Lecture 3

Complex hyperbolic applications

Course *Block-structured Adaptive Mesh Refinement in C++*

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Outline

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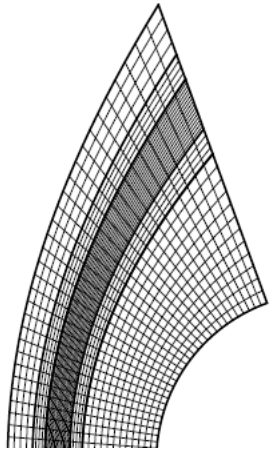
SAMR on boundary aligned meshes

Analytic or stored geometric mapping of the coordinates (graphic from [Yamaleev and Carpenter, 2002])

- Topology remains unchanged and thereby entire SAMR algorithm
- Patch solver and interpolation need to consider geometry transformation
- Handles boundary layers well



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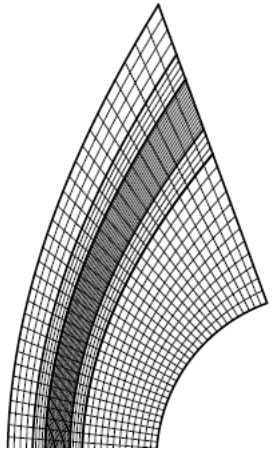

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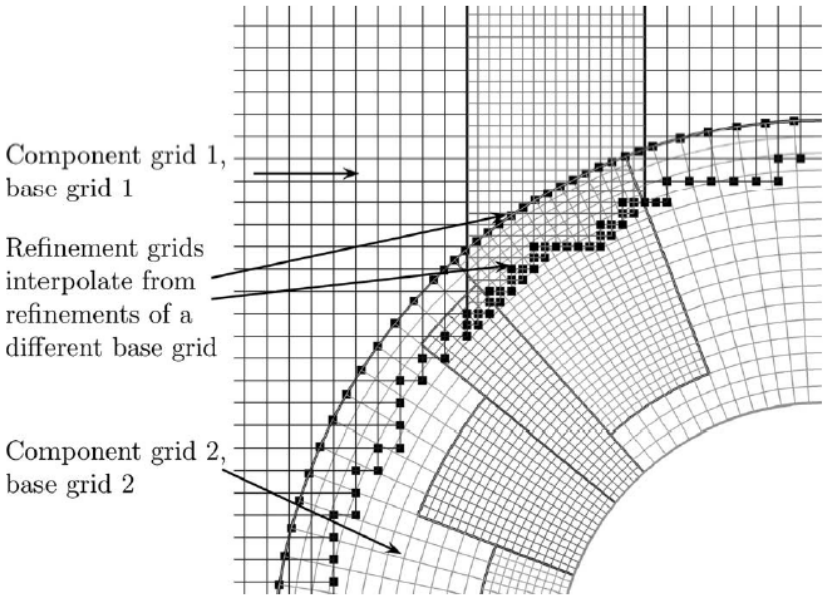


Overlapping adaptive meshes [Henshaw and Schwendeman, 2003], [Meakin, 1995]

- Idea is to use a non-Cartesian structured grids only near boundary
- Very suitable for moving objects with boundary layers
- Interpolation between meshes is usually non-conservative

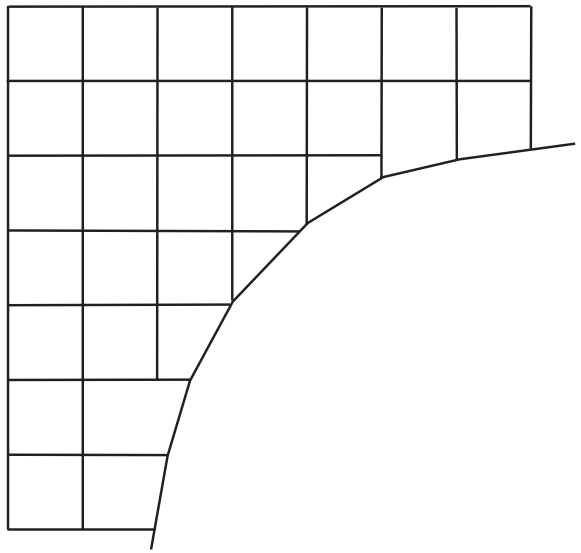
Cut-cell techniques

Accurate embedded boundary method

$$V_j^{n+1}\mathbf{Q}_j^{n+1} = V_j^n\mathbf{Q}_j^n - \Delta t \left(A_{j+1/2}^{n+1/2}\,\mathbf{F}(\mathbf{Q}, j) - A_{j-1/2}^{n+1/2}\,\mathbf{F}(\mathbf{Q}, j-1) \right)$$

Methods that represent the boundary sharply:

- Cut-cell approach constructs appropriate finite volumes
- Conservative by construction. Correct boundary flux



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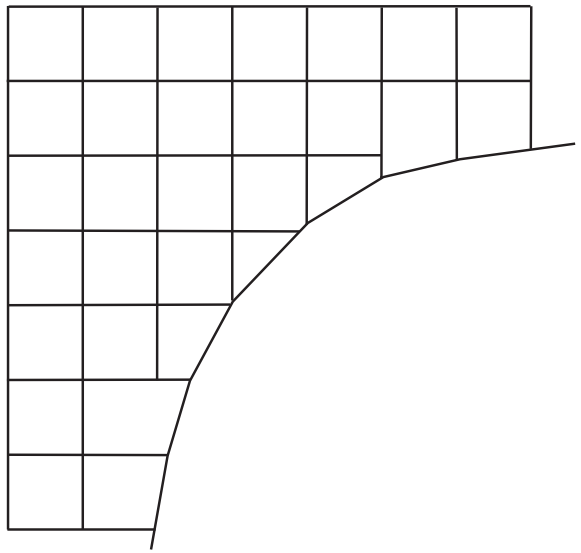

Methods that represent the boundary sharply:

- Cut-cell approach constructs appropriate finite volumes
- Conservative by construction. Correct boundary flux
- Key question: How to avoid small-cell time step restriction in explicit metyhods?
  - Cell merging: [Quirk, 1994a]
- Usually explicit geometry representation used [Aftosmis, 1997], but can also be implicit, cf. [Nourgaliev et al., 2003], [Murman et al., 2003]

Embedded boundary techniques

Volume of fluid methods that resemble a cut-cell technique on purely Cartesian mesh

- Redistribution of boundary flux achieves conservation and bypasses time step restriction: [Pember et al., 1999], [Berger and Helzel, 2002]

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Methods that diffuse the boundary in one cell (good overview in [Mittal and Iaccarino, 2005]):

- Related to the immersed boundary method by Peskin, cf. [Roma et al., 1999]
- Boundary prescription often by internal ghost cell values, cf. [Tseng and Ferziger, 2003]
- Not conservative by construction but conservative correction possible
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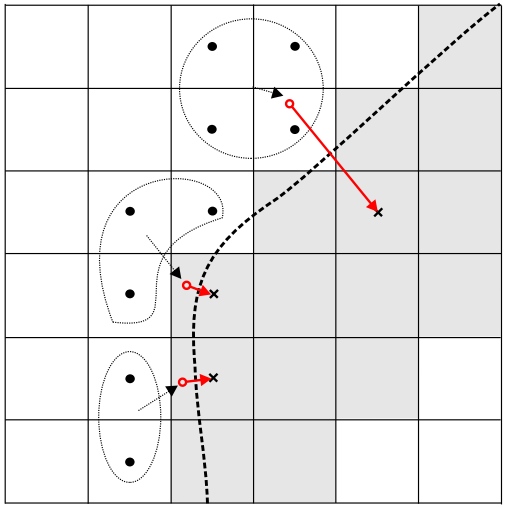
K. J. Richards et al., On the use of the immersed boundary method for engine modeling

Level-set method for boundary embedding



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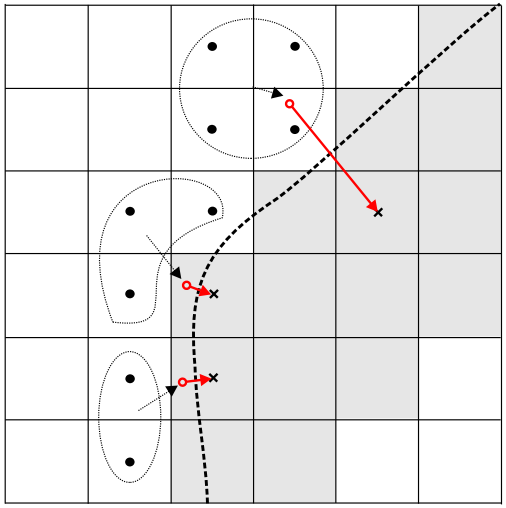

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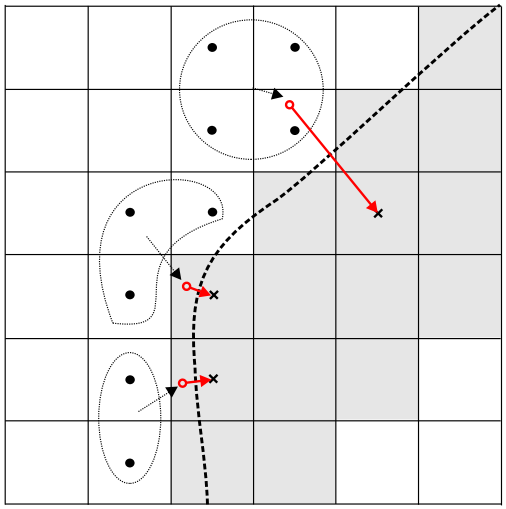

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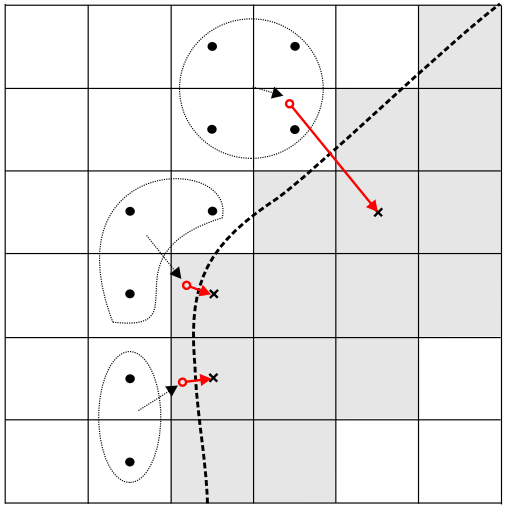

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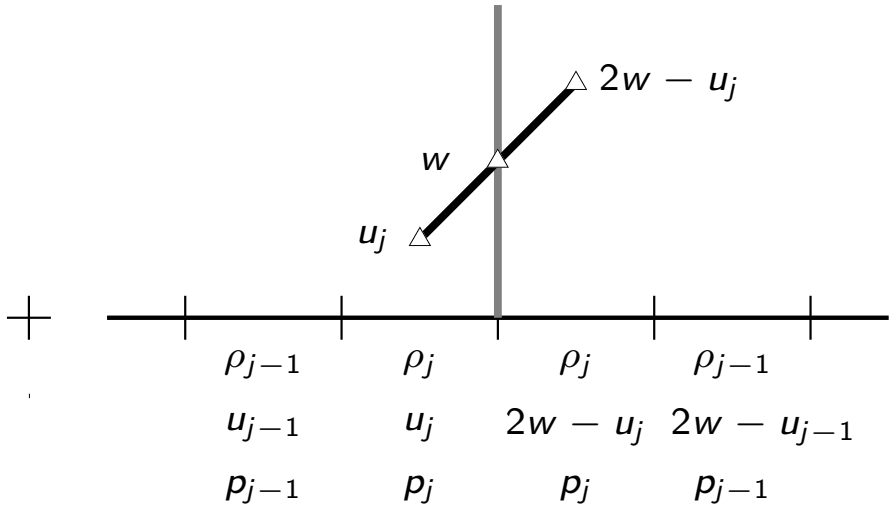
Interpolate / constant value extrapolate values at

$$\tilde{\mathbf{x}} = \mathbf{x} + 2\varphi\mathbf{n}$$



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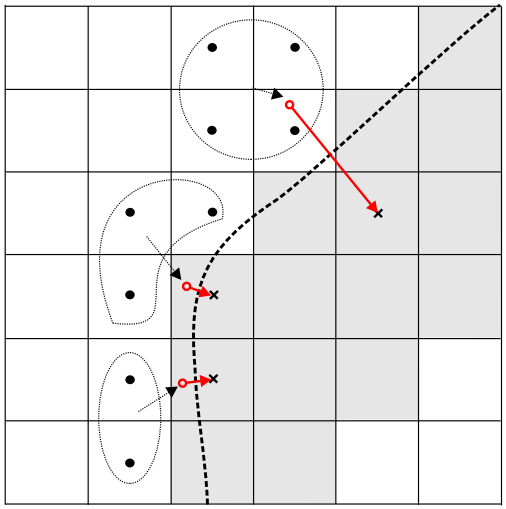

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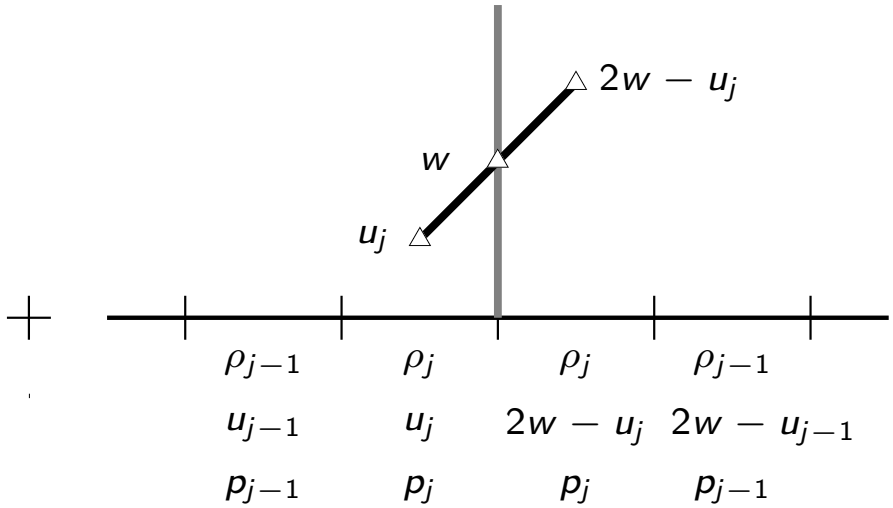
Velocity in ghost cells

$$\begin{aligned}\mathbf{u}' &= (2\mathbf{w}\cdot\mathbf{n} - \mathbf{u}\cdot\mathbf{n})\mathbf{n} + (\mathbf{u}\cdot\mathbf{t})\mathbf{t} \\ &= 2\left((\mathbf{w}-\mathbf{u})\cdot\mathbf{n}\right)\mathbf{n} + \mathbf{u}\end{aligned}$$



Closest point transform algorithm

The signed distance φ to a surface $\mathcal{I}$ satisfies the eikonal equation [Sethian, 1999]

$$|\nabla\varphi| = 1 \quad \text{with} \quad \varphi\big|_{\mathcal{I}} = 0$$

Solution smooth but non-diferentiable across characteristics.

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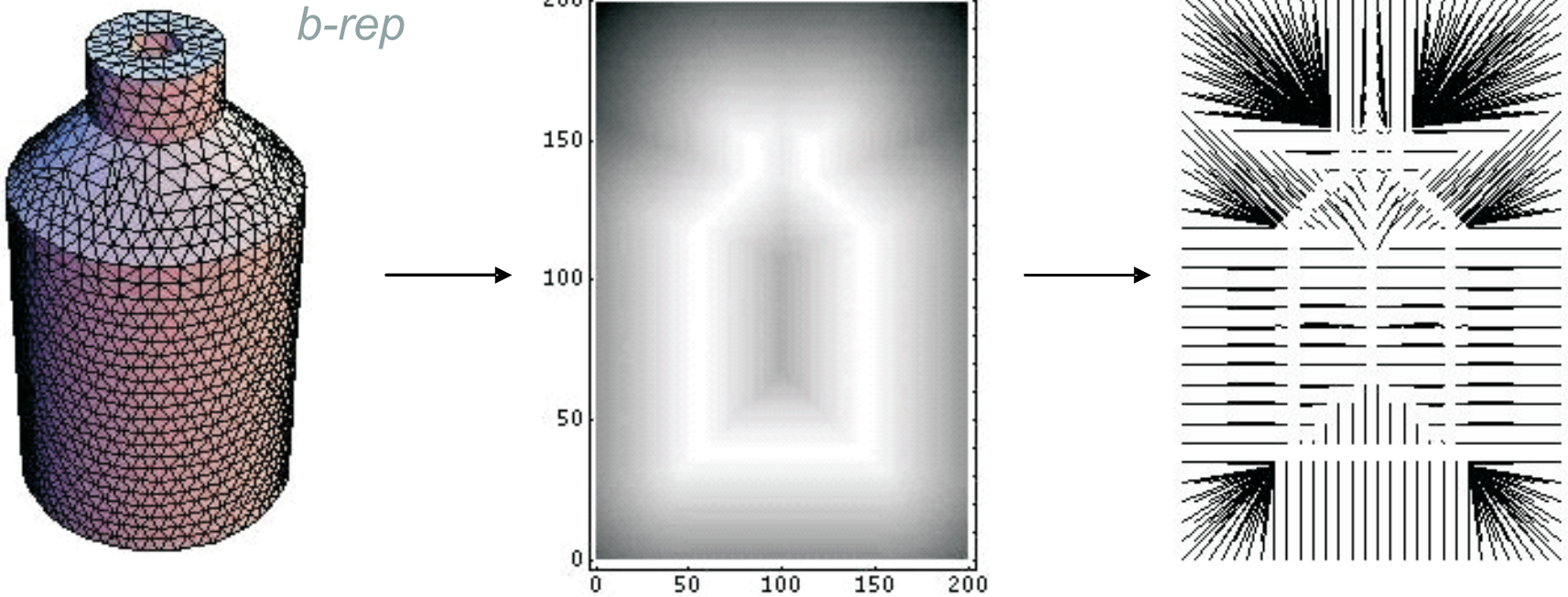
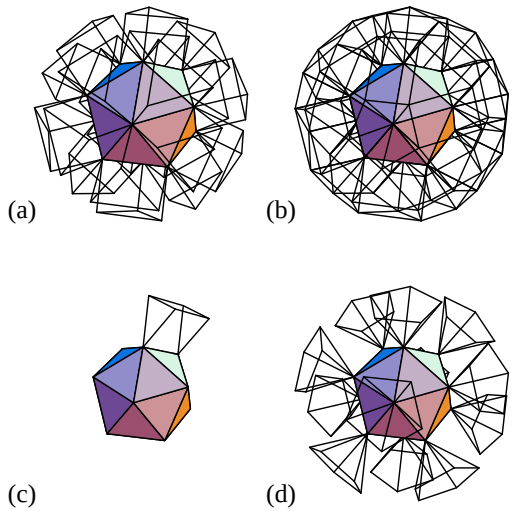


