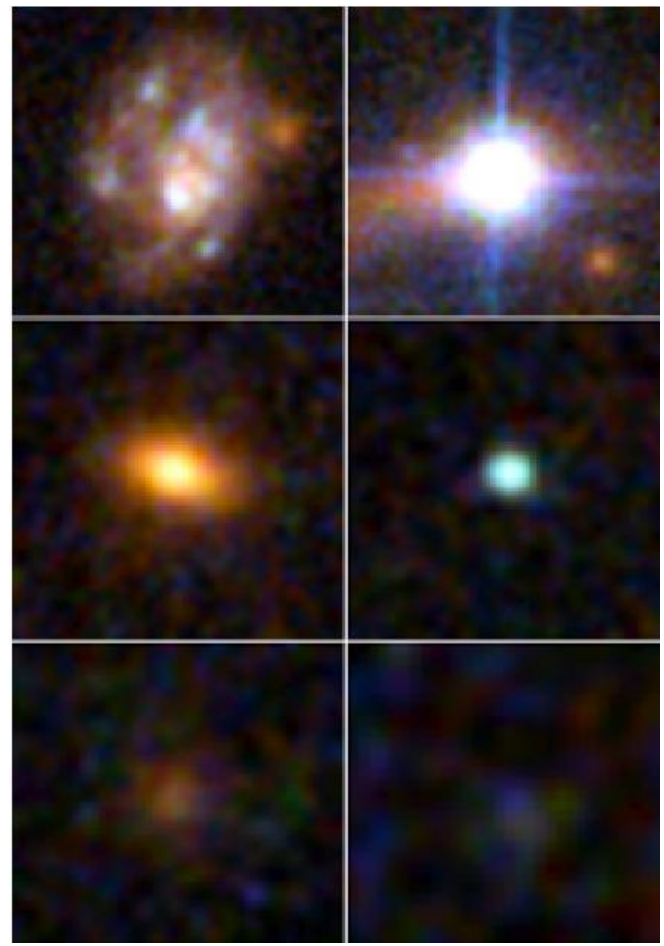
It is not easy task

See the figure:

(a) Easy

(b) More complicated

(c) How to classify?



Extreme events

- Two types of events:
  - Deep drought (2010: Amazon drought)

Extreme events

- Two types of events:
  - Intense rain fall (Nov/2008: Santa Catarina state, Brazil)