Surface mesh $\mathcal{I}$ Distance φ Normal to closest point

The characteristic / scan conversion algorithm

1. Build the characteristic polyhedrons for the surface mesh

Characteristic polyhedra for faces, edges, and vertices



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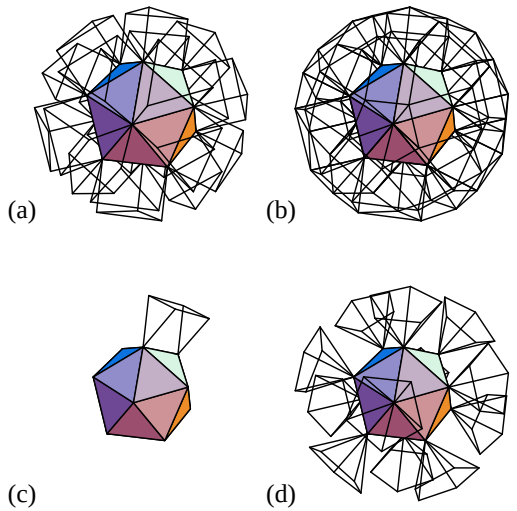
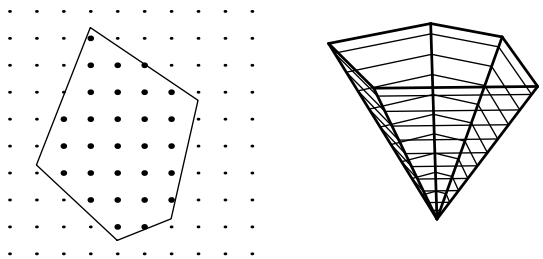


Slicing and scan conversion of apolygon



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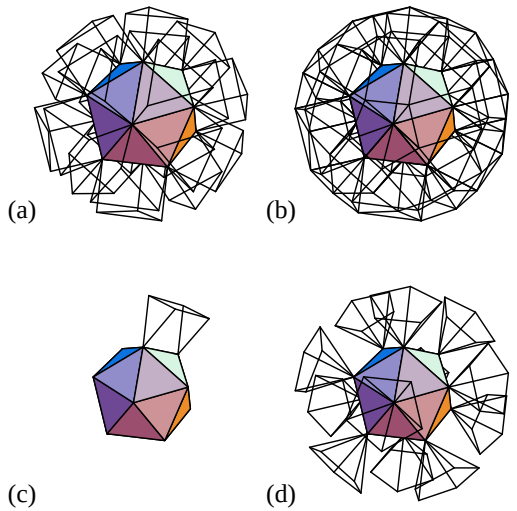

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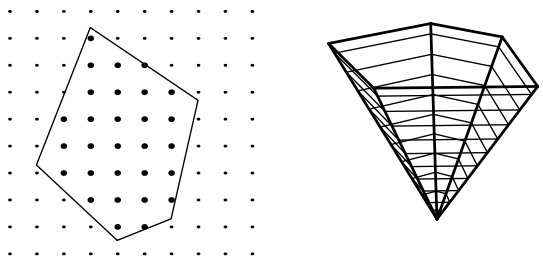

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   - $O(m)$ to build the b-rep and the polyhedra.
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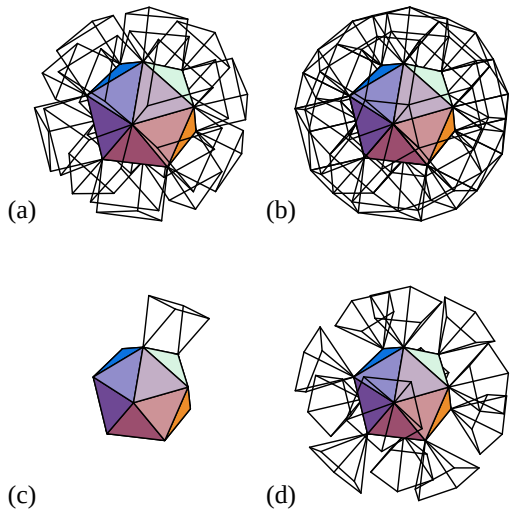

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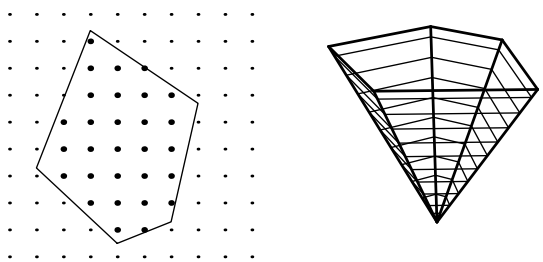

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4. Problem reduction by evaluation only within specified max. distance

[Mauch, 2003], see also
[Deiterding et al., 2006]

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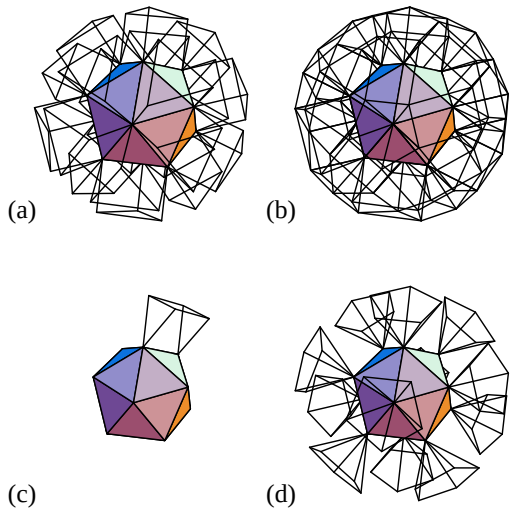

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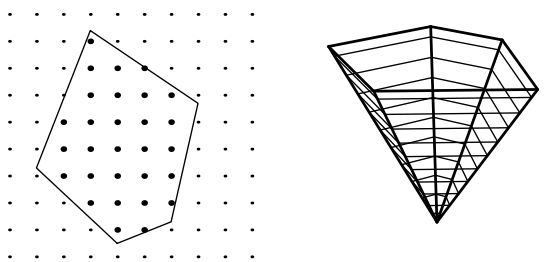


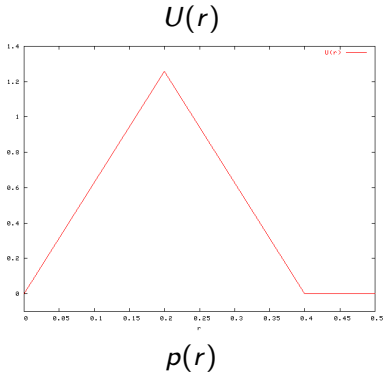
Accuracy test: stationary vortex

Construct non-trivial *radially symmetric* and *stationary* solution by balancing hydrodynamic pressure and centripetal force per volume element, i.e.

$$\frac{d}{dr}p(r) = \rho(r)\frac{U(r)^2}{r}$$

For $\rho_0 \equiv 1$ and the velocity field

$$U(r) = \alpha \cdot \begin{cases} 2r/R & \text{if } 0 < r < R/2, \\ 2(1 - r/R) & \text{if } R/2 \leq r \leq R, \\ 0 & \text{if } r > R, \end{cases}$$



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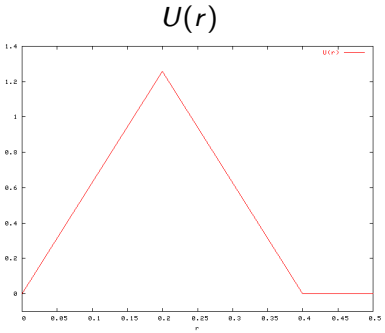

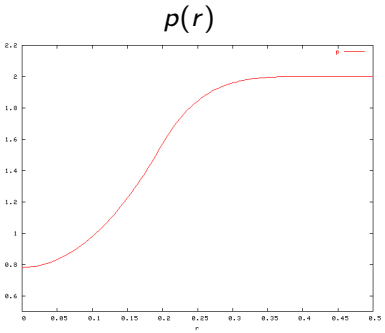

one gets with boundary condition $p(R) = p_0 \equiv 2$ the pressure distribution

$$p(r) = p_0 + 2\rho_0\alpha^2 \cdot \begin{cases} r^2/R^2 + 1 - 2\log 2 & \text{if } 0 < r < R/2, \\ r^2/R^2 + 3 - 4r/R + 2\log(r/R) & \text{if } R/2 \le r \le R, \\ 0 & \text{if } r > R. \end{cases}$$

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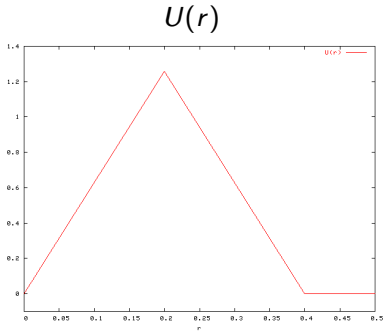

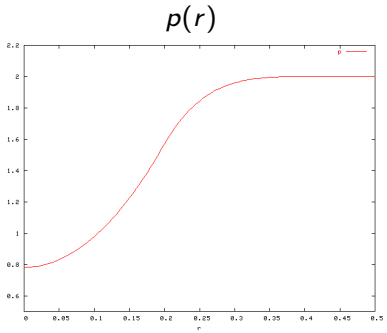

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Entire solution for Euler equations reads

$$\rho(x_1, x_2, t) = \rho_0,\ u_1(x_1, x_2, t) = -U(r)\sin\phi,\ u_2(x_1, x_2, t) = U(r)\cos\phi,\ p(x_1, x_2, t) = p(r)$$

for all $t \ge 0$ with $r = \sqrt{(x_1 - x_{1,c})^2 + (x_2 - x_{2,c})^2}$ and $\phi = \arctan\dfrac{x_2 - x_{2,c}}{x_1 - x_{1,c}}$

Stationary vortex: results

Compute one full rotation, Roe solver, embedded slip wall boundary conditions
$x_{1,c} = 0.5,\ x_{2,c} = 0.5,\ R = 0.4,\ t_{end} = 1,\ \Delta h = \Delta x_1 = \Delta x_2 = 1/N,\ \alpha = R\pi$

No embedded boundary

N	Wave Propagation		Godunov Splitting	
	Error	Order	Error	Order
20	0.0111235		0.0182218	
40	0.0037996	1.55	0.0090662	1.01
80	0.0013388	1.50	0.0046392	0.97
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Marginal shear flow along embedded boundary, $\alpha = R\pi,\ R_G = R,\ U_W = 0$

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	Error	Order	Mass loss	Error	Order	Mass loss
20	0.0120056		0.0079236	0.0144203		0.0020241
40	0.0035074	1.78	0.0011898	0.0073070	0.98	0.0001300
80	0.0014193	1.31	0.0001588	0.0038401	0.93	-0.0001036
160	0.0005032	1.50	5.046e-05	0.0018988	1.02	-2.783e-06

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20	0.0423925		0.0423925	0.0271446		0.0271446
40	0.0358735	0.24	0.0358735	0.0242260	0.16	0.0242260
80	0.0212340	0.76	0.0212340	0.0128638	0.91	0.0128638
160	0.0121089	0.81	0.0121089	0.0070906	0.86	0.0070906

Verification: shock reflection

- Reflection of a Mach 2.38 shock in nitrogen at 43^o wedge
- 2nd order MUSCL scheme with Roe solver, 2nd order multidimensional wave propagation method

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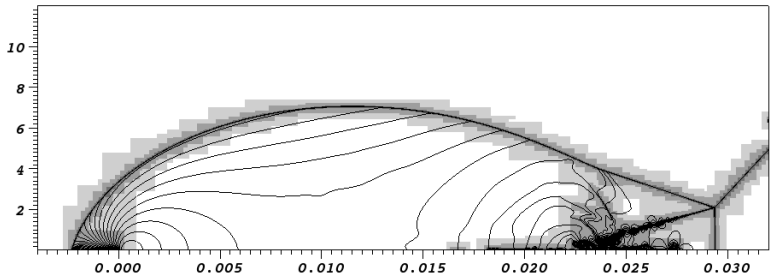
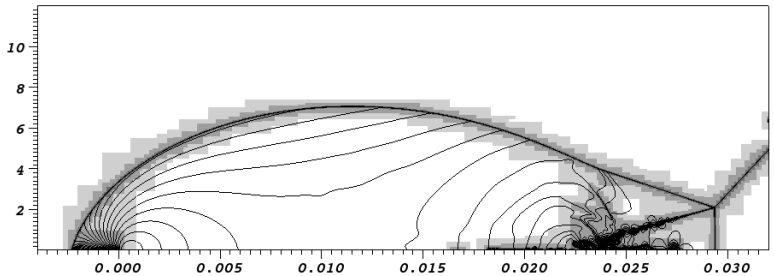


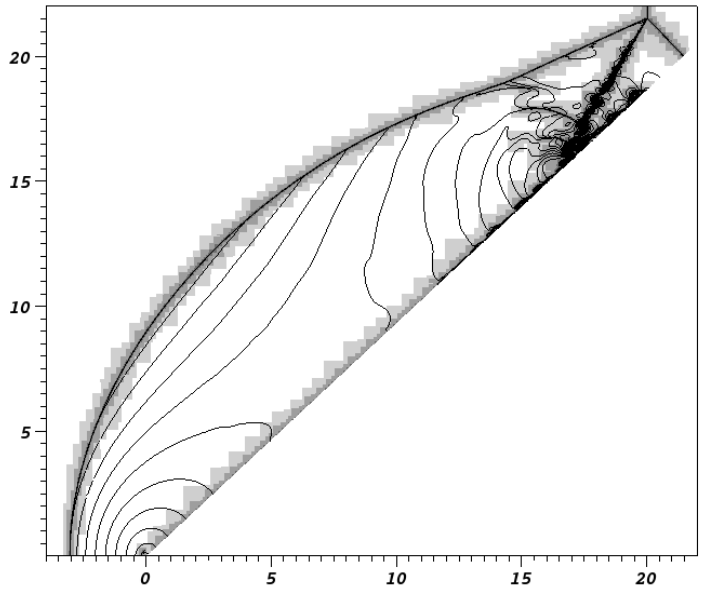
Cartesian base grid 360×160 cells on domain of $36\,\mathrm{mm} \times 16\,\mathrm{mm}$ with up to 3 refinement levels with $r_l = 2, 4, 4$ and $\Delta x_{1,2} = 3.125 \mu m$, $38\,\mathrm{h}$ CPU

Verification: shock reflection

- Reflection of a Mach 2.38 shock in nitrogen at 43^o wedge
- 2nd order MUSCL scheme with Roe solver, 2nd order multidimensional wave propagation method

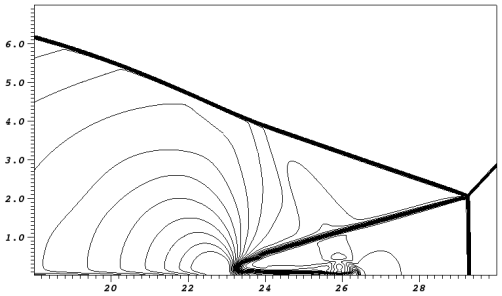


Cartesian base grid 360 $\times$ 160 cells on domain of $36\,\mathrm{mm} \times 16\,\mathrm{mm}$ with up to 3 refinement levels with $r_l = 2, 4, 4$ and $\Delta x_{1,2} = 3.125 \mu m$, 38 h CPU

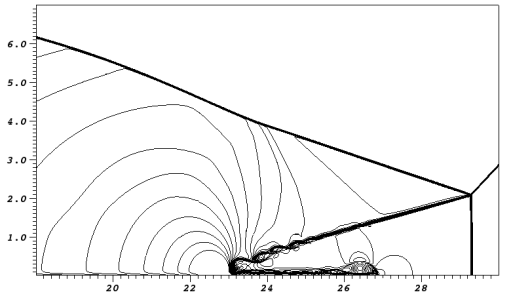


GFM base grid 390 $\times$ 330 cells on domain of $26\,\mathrm{mm} \times 22\,\mathrm{mm}$ with up to 3 refinement levels with $r_l = 2, 4, 4$ and $\Delta x_{e,1,2} = 2.849 \mu m$, 200 h CPU

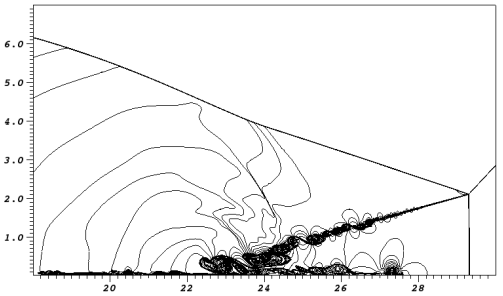
Shock reflection: SAMR solution for Euler equations



$\Delta x = 25\,\mathrm{mm}$

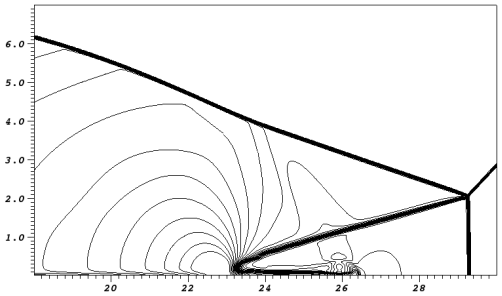


$\Delta x = 12.5\,\mathrm{mm}$



$\Delta x = 3.125\,\mathrm{mm}$

Shock reflection: SAMR solution for Euler equations



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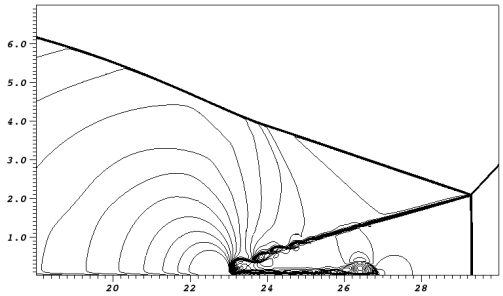
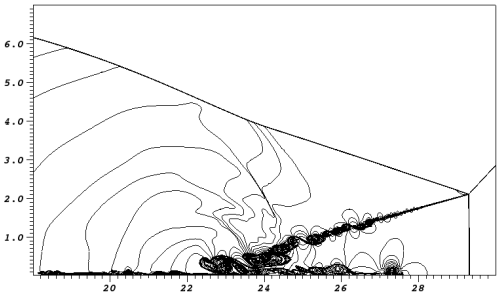


$\Delta x = 12.5\,\mathrm{mm}$



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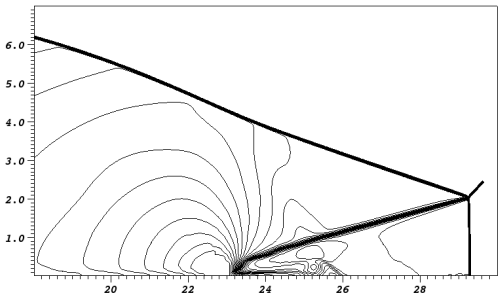
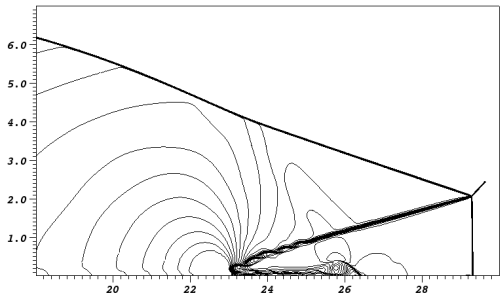
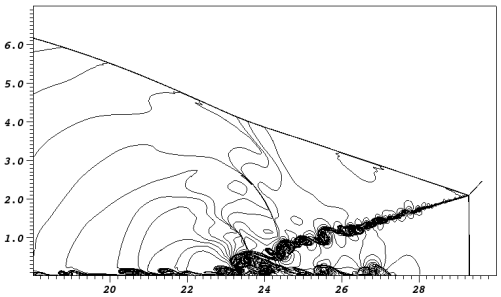


$\Delta x_e = 22.8\,\mathrm{mm}$

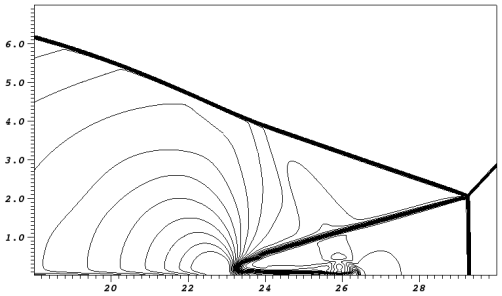


$\Delta x_e = 11.4\,\mathrm{mm}$

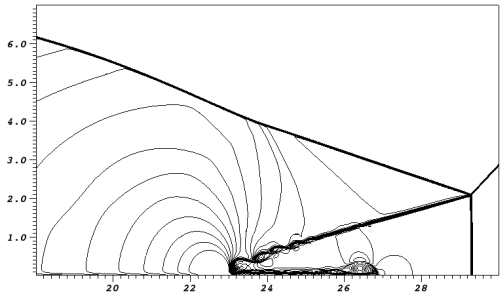


$\Delta x_e = 2.849\,\mathrm{mm}$

Shock reflection: SAMR solution for Euler equations



$\Delta x = 25\,\mathrm{mm}$



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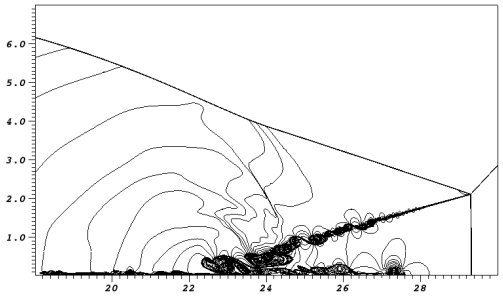

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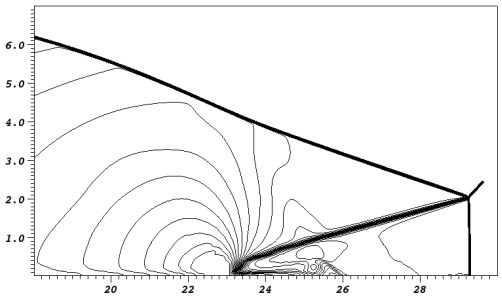

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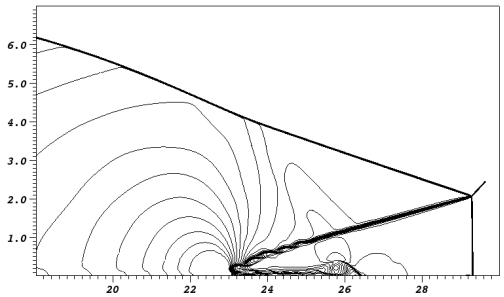

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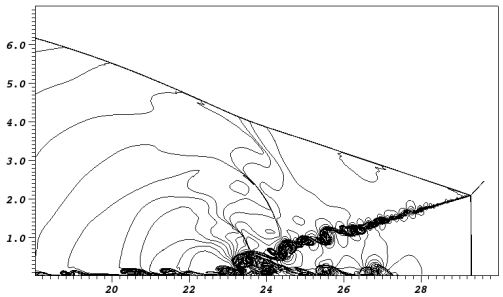
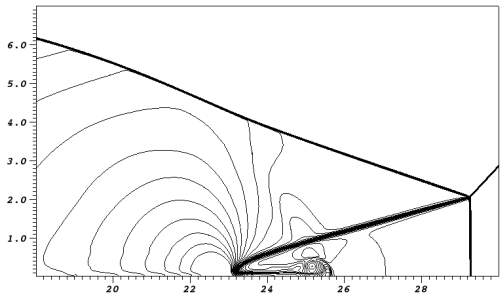


$\Delta x_e = 2.849\,\mathrm{mm}$

2nd order MUSCL scheme with Van Leer FVS, dimensional splitting



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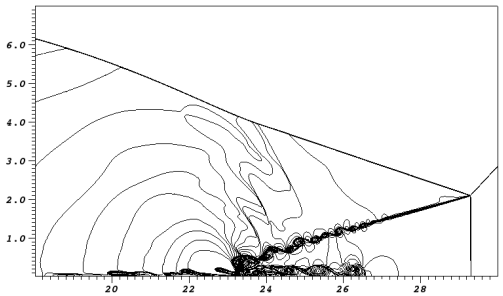

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Shock reflection: solution for Navier-Stokes equations

- No-slip boundary conditions enforced
- Conservative 2nd order centered differences to approximate stress tensor and heat flow

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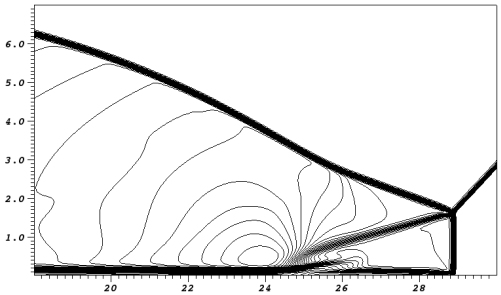
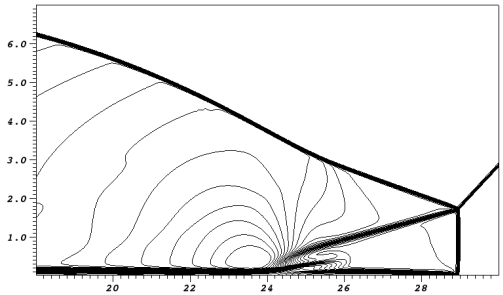


$\Delta x = 50\,\mathrm{mm}$



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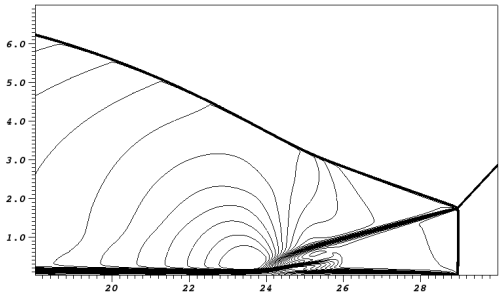

$\Delta x = 12.5\,\mathrm{mm}$, SAMR

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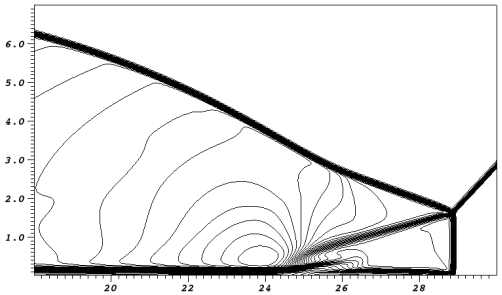

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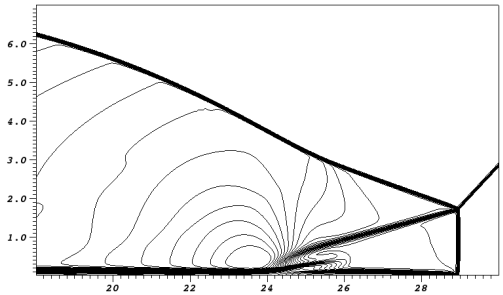

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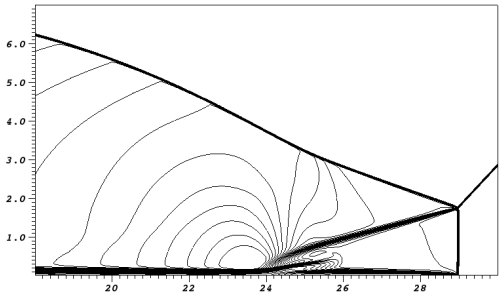
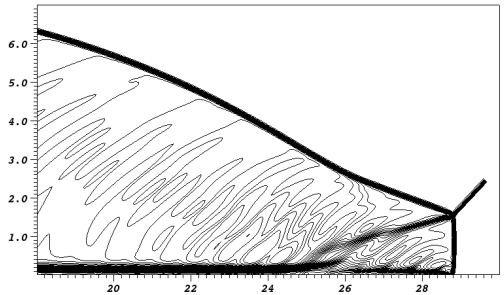
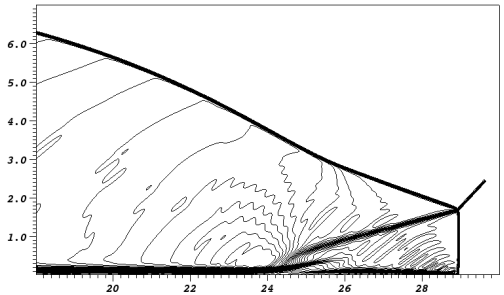


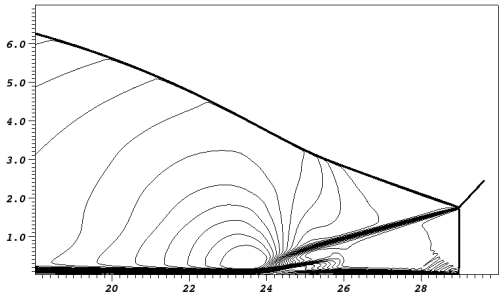
$\Delta x = 12.5\,\mathrm{mm}$, SAMR



$\Delta x_e = 45.6\,\mathrm{mm}$



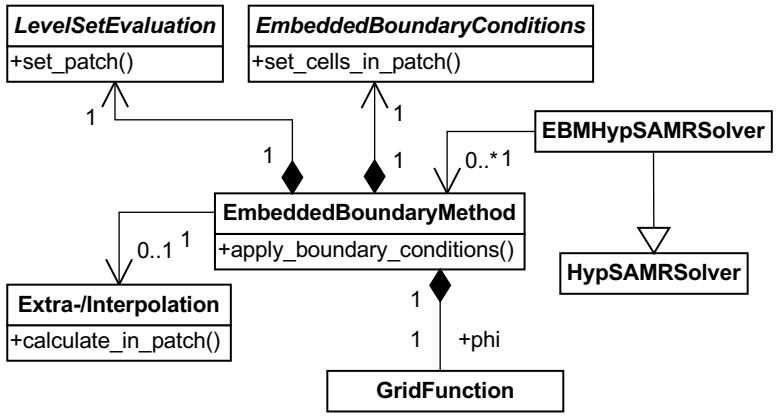
$\Delta x_e = 22.8\,\mathrm{mm}$



$\Delta x_e = 11.4\,\mathrm{mm}$, SAMR

Embedded boundary method

- Multiple independent EmbeddedBoundaryMethod objects possible
- Specialization of GFM boundary conditions



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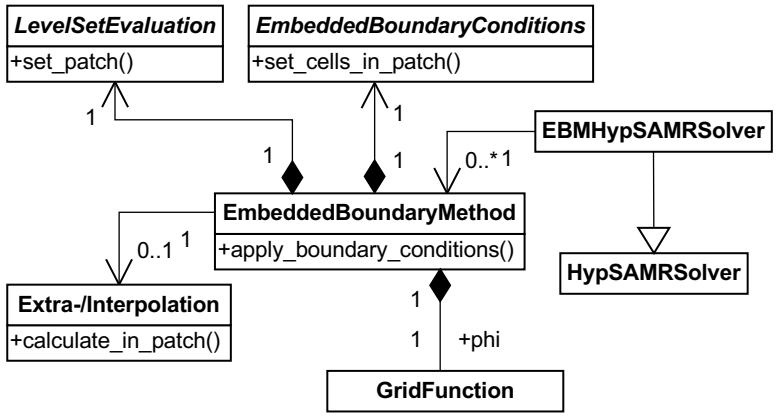

- Multiple independent EmbeddedBoundaryMethod objects possible
- Specialization of GFM boundary conditions
- The generic embedded boundary method is implemented in **GhostFluidMethod<VectorType, dim >** and has a **GFMLevelSet<DataType, dim >** and **GFMBoundary<VectorType, dim >** object.

  `code/amroc/doc/html/amr/classGhostFluidMethod.html` `code/amroc/doc/html/amr/classGFMLevelSet.html`

  `code/amroc/doc/html/amr/classGFMBoundary_3_01VectorType_00_012_01_4.html`
- Multiple **GhostFluidMethod<VectorType, dim >** can be registered with **AMRGFMSolver<VectorType, FixupType, FlagType, dim >** and are called as part of the boundary condition setting routine.

  `code/amroc/doc/html/amr/classAMRGFMSolver.html`
- Interface classes to **GFMLevelSet<DataType, dim >** and **GFMBoundary<VectorType, dim >** are available amroc/amr/F77Interfaces and in amroc/amr/Interfaces

  `code/amroc/doc/html/amr/classF77GFMBoundary.html` `code/amroc/doc/html/amr/classSchemeGFMBoundary.html` make the approach available to all current solvers
- For instance `code/amroc/doc/html/clp/ClpStdGFMProblem_8h.html` or `code/amroc/doc/html/weno/WENOStdGFMProblem_8h.html` in Problem.h uses AMRGFMSolver<>

Outline

Governing equations for premixed combustion

Euler equations with reaction terms

$$\frac{\partial \rho_i}{\partial t} + \frac{\partial}{\partial x_n}\left(\rho_i u_n\right) = \dot{\omega}_i \ , \quad i = 1, \ldots, K$$

$$\frac{\partial}{\partial t}\left(\rho u_k\right) + \frac{\partial}{\partial x_n}\left(\rho u_k u_n + \delta_{kn} p\right) = 0 \ , \quad k = 1, \ldots, d$$

$$\frac{\partial}{\partial t}\left(\rho E\right) + \frac{\partial}{\partial x_n}\left(u_n(\rho E + p)\right) = 0$$

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Ideal gas law and Dalton's law for gas-mixtures

$$p(\rho_1, \ldots, \rho_K, T) = \sum_{i=1}^{K} p_i = \sum_{i=1}^{K} \rho_i \frac{\mathcal{R}}{W_i} T = \rho \frac{\mathcal{R}}{W} T \quad \text{with} \quad \sum_{i=1}^{K} \rho_i = \rho \, , \, Y_i = \frac{\rho_i}{\rho}$$

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Caloric equation

$$h(Y_1, \ldots, Y_K, T) = \sum_{i=1}^{K} Y_i h_i(T) \quad \text{with} \quad h_i(T) = h_i^0 + \int_0^T c_{pi}(s) ds$$

Governing equations for premixed combustion

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Computation of $T = T(\rho_1, \ldots, \rho_K, e)$ from implicit equation

$$\sum_{i=1}^{K} \rho_i \, h_i(T) - \mathcal{R} T \sum_{i=1}^{K} \frac{\rho_i}{W_i} - \rho e = 0$$

for *thermally perfect* gases with $\gamma_i(T) = c_{pi}(T)/c_{vi}(T)$

Chemistry

Arrhenius-Kinetics:

$$\dot{\omega}_i = \sum_{j=1}^{M} (\nu_{ji}^r - \nu_{ji}^f) \left[k_j^f \prod_{n=1}^{K} \left(\frac{\rho_n}{W_n} \right)^{\nu_{jn}^f} - k_j^r \prod_{n=1}^{K} \left(\frac{\rho_n}{W_n} \right)^{\nu_{jn}^r} \right] \quad i = 1, \ldots, K$$

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- Parsing of mechanisms with Chemkin-II
- Evalutation of $\dot{\omega}_i$ with automatically generated optimized Fortran-77 functions in the line of Chemkin-II

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- Parsing of mechanisms with Chemkin-II
- Evalutation of $\dot{\omega}_i$ with automatically generated optimized Fortran-77 functions in the line of Chemkin-II

Integration of reaction rates: ODE integration in $\mathcal{S}^{(\cdot)}$ for Euler equations with chemical reaction

- Standard implicit or semi-implicit ODE-solver subcycles within each cell
- ρ, e, u_k remain unchanged!

$$\partial_t \rho_i = W_i \dot{\omega}_i(\rho_1, \ldots, \rho_K, T) \qquad i = 1, \ldots, K$$

Use Newton or bisection method to compute T iteratively.