135 people died
78.000 homeless

Data science – tools

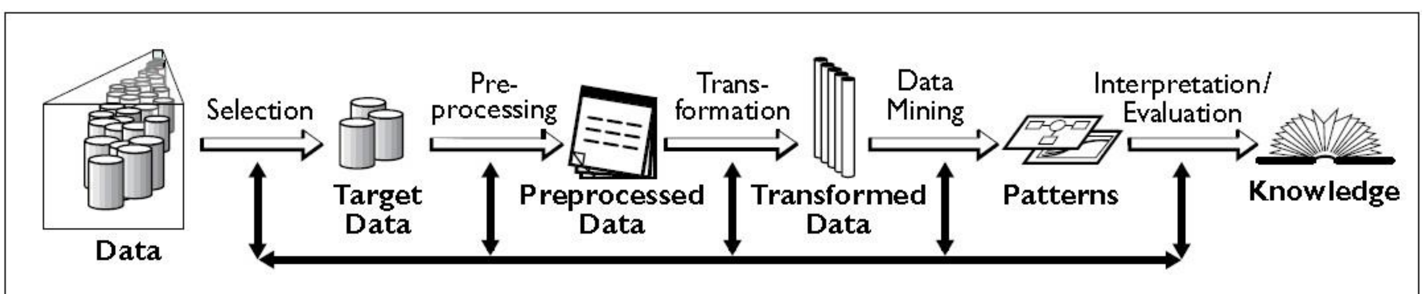
- From statistics

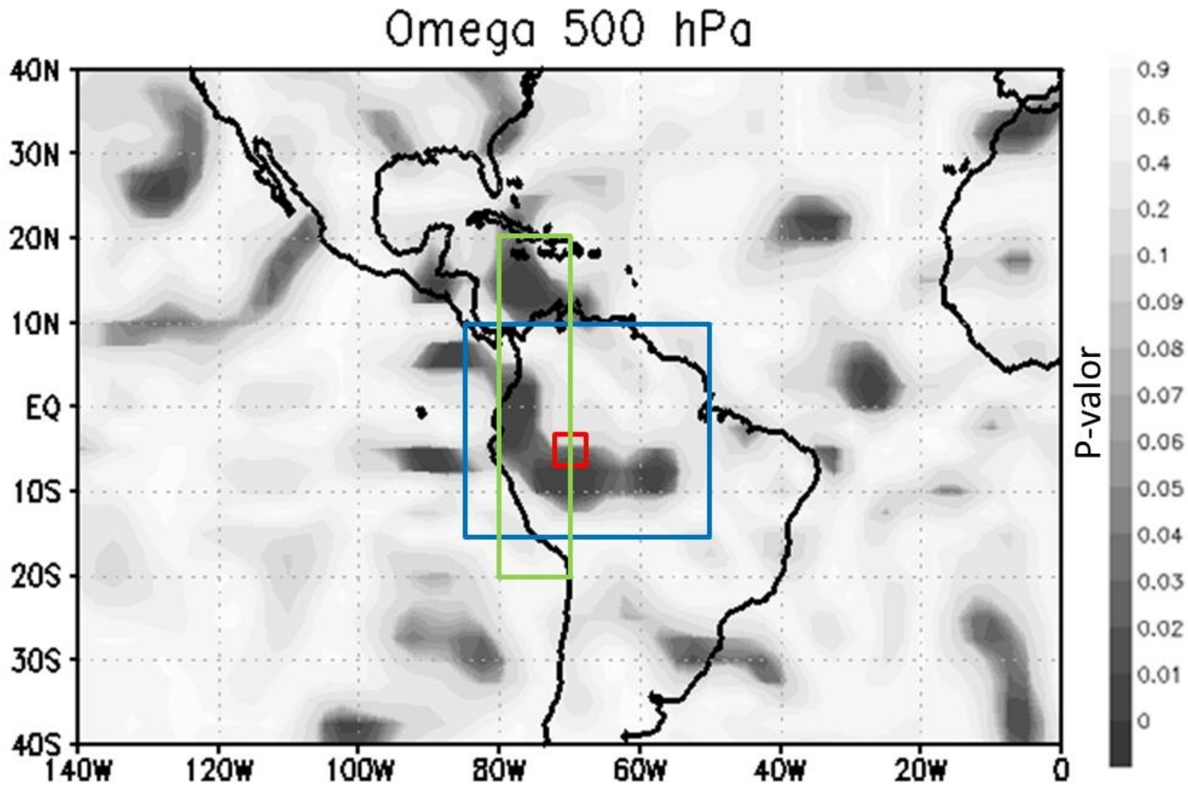