Non-equilibrium mechanism for hydrogen-oxgen combustion

				A [cm, mol, s]	β	E_{act} [cal mol^{-1}]
1.	$H + O_2$	$\longrightarrow$	$O + OH$	1.86×10^{14}	0.00	16790.
2.	$O + OH$	$\longrightarrow$	$H + O_2$	1.48×10^{13}	0.00	680.
3.	$H_2 + O$	$\longrightarrow$	$H + OH$	1.82×10^{10}	1.00	8900.
4.	$H + OH$	$\longrightarrow$	$H_2 + O$	8.32×10^{09}	1.00	6950.
5.	$H_2O + O$	$\longrightarrow$	$OH + OH$	3.39×10^{13}	0.00	18350.
6.	$OH + OH$	$\longrightarrow$	$H_2O + O$	3.16×10^{12}	0.00	1100.
7.	$H_2O + H$	$\longrightarrow$	$H_2 + OH$	9.55×10^{13}	0.00	20300.
8.	$H_2 + OH$	$\longrightarrow$	$H_2O + H$	2.19×10^{13}	0.00	5150.
9.	$H_2O_2 + OH$	$\longrightarrow$	$H_2O + HO_2$	1.00×10^{13}	0.00	1800.
10.	$H_2O + HO_2$	$\longrightarrow$	$H_2O_2 + OH$	2.82×10^{13}	0.00	32790.
...	...		...	...	...	...
30.	$OH + M$	$\longrightarrow$	$O + H + M$	7.94×10^{19}	-1.00	103720.
31.	$O_2 + M$	$\longrightarrow$	$O + O + M$	5.13×10^{15}	0.00	115000.
32.	$O + O + M$	$\longrightarrow$	$O_2 + M$	4.68×10^{15}	-0.28	0.
33.	$H_2 + M$	$\longrightarrow$	$H + H + M$	2.19×10^{14}	0.00	96000.
34.	$H + H + M$	$\longrightarrow$	$H_2 + M$	3.02×10^{15}	0.00	0.

Third body efficiencies: $f(O_2) = 0.40$, $f(H_2O) = 6.50$

C. K. Westbrook. Chemical kinetics of hydrocarbon oxidation in gaseous detonations. *J. Combustion and Flame*, 46:191–210, 1982.

Riemann solver for combustion

(S1) Calculate standard Roe-averages $\hat{\rho}$, $\hat{u}_n$, $\hat{H}$, $\hat{Y}_i$, $\hat{T}$.

(S2) Compute $\hat{\gamma} := \hat{c}_p/\hat{c}_v$ with $\hat{c}_{\{p/v\}i} = \dfrac{1}{T_R - T_L} \displaystyle\int_{T_L}^{T_R} c_{\{p,v\}i}(\tau)\, d\tau$.

(S3) Calculate $\hat{\phi}_i := (\hat{\gamma} - 1)\left(\frac{\hat{\mathbf{u}}^2}{2} - \hat{h}_i\right) + \hat{\gamma}\, R_i\, \hat{T}$ with standard Roe-averages $\hat{e}_i$ or $\hat{h}_i$.

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$\mathbf{F}_{Roe}(\mathbf{q}_L, \mathbf{q}_R) = \frac{1}{2}\left(\mathbf{f}(\mathbf{q}_L) + \mathbf{f}(\mathbf{q}_R) - \sum_{\iota=1}^{3} |\tilde{s}_\iota|\mathcal{W}_\iota\right)$

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(S11) Positivity correction: Replace $\mathbf{F}_i$ by
$\mathbf{F}^\star_i = \mathbf{F}_\rho \cdot \begin{cases} Y^l_i\,, & \mathbf{F}_\rho \ge 0\,, \\ Y^r_i\,, & \mathbf{F}_\rho < 0\,. \end{cases}$

(S12) Evaluate maximal signal speed by $S = \max(|s_1|, |s_3|)$.

Riemann solver for combustion: carbuncle fix

Entropy corrections

Riemann solver for combustion: carbuncle fix

Entropy corrections [Harten, 1983]

1. $|\tilde{s}_\iota| = \begin{cases} |s_\iota| & \text{if} |s_\iota| \geq 2\eta \\ \frac{|s_\iota^2|}{4\eta} + \eta & \text{otherwise} \end{cases}$

$\eta = \frac{1}{2} \max_\iota \left\{ |s_\iota(\mathbf{q}_R) - s_\iota(\mathbf{q}_L)| \right\}$

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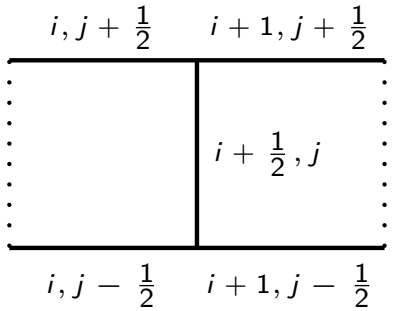
2D modification of entropy correction [Sanders et al., 1998]:



$$\tilde{\eta}_{i+1/2,j} = \max \left\{ \eta_{i+1/2,j}, \eta_{i,j-1/2}, \ \eta_{i,j+1/2}, \eta_{i+1,j-1/2}, \eta_{i+1,j+1/2} \right\}$$

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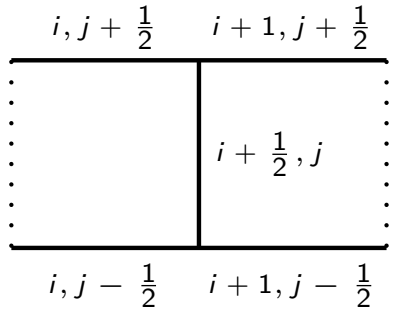

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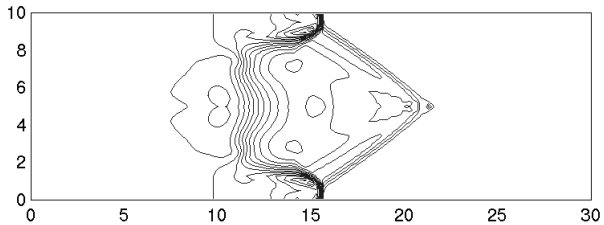
Carbuncle phenomenon

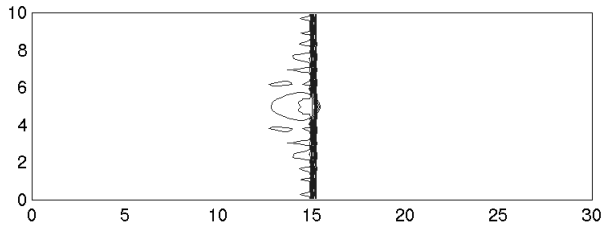
- [Quirk, 1994b]
- Test from [Deiterding, 2003]

`code/amroc/doc/html/apps/clawpack_2applications_2euler_znd_22d_2Carbuncle_2src_2Problem_8h_source.html`

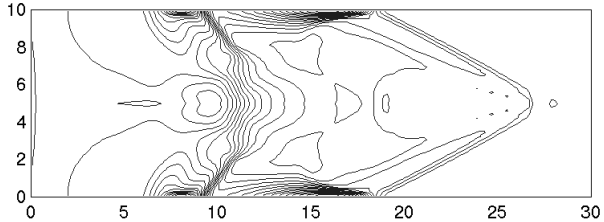
Roe + EC 1.



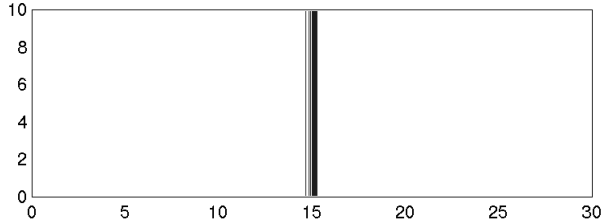
Exact Riemann solver



Roe + EC 2.



SW FVS, VL FVS, HLL, Roe + EC 2.+2D



Detonations - motivation for SAMR

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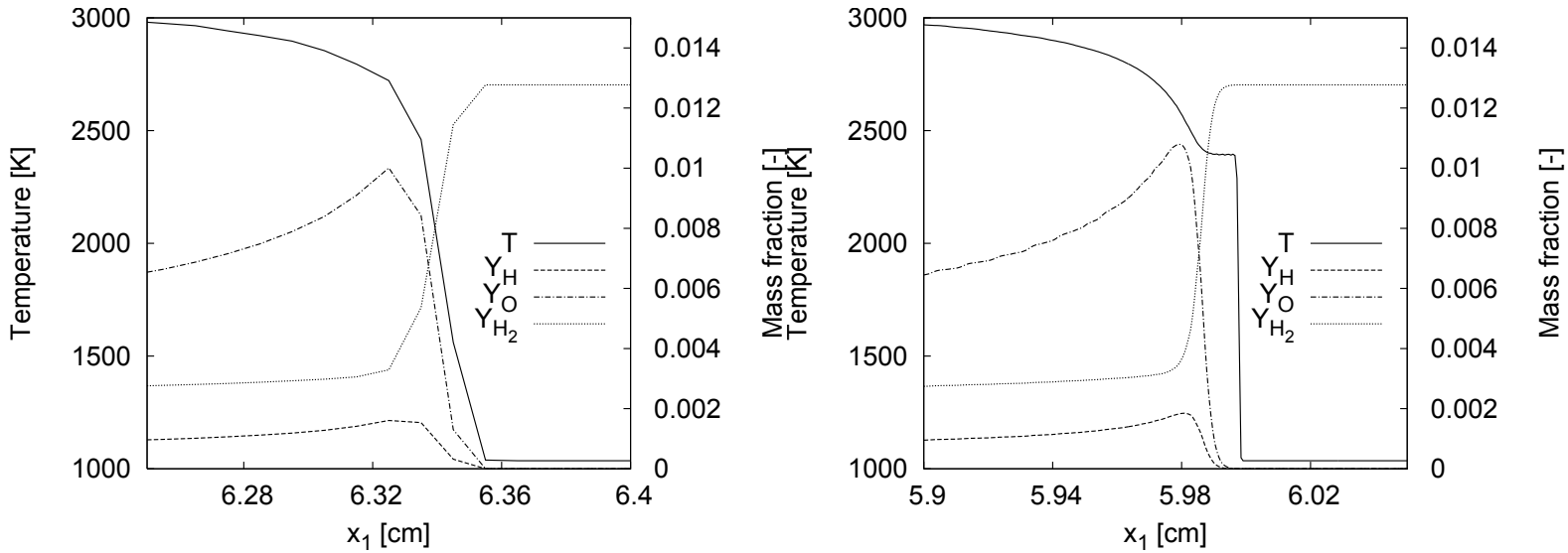

Approximation of $H_2 : O_2$ detonation at $\sim 1.5\,\mathrm{Pts}/l_{ig}$ (left) and $\sim 24\,\mathrm{Pts}/l_{ig}$ (right)

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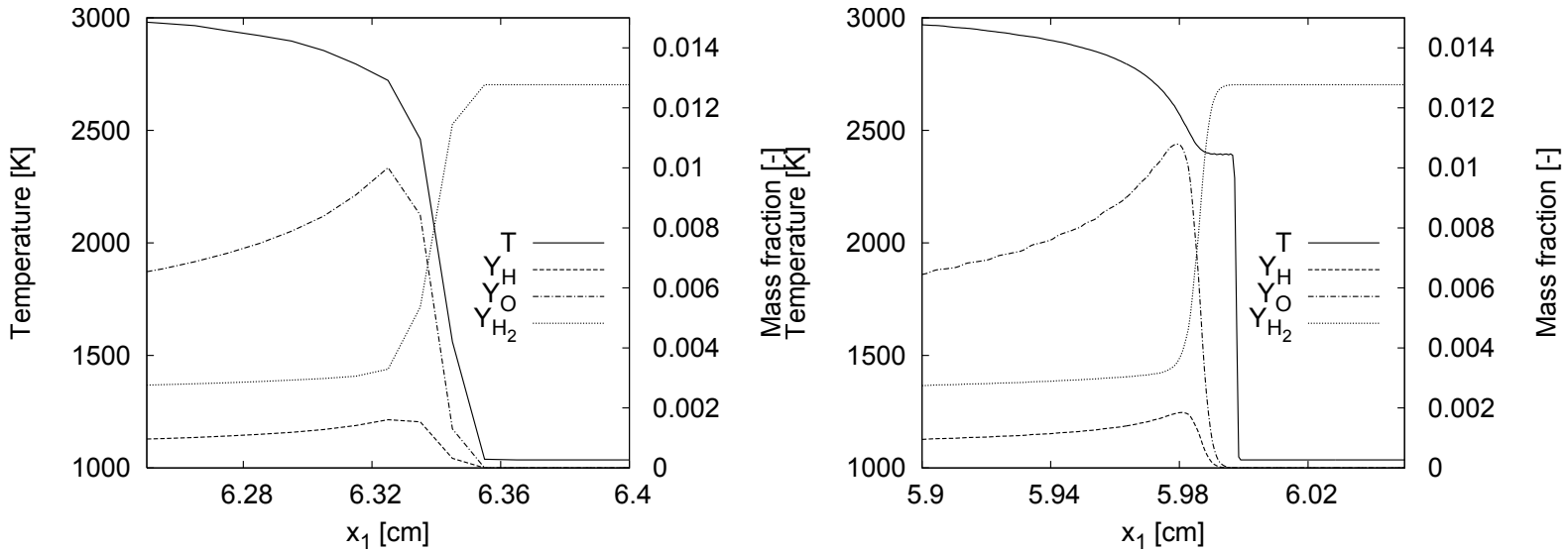
- Uniform grids for typical geometries: $> 10^7\,\mathrm{Pts}$ in 2D, $> 10^9\,\mathrm{Pts}$ in 3D $\longrightarrow$ Self-adaptive finite volume method (AMR)



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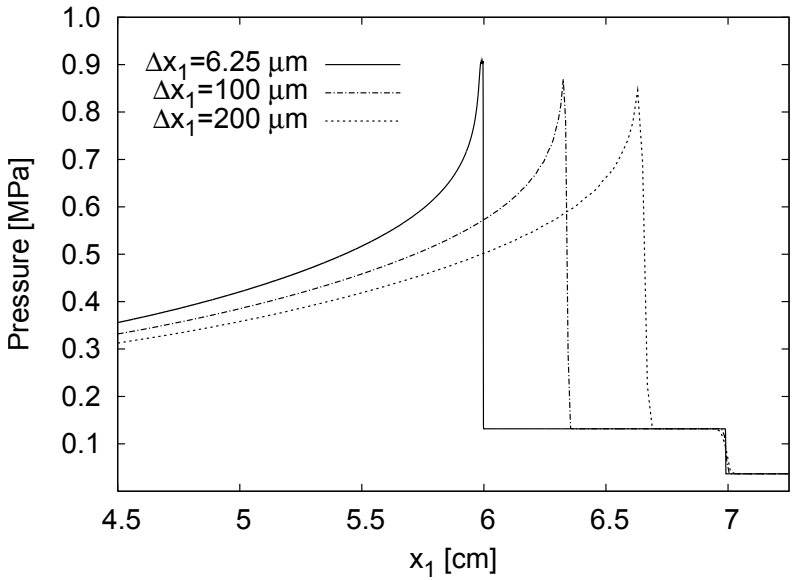

Left: Comparison of pressure distribution $t = 170\,\mu\mathrm{s}$ after shock reflection.

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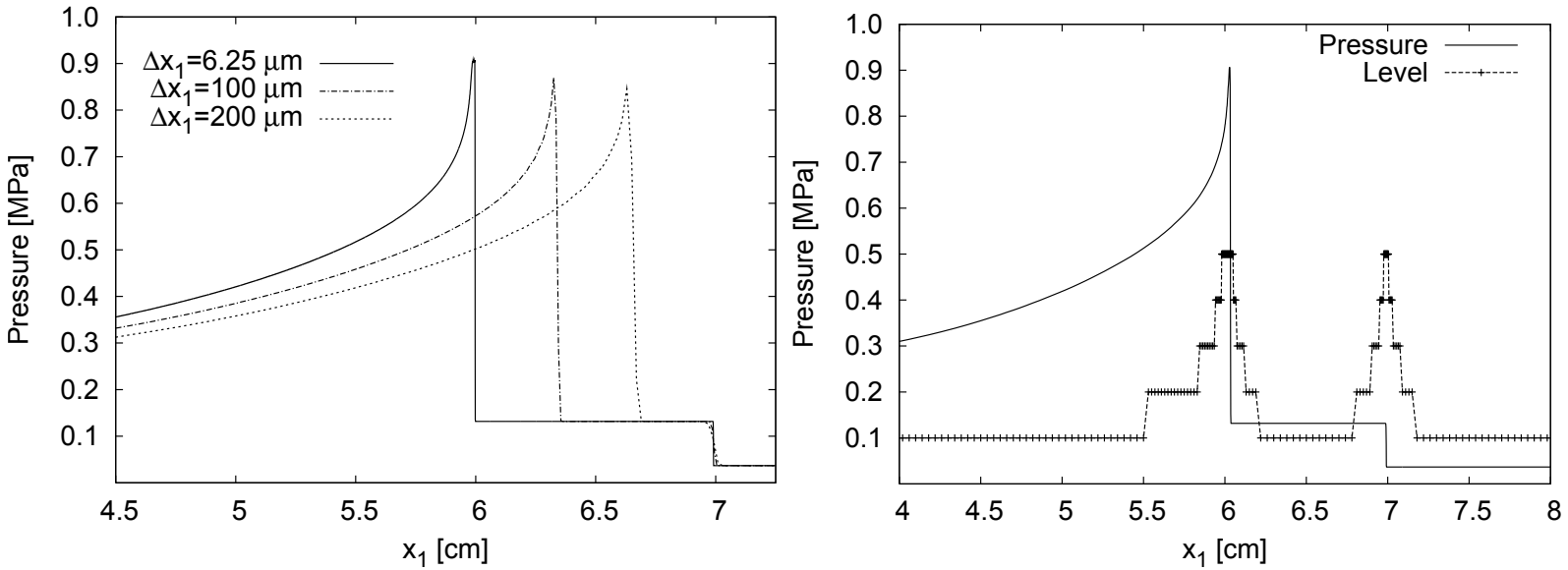

Left: Comparison of pressure distribution $t = 170\,\mu s$ after shock reflection.Right: Domains of refinement levels

Detonation ignition in 1d - adaptive vs. uniform

Uniformly refined vs. dynamic adaptive simulations (Intel Xeon 3.4 GHz CPU)

	Uniform			Adaptive			
$\Delta x_1[\mu m]$	Cells	$t_m[\mu s]$	Time [s]	l_{max}	r_l	$t_m[\mu s]$	Time [s]
400	300	166.1	31				
200	600	172.6	90	2	2	172.6	99
100	1200	175.5	277	3	2,2	175.8	167
50	2400	176.9	858	4	2,2,2	177.3	287
25	4800	177.8	2713	4	2,2,4	177.9	393
12.5	9600	178.3	9472	5	2,2,2,4	178.3	696
6.25	19200	178.6	35712	5	2,2,4,4	178.6	1370

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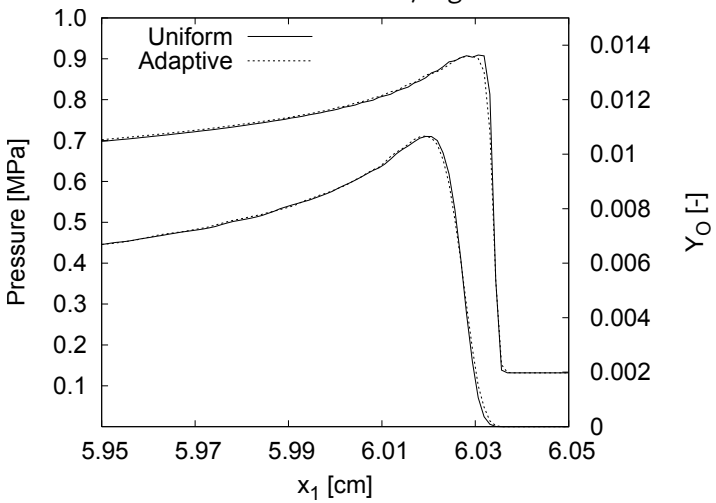

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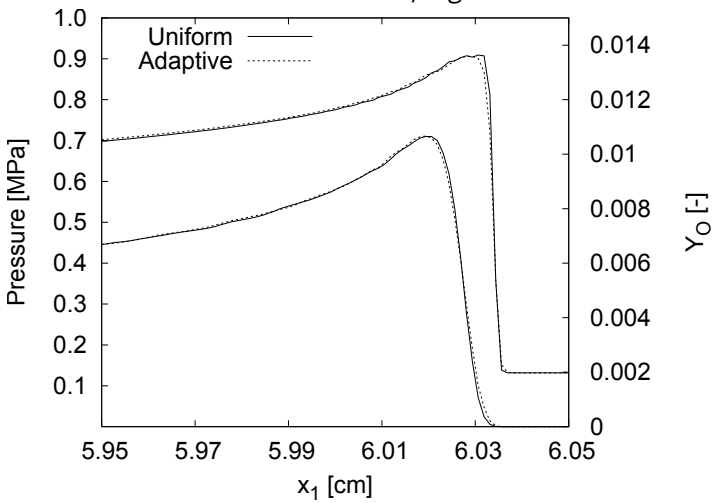

Refinement criteria:

Y_i	$S_{Y_i} \cdot 10^{-4}$	$\eta^r_{Y_i} \cdot 10^{-3}$
O_2	10.0	2.0
H_2O	7.8	8.0
H	0.16	5.0
O	1.0	5.0
OH	1.8	5.0
H_2	1.3	2.0

$\epsilon_\rho = 0.07\,\mathrm{kg\,m^{-3}}$, $\epsilon_p = 50\,\mathrm{kPa}$

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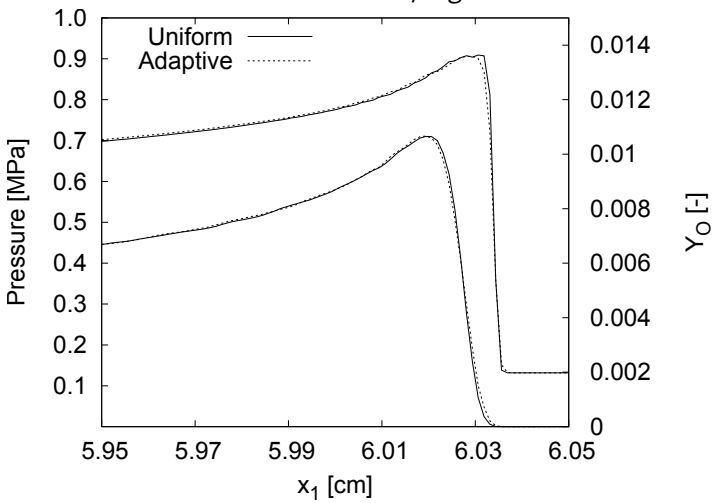

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`code/amroc/doc/html/apps/clawpack_2applications_2euler__chem_21d_2ReflecDet_2src_2Problem_8h_source.html`

Shock-induced combustion around a sphere

- Spherical projectile of radius $1.5\,\mathrm{mm}$ travels with constant velocity $v_l = 2170.6\,\mathrm{m/s}$ through $\mathrm{H_2} : \mathrm{O_2} : \mathrm{Ar}$ mixture (molar ratios 2:1:7) at $6.67\,\mathrm{kPa}$ and $T = 298\,\mathrm{K}$
- Cylindrical symmetric simulation on AMR base mesh of 70×40 cells

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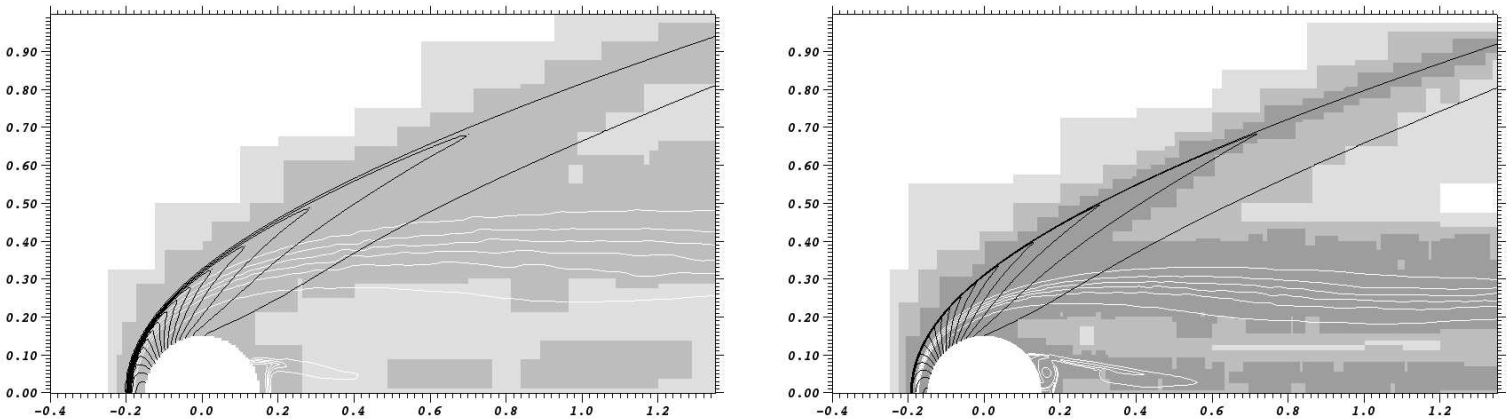
- Comparison of 3-level computation with refinement factors 2,2 ($\sim 5\,\mathrm{Pts}/l_{ig}$) and a 4-level computation with refinement factors 2,2,4 ($\sim 19\,\mathrm{Pts}/l_{ig}$) at $t = 350\,\mu\mathrm{s}$



Iso-contours of p (black) and Y_{H_2} (white) on refinement domains for 3-level (left) and 4-level computation (right)

`code/amroc/doc/html/apps/clawpack_2applications_2euler__chem_22d_2Sphere_2src_2Problem_8h_source.html`

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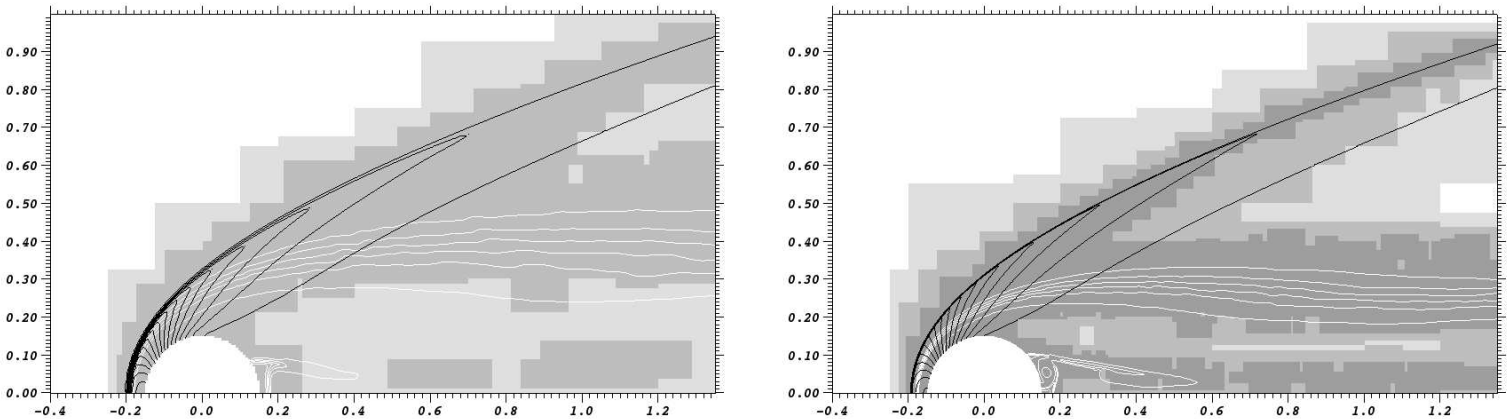
- Higher resolved computation captures combustion zone visibly better and at slightly different position (see below)



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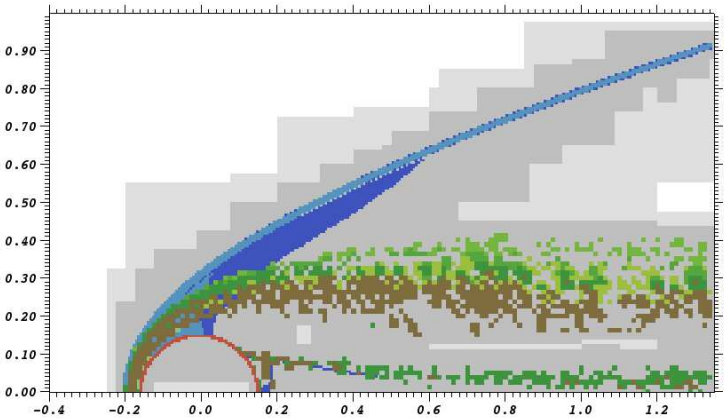
Combustion around a sphere - adaptation

Refinement indicators on $l = 2$ at $t = 350\,\mu\mathrm{s}$. Blue: ϵ_ρ, light blue: ϵ_p, green shades: $\eta^r_{Y_i}$, red: embedded boundary



Refinement criteria:

Y_i	$S_{Y_i} \cdot 10^{-4}$	$\eta^r_{Y_i} \cdot 10^{-4}$
O_2	10.0	4.0
H_2O	5.8	3.0
H	0.2	10.0
O	1.4	10.0
OH	2.3	10.0
H_2	1.3	4.0

$\epsilon_\rho = 0.02\,\mathrm{kg\,m^{-3}}$, $\epsilon_p = 16\,\mathrm{kPa}$

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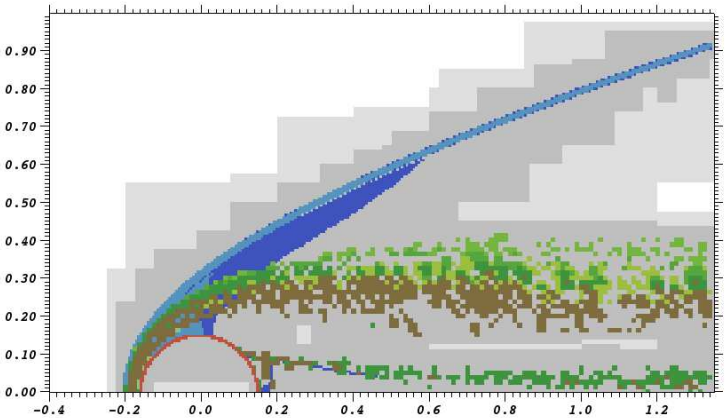

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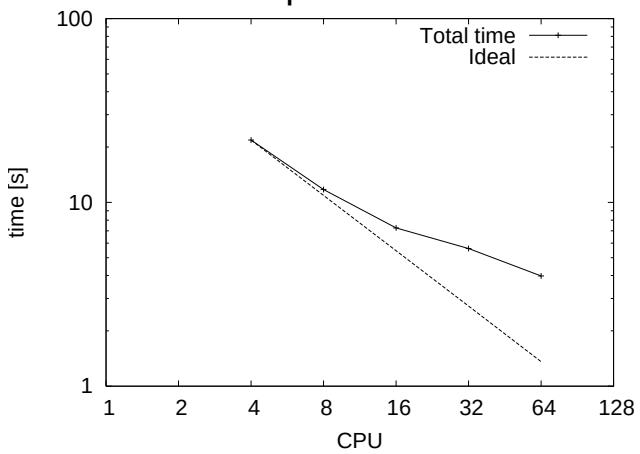
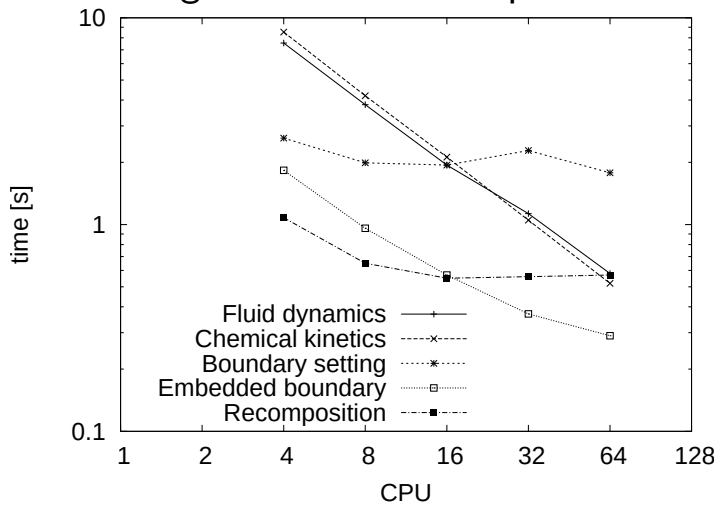
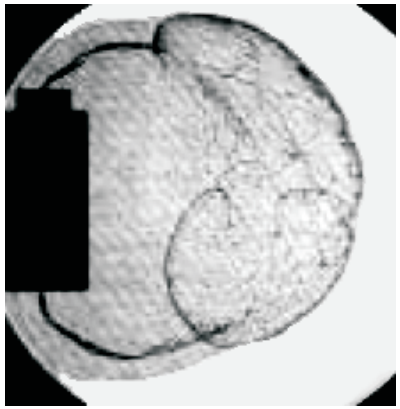


Scaling of different code portions

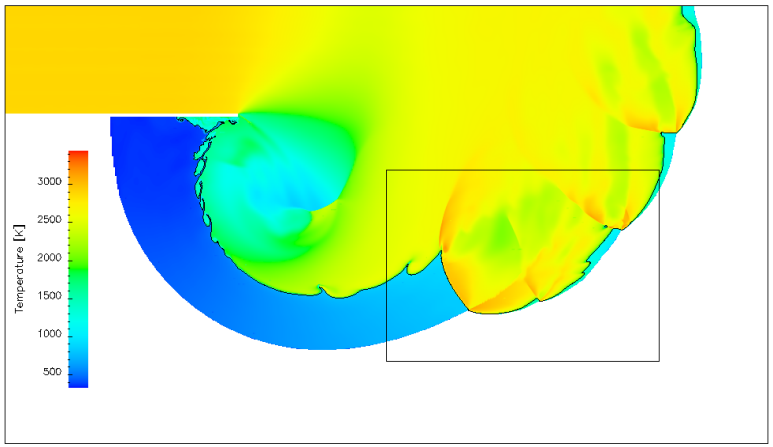


Detonation diffraction

- CJ detonation for $H_2 : O_2 : Ar/2 : 1 : 7$ at $T_0 = 298\,\mathrm{K}$ and $p_0 = 10\,\mathrm{kPa}$. Cell width $\lambda_c = 1.6\,\mathrm{cm}$
- Adaption criteria (similar as before):
  1. Scaled gradients of ρ and p
  2. Error estimation in Y_i by Richardson extrapolation
- $25\,\mathrm{Pts}/l_{ig}$. 5 refinement levels (2,2,2,4).
- Adaptive computations use up to $\sim 2.2\,\mathrm{M}$ instead of $\sim 150\,\mathrm{M}$ cells (uniform grid)
- $\sim 3850\,\mathrm{h}$ CPU ($\sim 80\,\mathrm{h}$ real time) on 48 nodes Athlon 1.4GHz

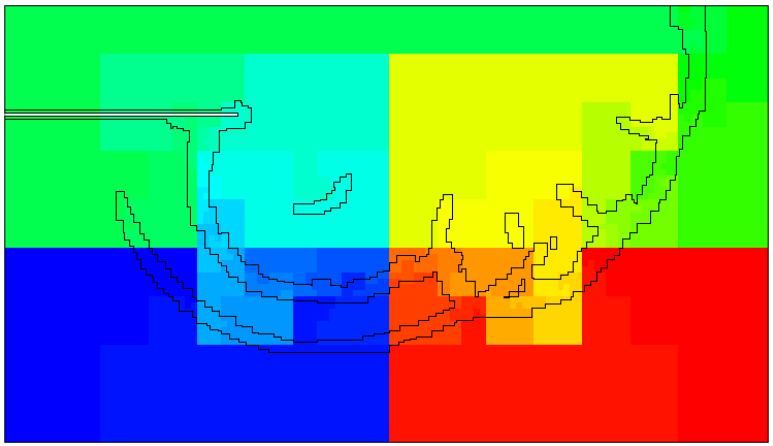


E. Schultz. *Detonation diffraction through an abrupt area expansion*. PhD thesis, California Institute of Technology, Pasadena, California, April 2000.