NATIONAL CANCER INSTITUTE

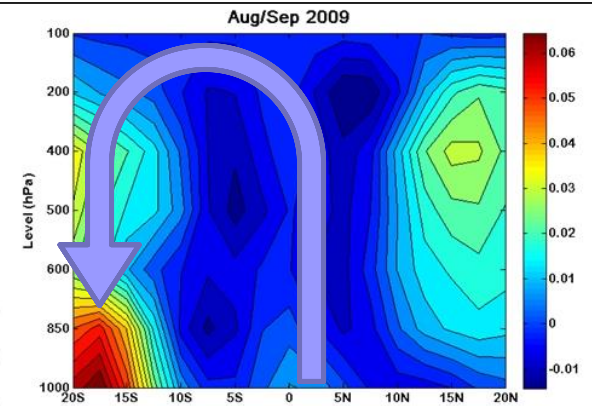
Biometric Research Branch

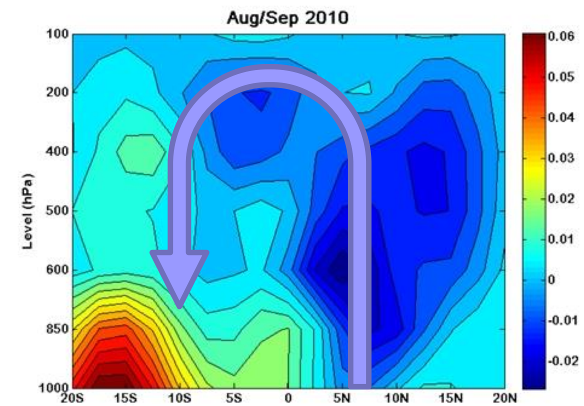
Division of Cancer Treatment and Diagnosis

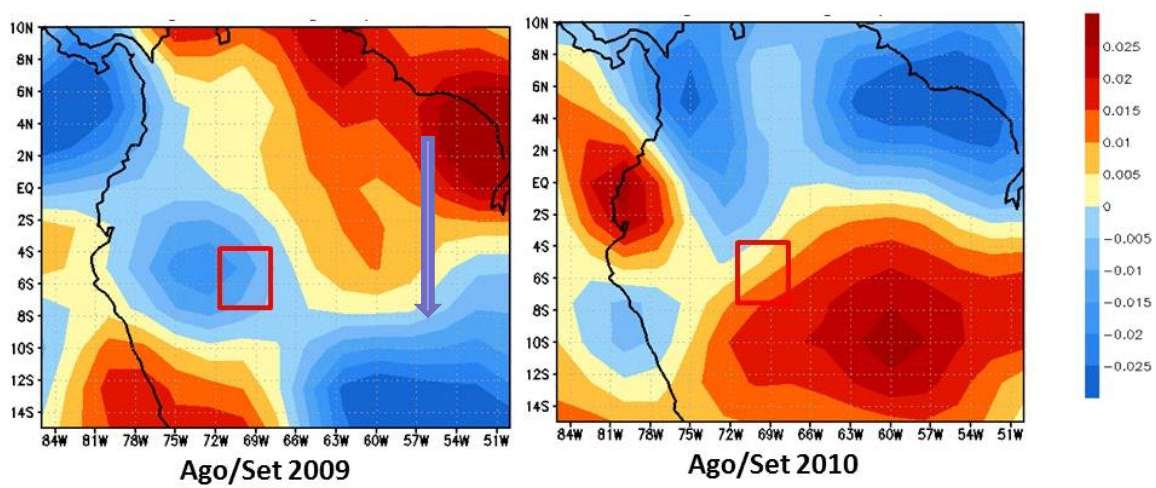
- From artificial inteligence

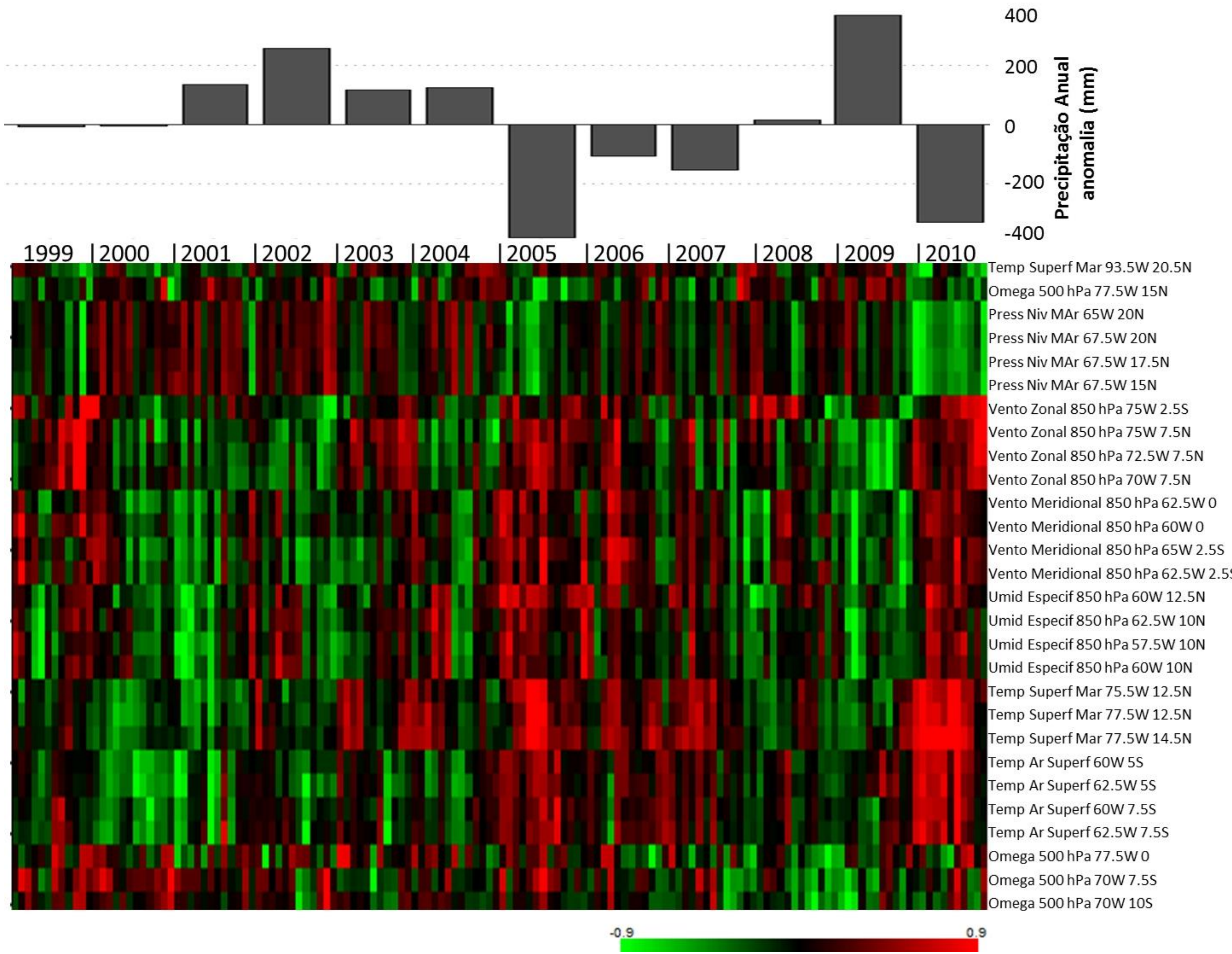



Omega 500 hPa
P-valor
40N
30N
20N
10N
EQ
10S
20S
30S
40S
140W
120W
100W
80W
60W
40W
20W
0
0.9
0.6
0.4
0.2
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0


Aug/Sep 2009
Level (hPa)
100
200
400
500
600
850
1000
20S
15S
10S
5S
0
5N
10N
15N
20N
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01