Detonation diffraction - adaptation

Final distribution to 48 nodes and density distribution on four refinement levels



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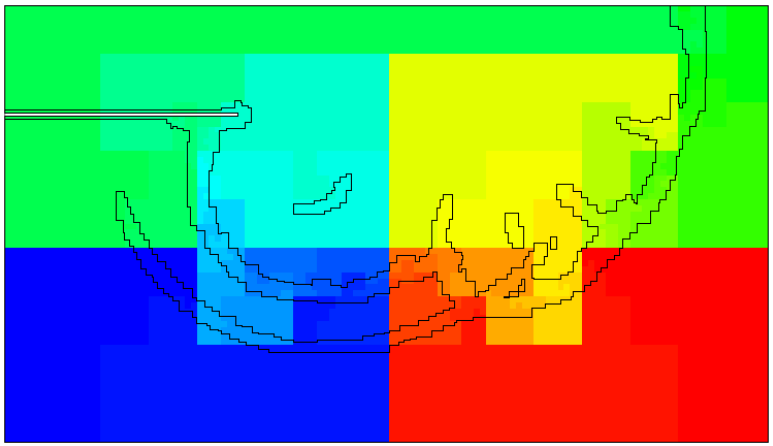

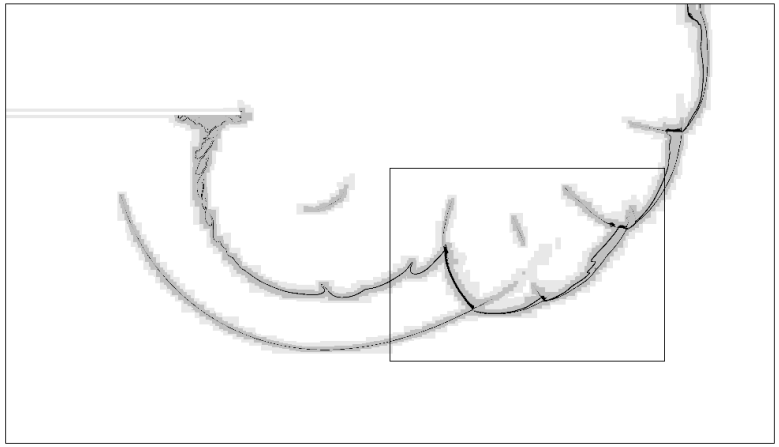

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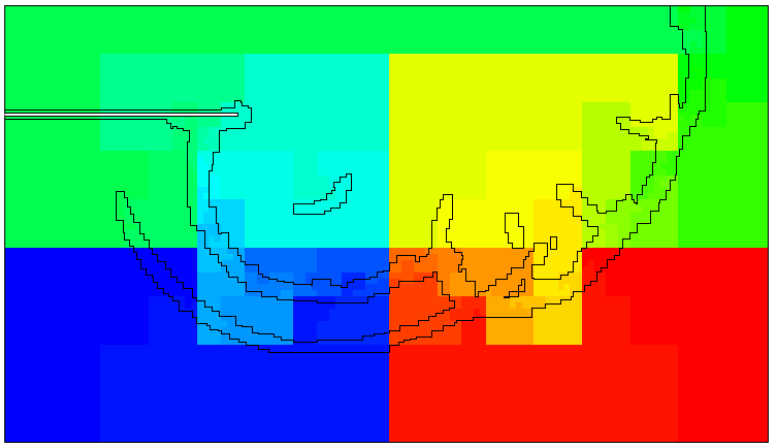

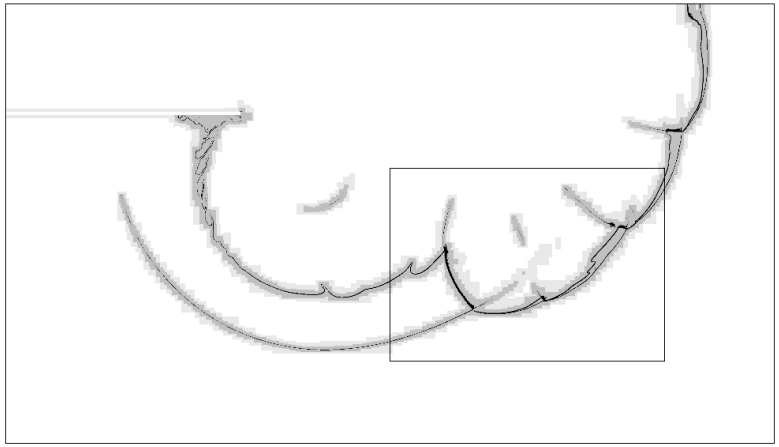

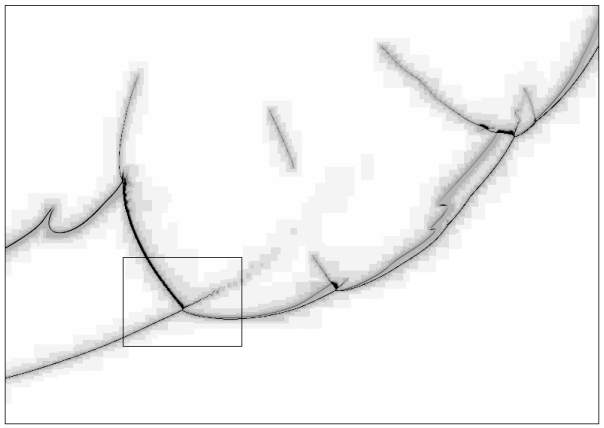

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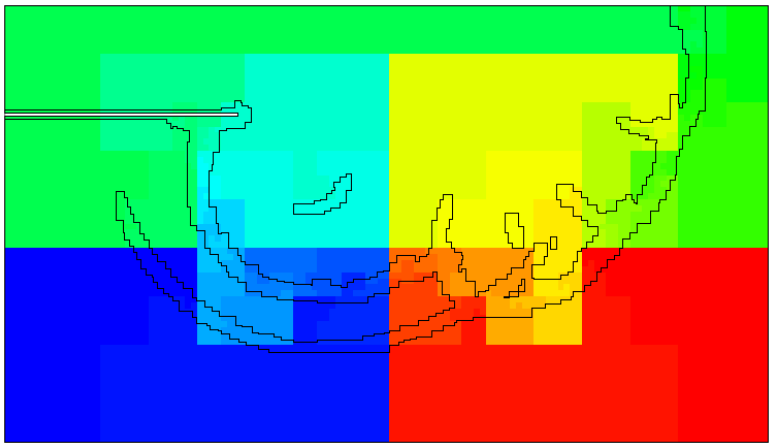

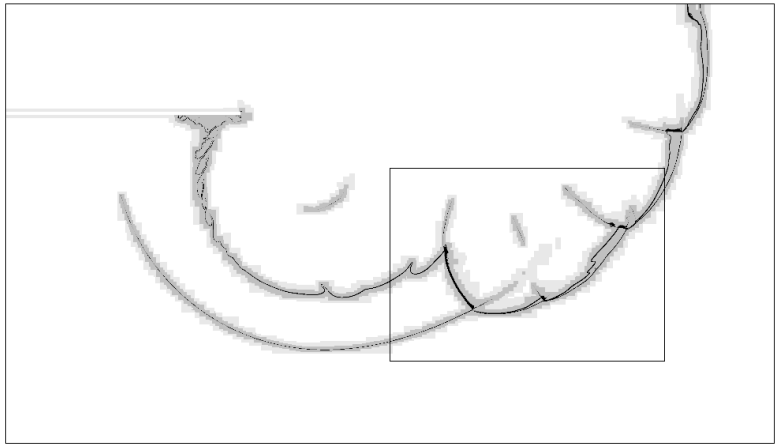

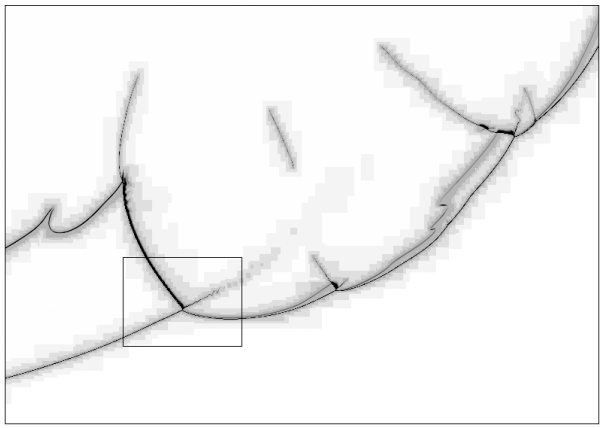

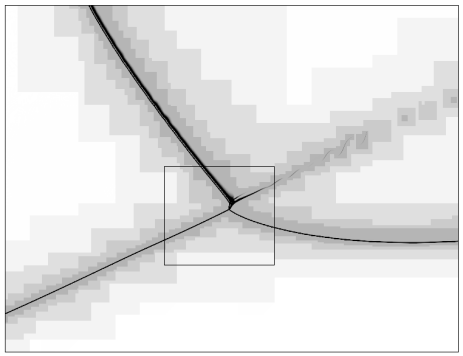

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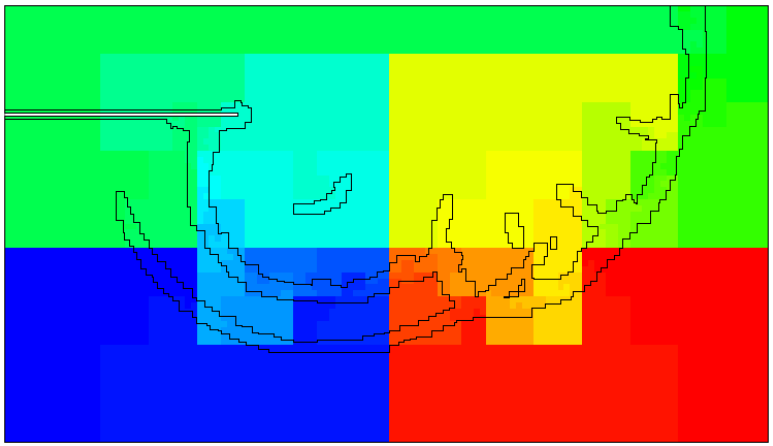

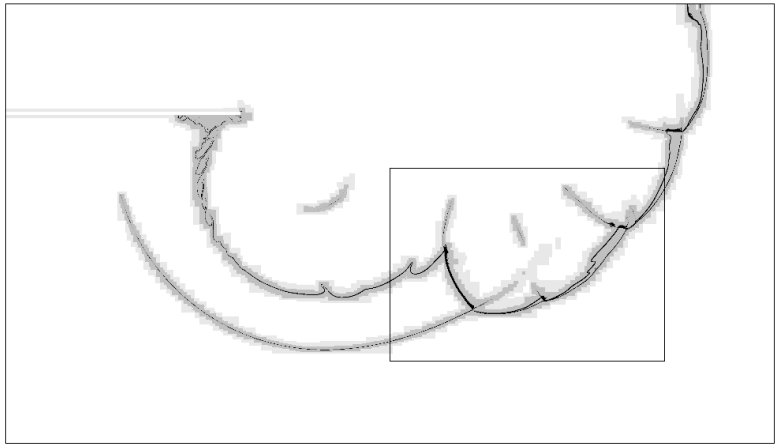

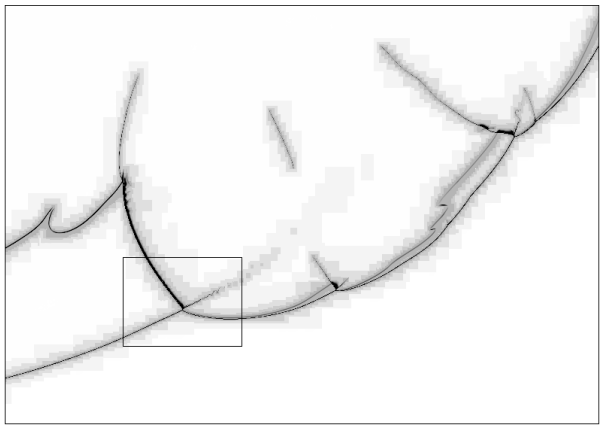

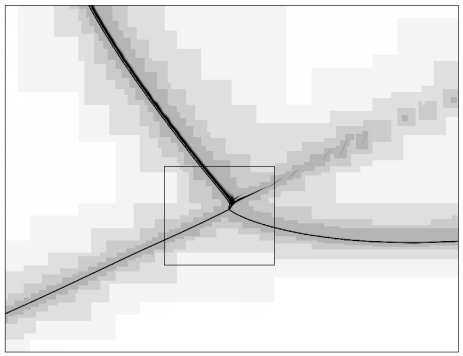

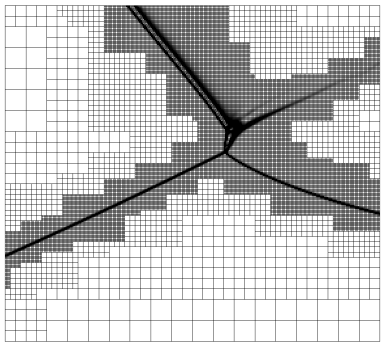

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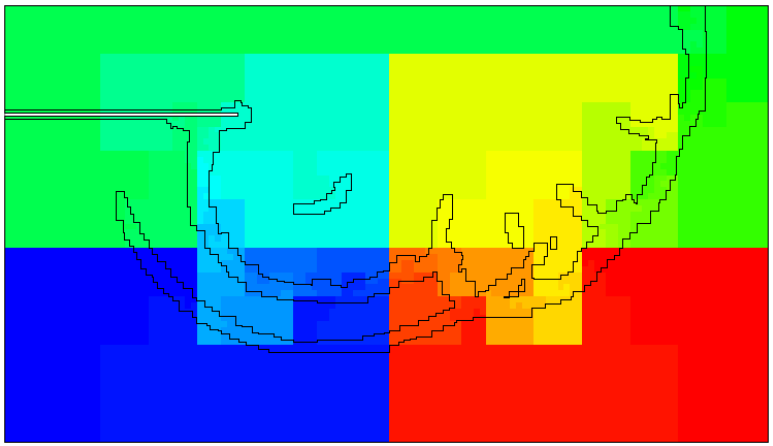
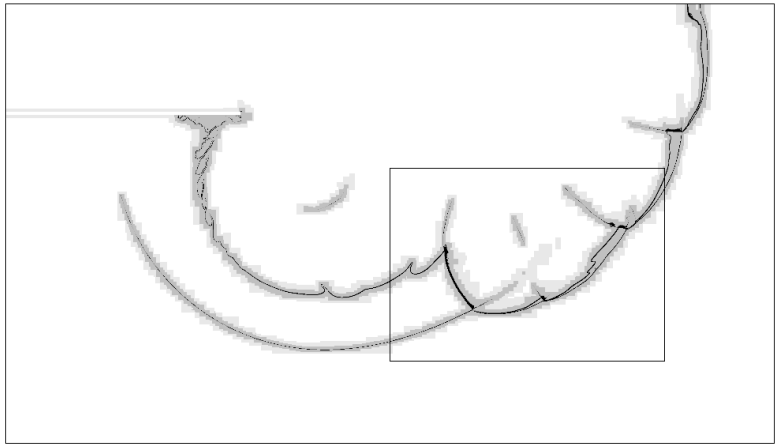
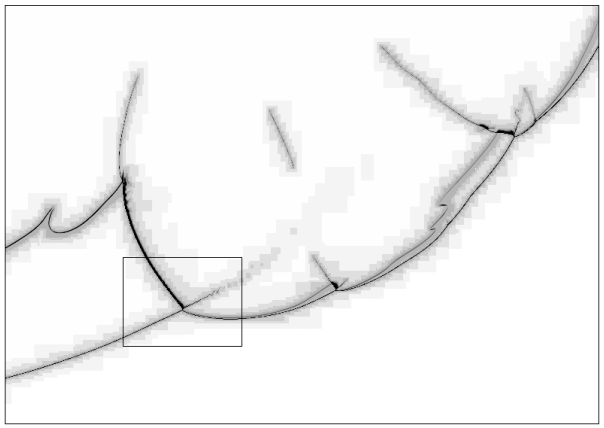
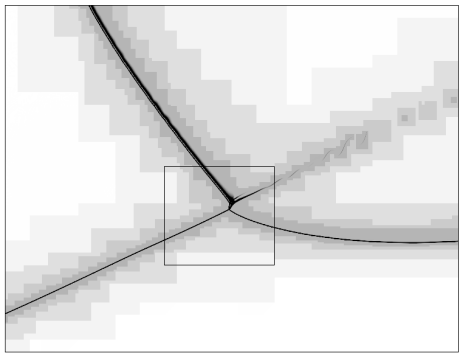
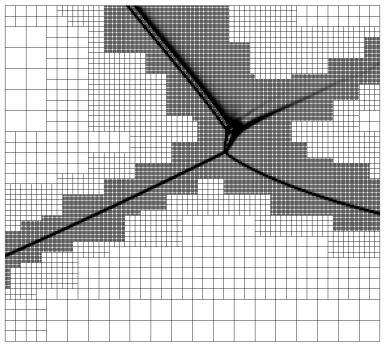








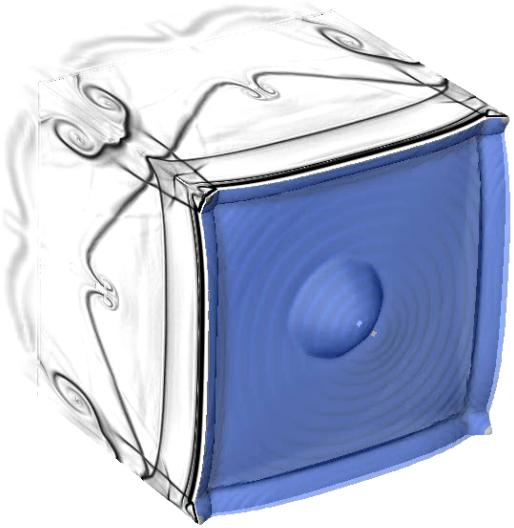


`code/amroc/doc/html/apps/clawpack_2applications_2euler__chem_22d_2Diffraction_2src_2Problem_8h_source.html`

Detonation cell structure in 3D

- Simulation of only one quadrant
- 44.8 Pts/l_{ig} for $\mathrm{H_2} : \mathrm{O_2} : \mathrm{Ar}$ CJ detonation
- SAMR base grid 400x24x24, 2 additional refinement levels (2, 4)
- Simulation uses $\sim 18\,\mathrm{M}$ cells instead of $\sim 118\,\mathrm{M}$ (unigrid)
- $\sim 51,000\,\mathrm{h}$ CPU on 128 CPU Compaq Alpha. $\mathcal{H}$: 37.6 %, $\mathcal{S}$: 25.1 %

Schlieren and isosurface of Y_{OH}



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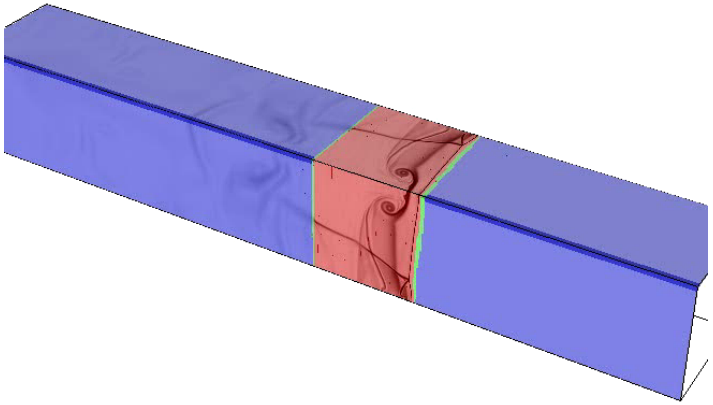

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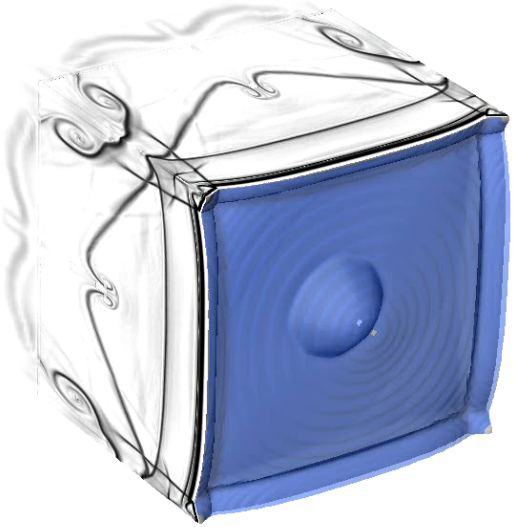

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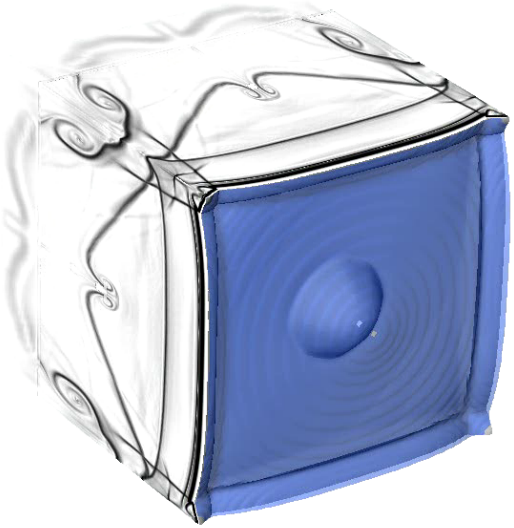

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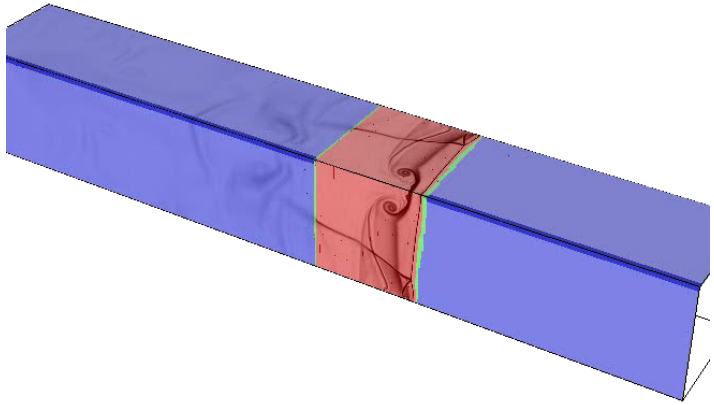
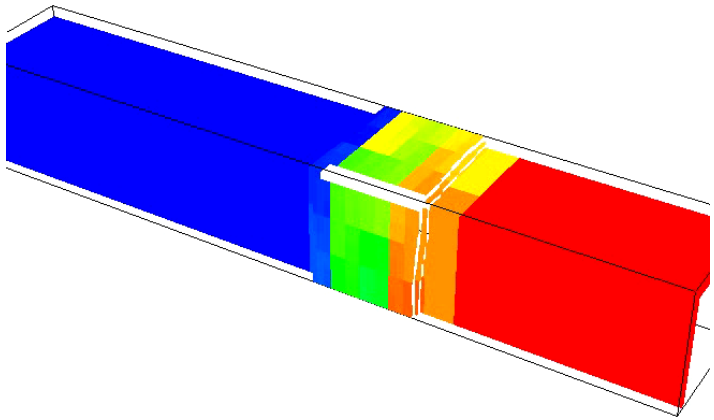


Distribution to 128 processors



code/amroc/doc/html/apps/clawpack_2applications_2euler__chem_23d_2StrehlowH2O2_2StatDet_2src_2Problem_8h_source.html

Outline

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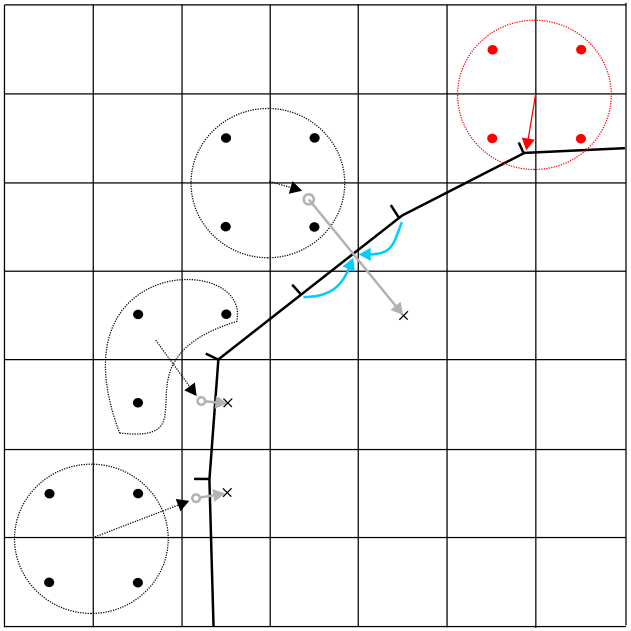

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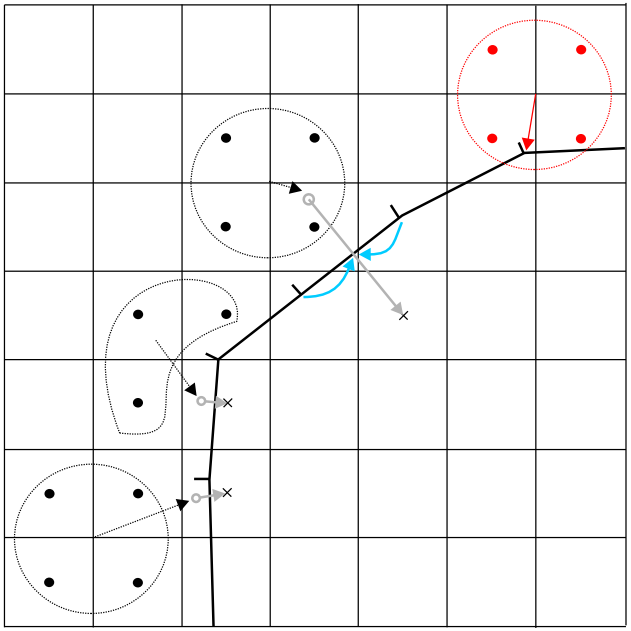

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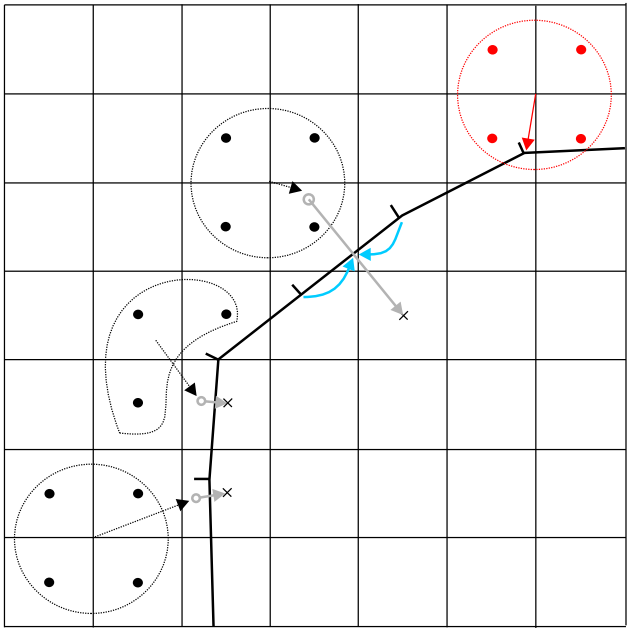

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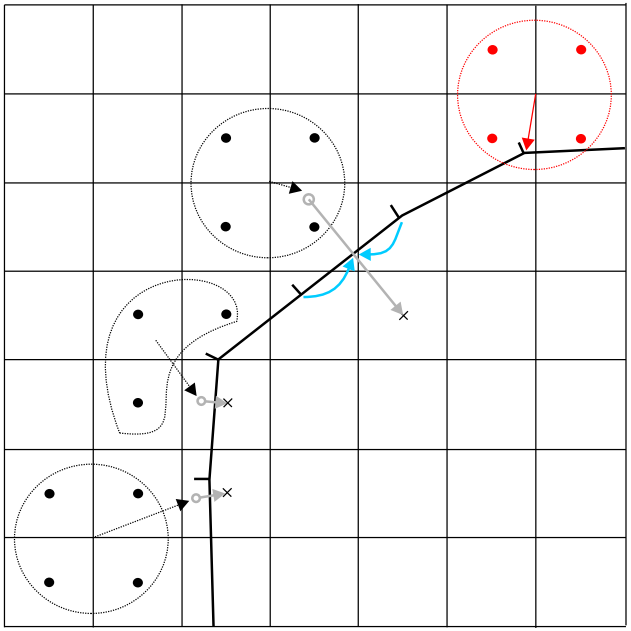
- One-sided construction of mirrored ghost cell and new FEM nodal point values
- FEM ansatz-function interpolation to obtain intermediate surface values
- Explicit coupling possible if geometry and velocities are prescribed for the more compressible medium [Specht, 2000]

$u_n^F := u_n^S(t)|_{\mathcal{I}}$
`UpdateFluid(` Δt `)`
$\sigma_{nn}^S := p^F(t + \Delta t)|_{\mathcal{I}}$
`UpdateSolid(` Δt `)`
$t := t + \Delta t$



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  - Boundary data is sent to solid when highest level available
- Inter-solver communication (point-to-point or globally) managed on the fly special coupling module

SAMR algorithm for FSI coupling

AdvanceLevel(l)

Repeat r_l times
Set ghost cells of $\mathbf{Q}^l(t)$

If time to regrid?
Regrid(l)
UpdateLevel(l)
If level $l+1$ exists?
Set ghost cells of $\mathbf{Q}^l(t+\Delta t_l)$
AdvanceLevel($l+1$)
Average $\mathbf{Q}^{l+1}(t+\Delta t_l)$ onto $\mathbf{Q}^l(t+\Delta t_l)$

$t := t + \Delta t_l$

SAMR algorithm for FSI coupling

```
AdvanceLevel(l)

 Repeat r_l times
   Set ghost cells of Q^l(t)
   CPT(φ^l, C^l, I, δ_l)
   If time to regrid?
       Regrid(l)
   UpdateLevel(Q^l, φ^l, C^l, u^S|_I, Δt_l)
   If level l+1 exists?
       Set ghost cells of Q^l(t+Δt_l)
       AdvanceLevel(l+1)
       Average Q^{l+1}(t+Δt_l) onto Q^l(t+Δt_l)

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```

- Call CPT algorithm before `Regrid(l)`
- Include also call to `CPT(·)` into `Recompose(l)` to ensure consistent level set data on levels that have changed

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       AdvanceLevel(l+1)
       Average Q^{l+1}(t+Δt_l) onto Q^l(t+Δt_l)
   If l = l_c?
       SendInterfaceData(p^F(t+Δt_l)|_I)
       If (t+Δt_l) < (t_0+Δt_0)?
           ReceiveInterfaceData(I, u^S|_I)
   t := t+Δt_l
```

- Call CPT algorithm before `Regrid(l)`
- Include also call to `CPT(·)` into `Recompose(l)` to ensure consistent level set data on levels that have changed
- Communicate boundary data on coupling level l_c