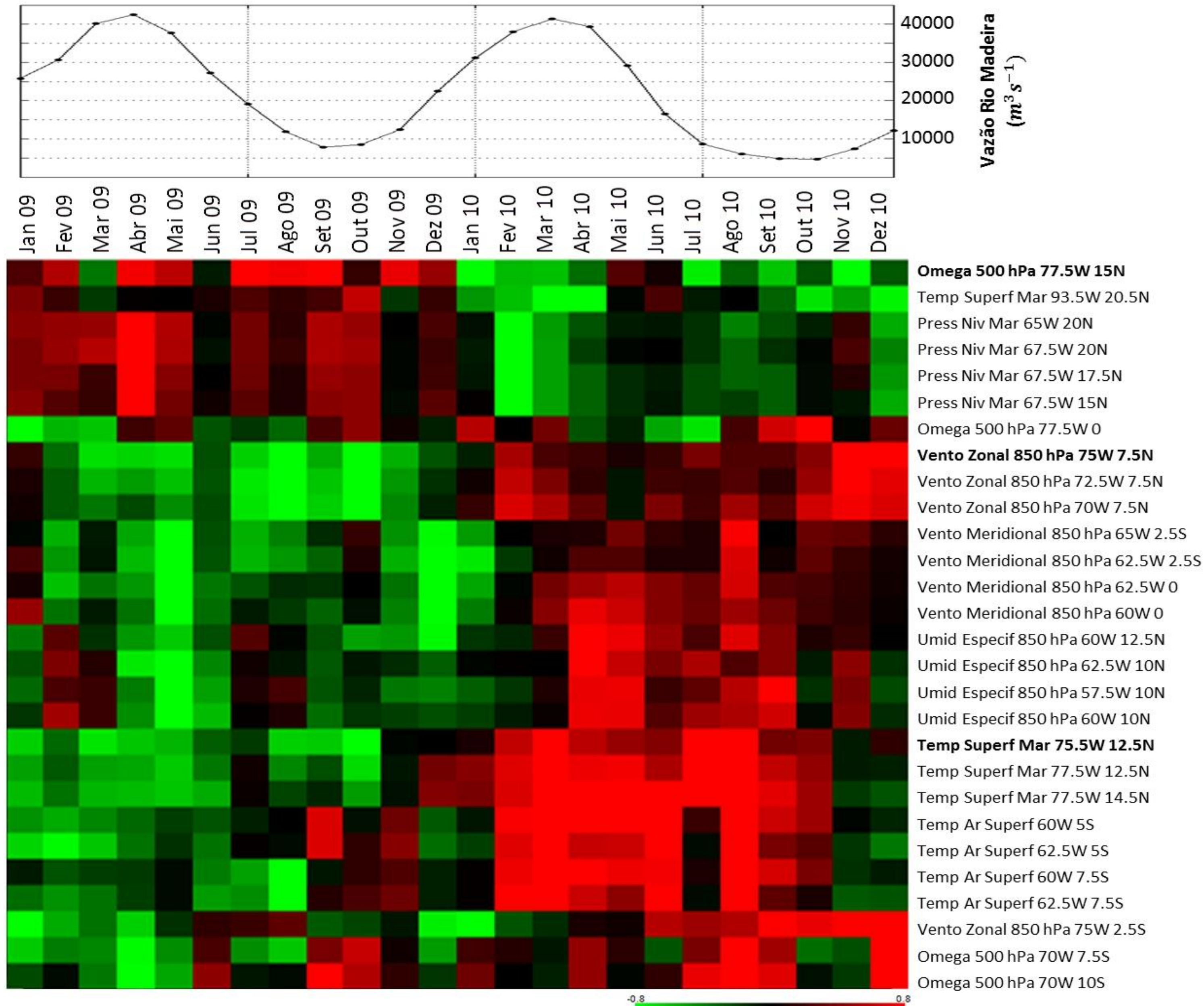

Aug/Sep 2010
Level (hPa)
100
200
400
500
600
850
1000
20S
15S
10S
5S
0
5N
10N
15N
20N
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02


10N
8N
6N
4N
2N
EQ
2S
4S
6S
8S
10S
12S
14S
84W
81W
78W
75W
72W
69W
66W
63W
60W
57W
54W
51W
Ago/Set 2009
Ago/Set 2010
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025



4 variáveis com menores p-valor das figuras anteriores

Amazon drought



4 variáveis com menores p-valor das figuras anteriores

Dimension reduction: Amazon drought

Class-1: p-values < 0,0005 → 104

Class-2: p-values < 0,005 → 182

Class-3: p-values < 0,001 → 825

Class-4: Selecting 10 features with lowest p-value AND features with p-value < 0,001→ 120

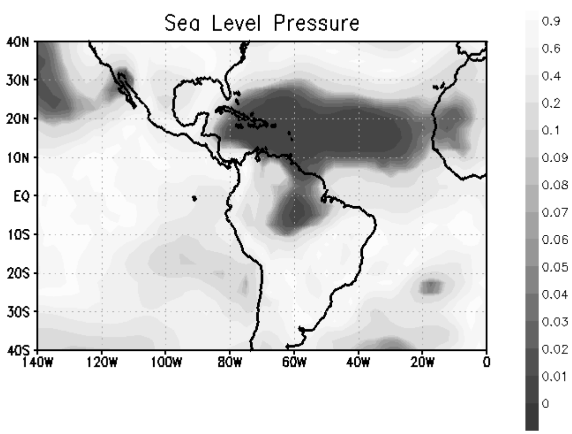
Class-5: Selecting 10 samples for each meteorological feature with smallest p-value → 280

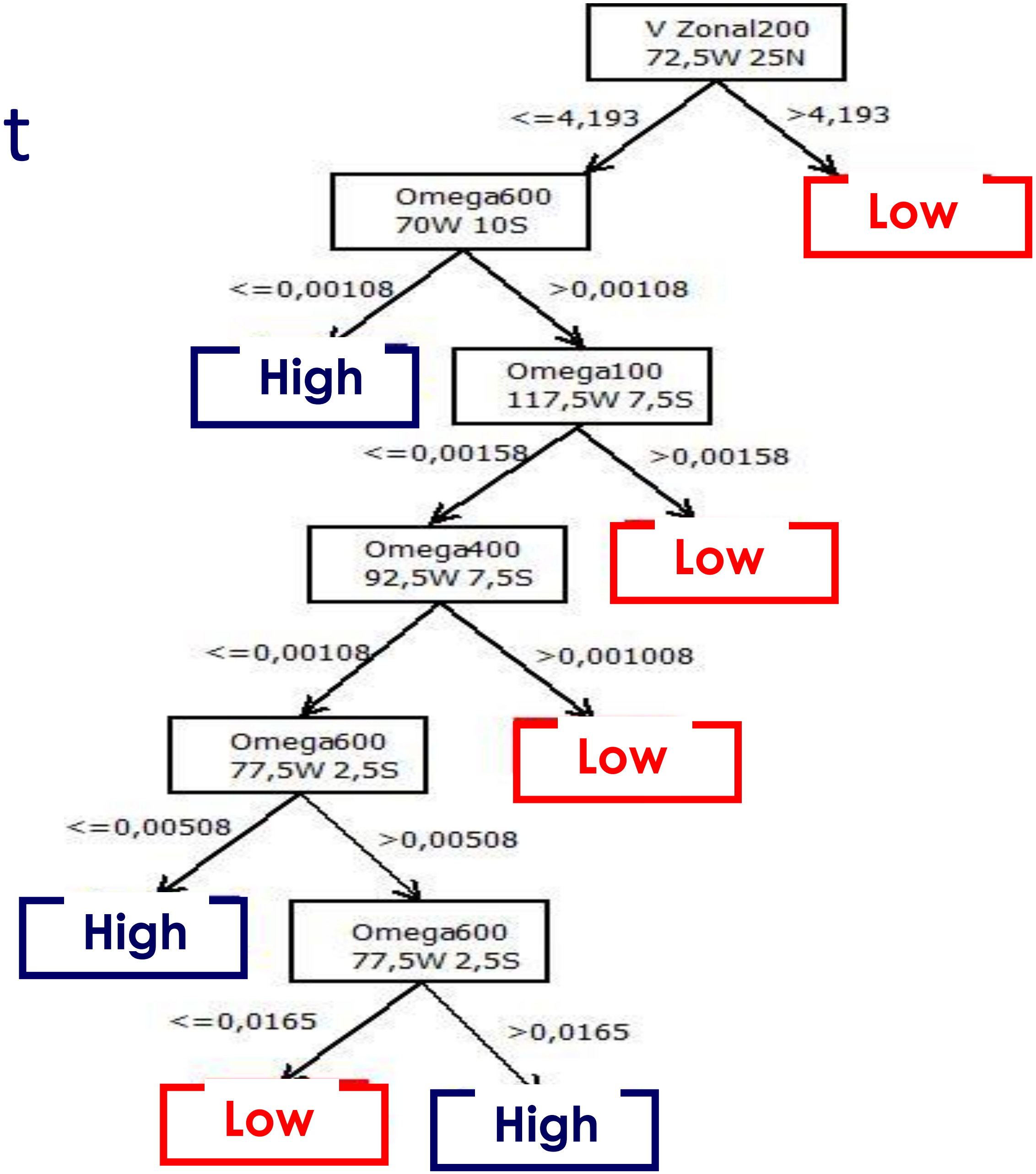
Dimension reduction: 10^8 → 10^2 or 10^3

Decision tree: Amazon drought

Class-4

Decision tree generated by J4.8 algorithm





Dimension reduction: Intense rain fall

Class-1: 50 features with lowest p-values → 104

Class-2: p-values < 0,001 → 179

Class-4: Selecting 10 features with lowest p-value AND features with p-value < 0,001→ 94

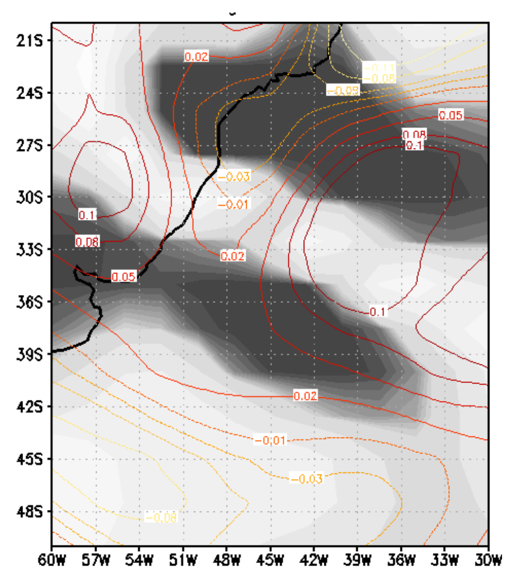
Class-5: Selecting 10 samples for each meteorological feature with smallest p-value → 50

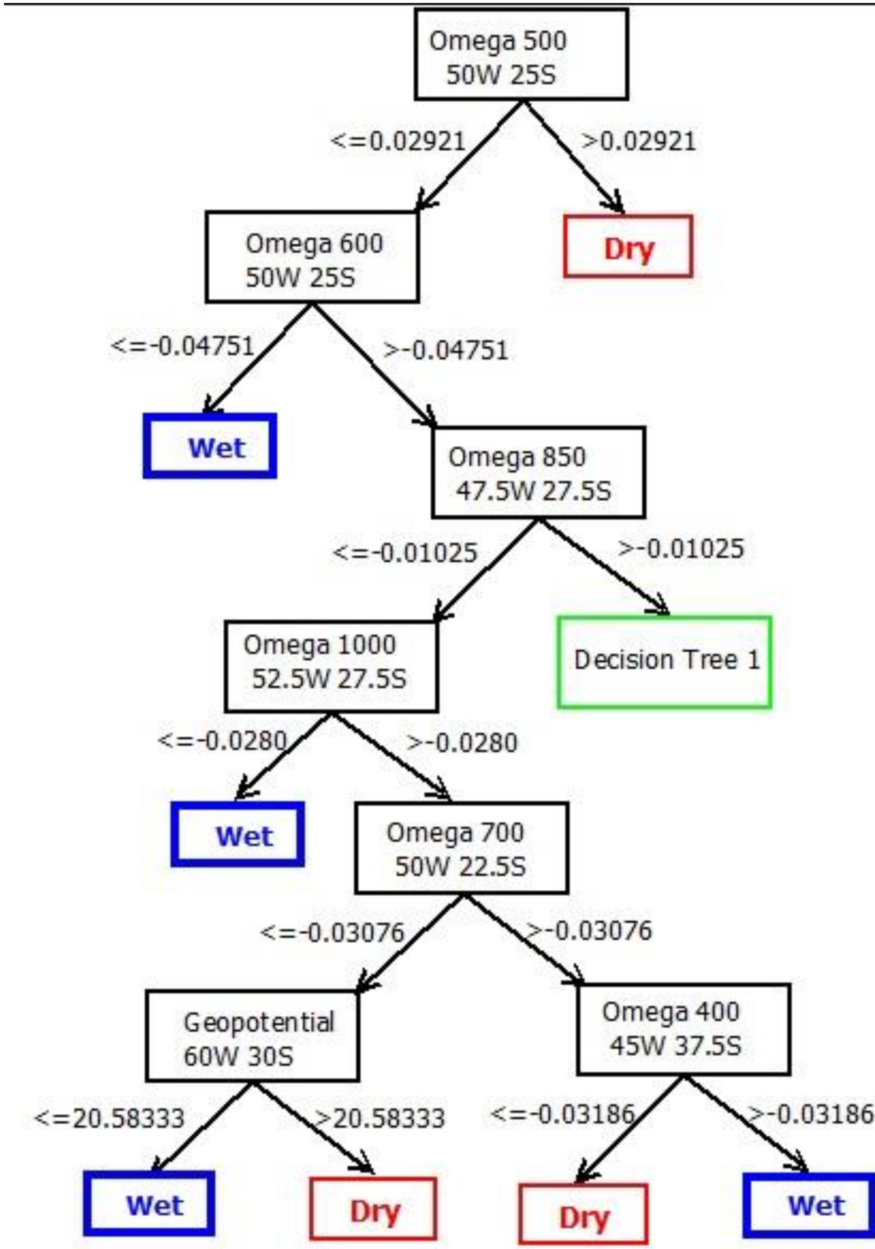
Dimension reduction: 10^8 → 10^2

Decision tree: Rain-fall

Class-4

Decision tree generated by J4.8 algorithm





Climate change research

- Predictability

Climate change research

- Predictability (… or quantifying uncertainties)
- What is that?
  - A set of PDE could be a prediction system
  - Question-1: How can I solve the PDE’s?

    Answering Question-1: Prediction
  - Question-2: How good is the prediction?

    Answering Question-2: Predictability
  - Predictability: quantifying uncertainties (computing the confidence interval)

Climate change research

- Predictability (… or quantifying uncertainties)
- Ensemble prediction
  - Time-integration of a set of initial conditions
  - From ensemble: statistical properties are computed
  - From the statistical properties:

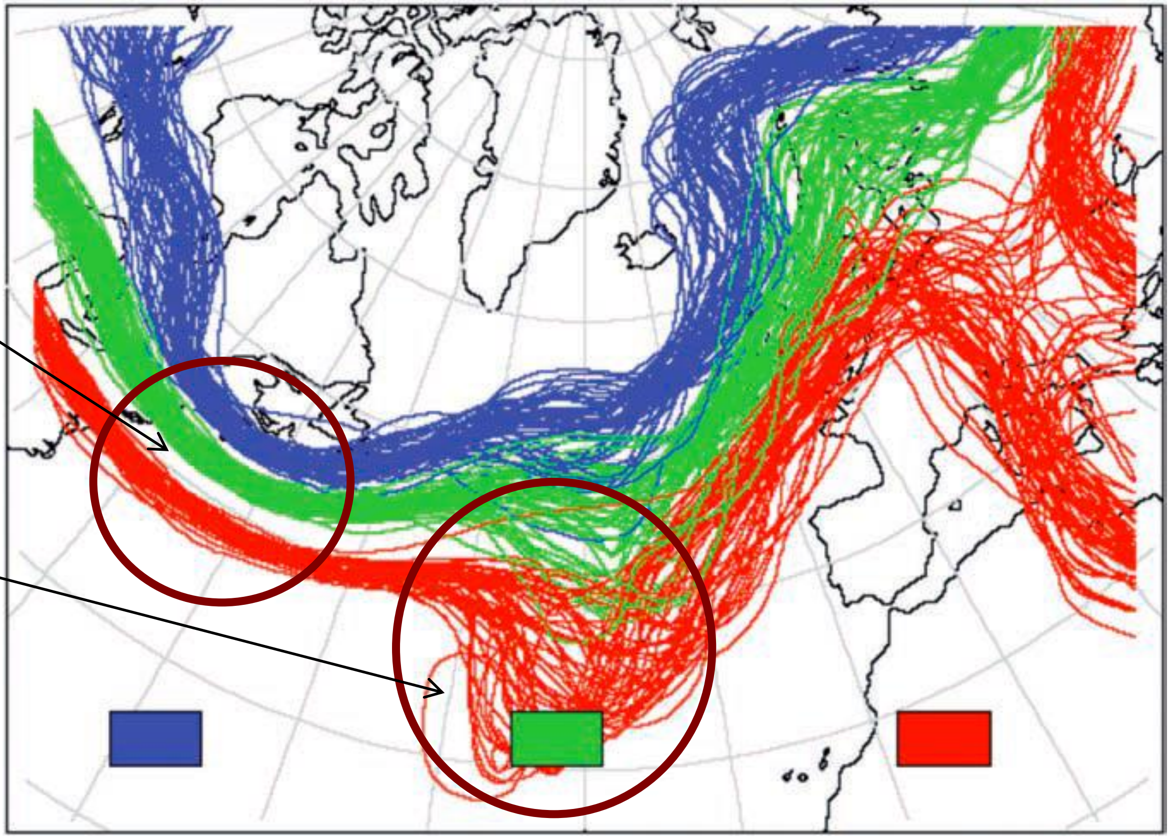
    A confidence interval can be calculated

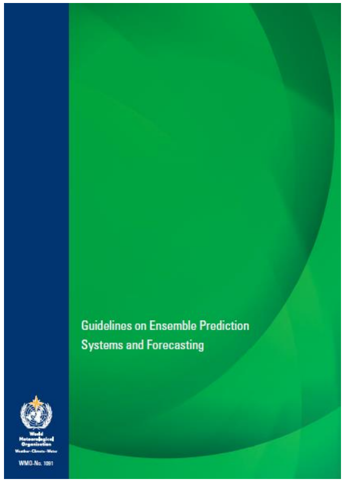
Climate change research

- Ensemble prediction

High predictability
(ensemble convergence)

Low predictability
(ensemble dispersion)





WMO's report describing/suggesting ensemble prediction

Climate change research

- Ensemble prediction
  - Computing the confidence interval