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       Set ghost cells of Q^l(t+Δt_l)
       AdvanceLevel(l+1)
       Average Q^{l+1}(t+Δt_l) onto Q^l(t+Δt_l)
   If l = l_c?
       SendInterfaceData(p^F(t+Δt_l)|_I)
       If (t+Δt_l) < (t_0+Δt_0)?
           ReceiveInterfaceData(I, u^S|_I)
   t := t+Δt_l
```

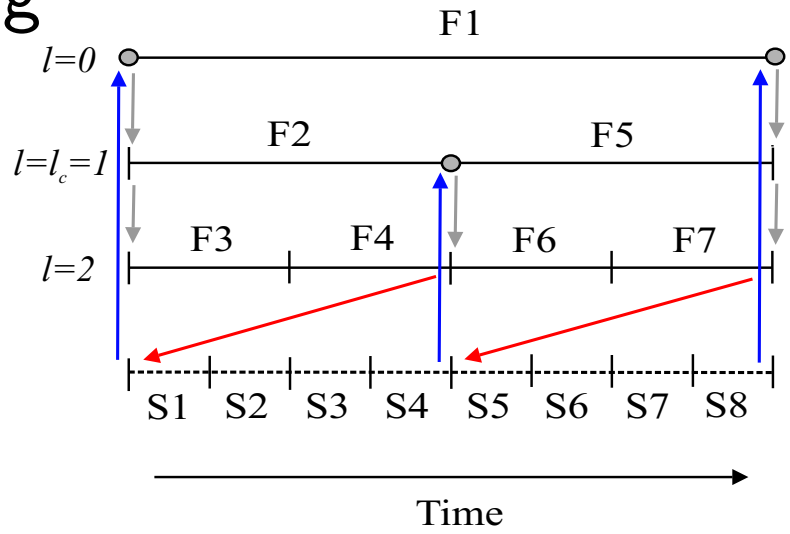

- Call CPT algorithm before `Regrid(l)`
- Include also call to `CPT(·)` into `Recompose(l)` to ensure consistent level set data on levels that have changed
- Communicate boundary data on coupling level l_c

Fluid and solid update / exchange of time steps

`FluidStep( )`

$\Delta\tau_F := \min_{l=0,\cdots,l_{\max}} (R_l \cdot$ `StableFluidTimeStep(`l`),` $\Delta\tau_S)$

$\Delta t_l := \Delta\tau_F / R_l$ `for` $l = 0, \cdots, L$

`ReceiveInterfaceData(`$\mathcal{I}$`,` $\mathbf{u}^S|_{\mathcal{I}}$`)`

`AdvanceLevel(0)`

with $R_l = \prod_{\iota=0}^{l} r_\iota$

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`SolidStep( )`

$\Delta\tau_S := \min(K \cdot R_{l_c} \cdot$ `StableSolidTimeStep(),` $\Delta\tau_F)$

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`SolidStep( )`

$\Delta\tau_S := \min(K \cdot R_{l_c} \cdot$ `StableSolidTimeStep(),` $\Delta\tau_F)$
`Repeat` R_{l_c} `times`
$t_{\text{end}} := t + \Delta\tau_S / R_{l_c}$, $\Delta t := \Delta\tau_S / (K R_{l_c})$

- Time step stays constant for R_{l_c} steps, which correponds to one fluid step at level 0

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`SolidStep( )`

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`Repeat` R_{l_c} `times`
  $t_{\text{end}} := t + \Delta\tau_S / R_{l_c}$, $\Delta t := \Delta\tau_S / (K R_{l_c})$
  `While` $t < t_{\text{end}}$
    `SendInterfaceData(`$\mathcal{I}(t)$`,` $\vec{u}^S|_{\mathcal{I}}(t)$`)`
    `ReceiveInterfaceData(`$p^F|_{\mathcal{I}}$`)`
    `UpdateSolid(`$p^F|_{\mathcal{I}}$`,` Δt`)`
    $t := t + \Delta t$
    $\Delta t := \min($`StableSolidTimeStep()`$, t_{\text{end}} - t)$

with $R_l = \prod_{\iota=0}^{l} r_\iota$

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Parallelization strategy for coupled simulations

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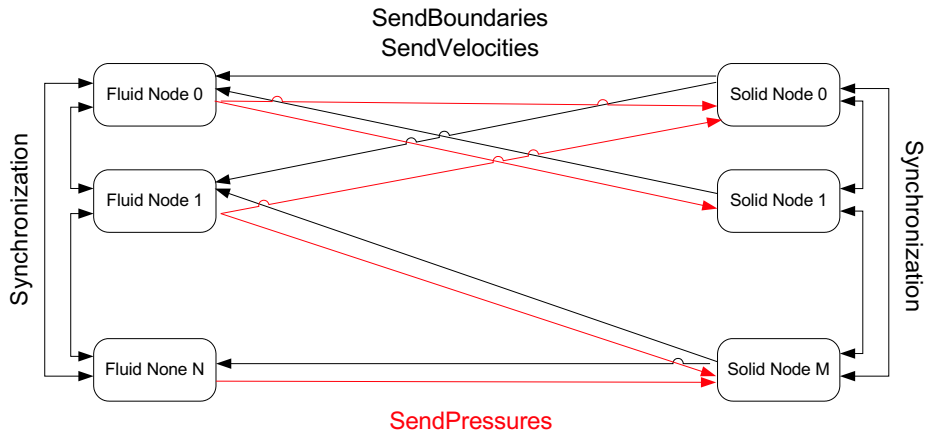

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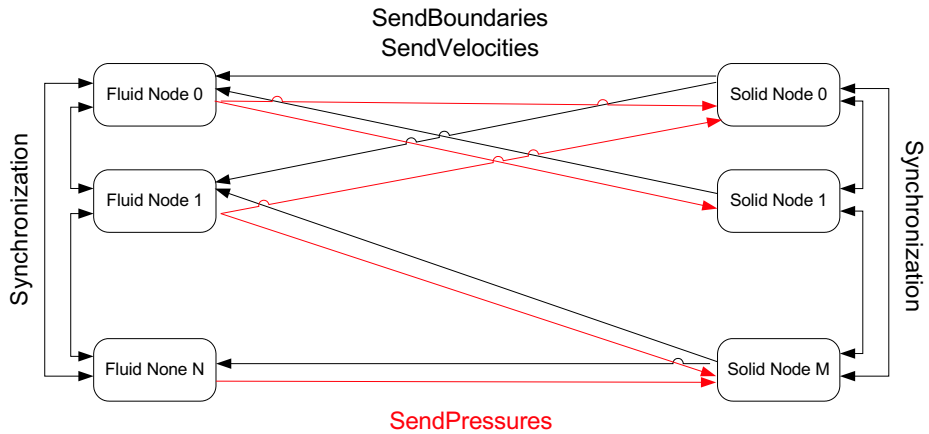

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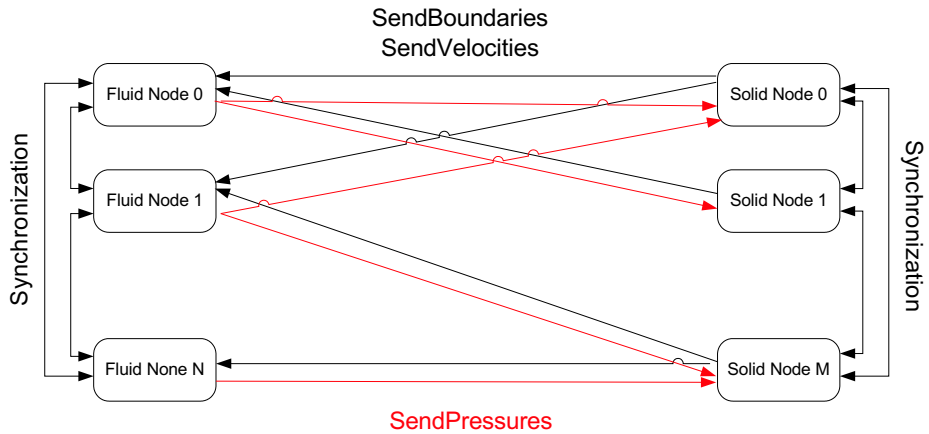
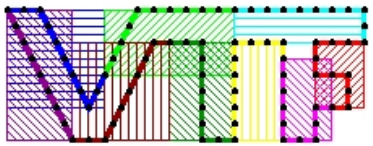
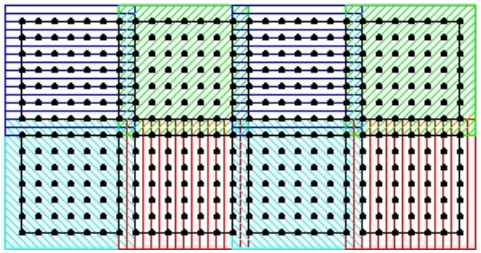
- Asynchronous communication ensures scalability
- Generic encapsulated implementation guarantees reusability



Eulerian/Lagrangian communication module

1. Put bounding boxes around each solid processors piece of the boundary and around each fluid processors grid





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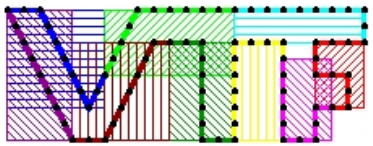

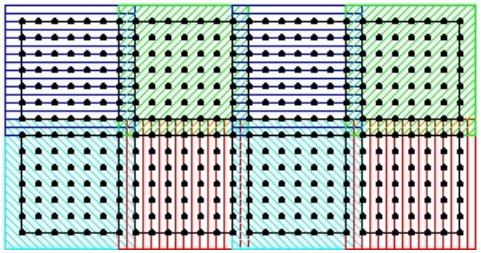

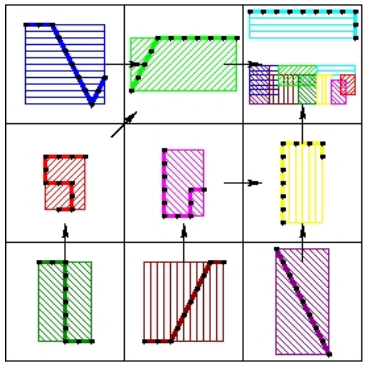

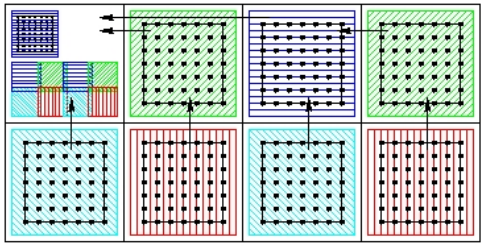

Eulerian/Lagrangian communication module

1. Put bounding boxes around each solid processors piece of the boundary and around each fluid processors grid
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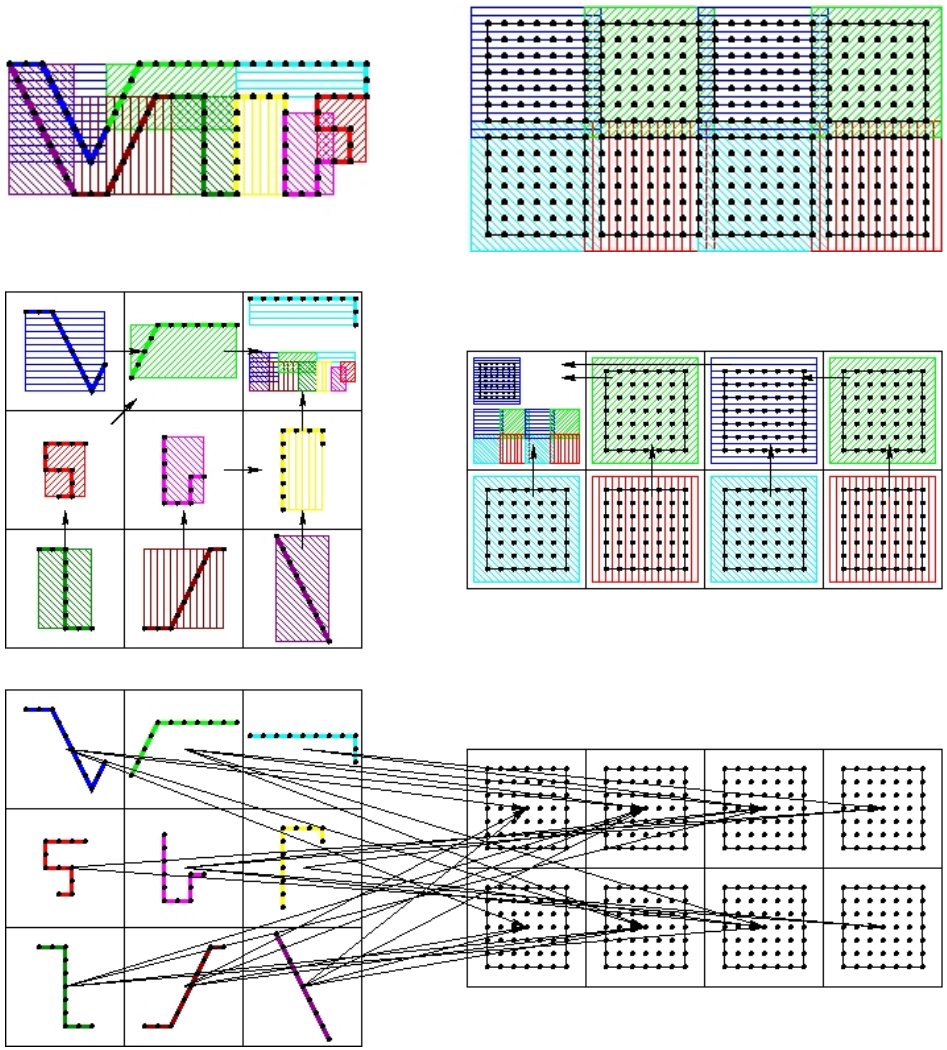
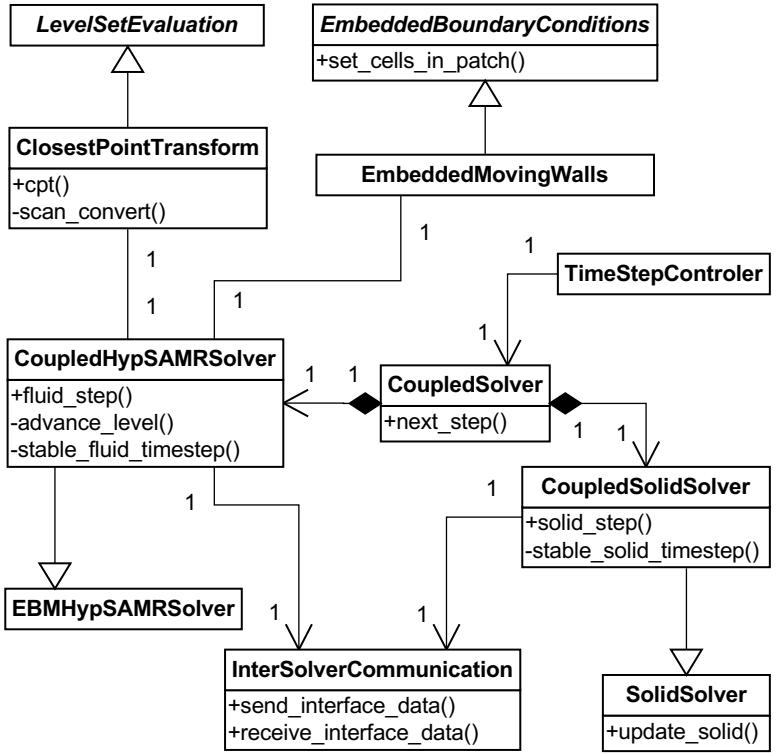


FSI coupling



- Coupling algorithm implemented in further derived HypSAMRSolver class
- Level set evaluation always with CPT algorithm
- Parallel communication through efficient non-blocking communication module ELC
- Time step selection for both solvers through CoupledSolver class

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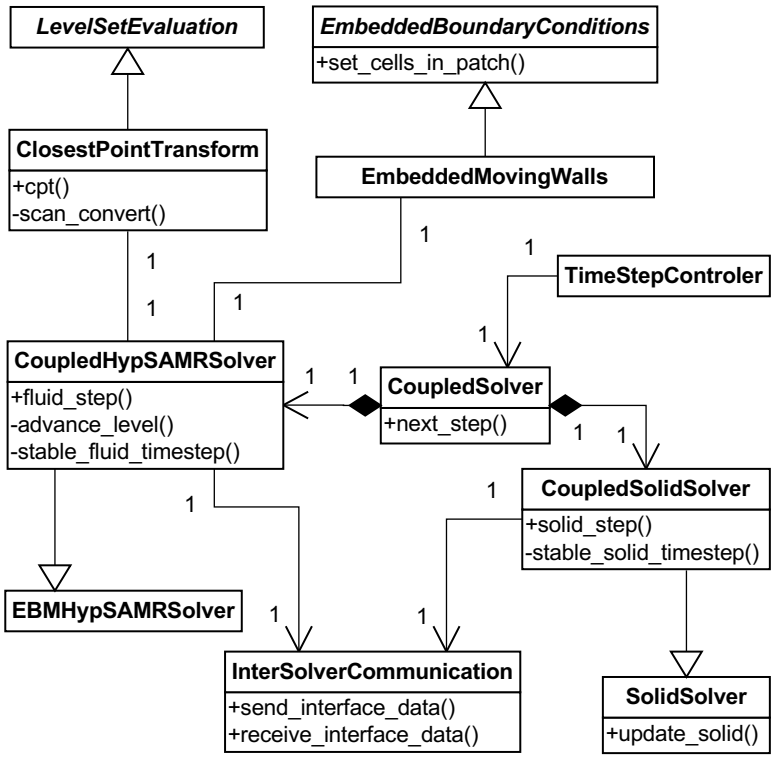

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- **AMRELCGFMSolver<VectorType, FixupType, FlagType, dim >** is the derived AMRSolver<>class. `code/amroc/doc/html/amr/classAMRELCGFMSolver.html`
- Uses the Eulerian interface of the Lagrangian communication routines

  `code/stlib/doc/html/elc/elc__page.html`
- and the closest point transform algorithm `code/stlib/doc/html/cpt/cpt__page.html` through the **CPTLevelSet<DataType, dim >**

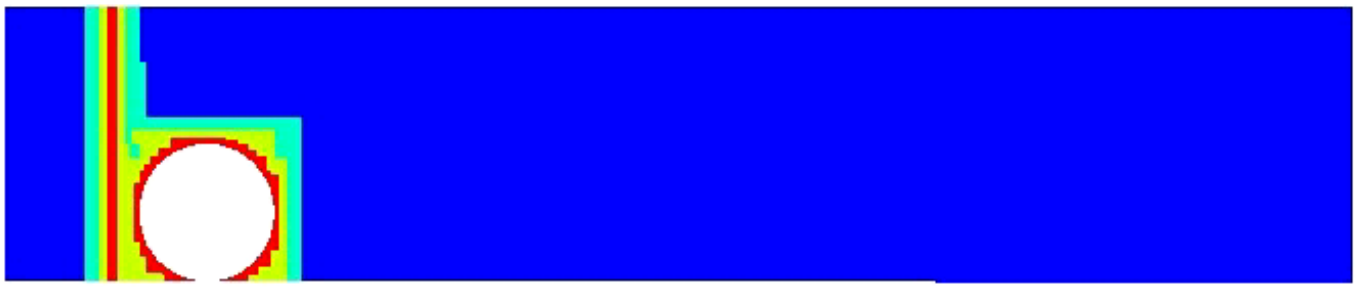
  `code/amroc/doc/html/amr/classCPTLevelSet.html`

Lift-up of a spherical body

Cylindrical body hit by Mach 3 shockwave, 2D test case by [Falcovitz et al., 1997]

Schlieren plot of density

Refinement levels



code/amroc/doc/html/apps/clawpack_2applications_2euler_22d_2SphereLiftOff_2src_2Problem_8h_source.html

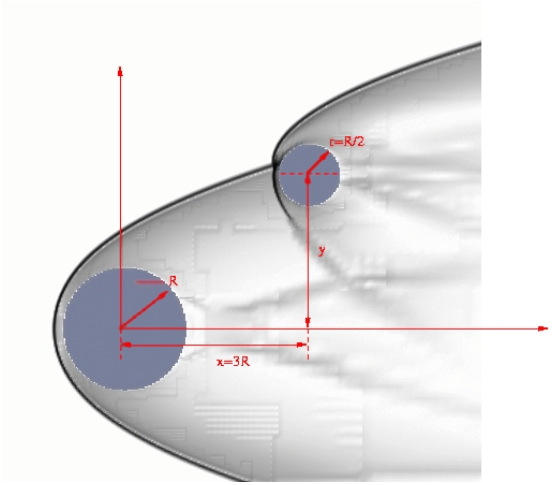
Proximal bodies in hypersonic flow

Flow modeled by Euler equations for a single polytropic gas with $p = (\gamma - 1)\,\rho e$

$$\partial_t \rho + \partial_{x_n}(\rho u_n) = 0\,, \quad \partial_t(\rho u_k) + \partial_{x_n}(\rho u_k u_n + \delta_{kn} p) = 0\,, \quad \partial_t(\rho E) + \partial_{x_n}(u_n(\rho E + p)) = 0$$

Numerical approximation with

- Finite volume flux-vector splitting scheme with MUSCL reconstruction, dimensional splitting



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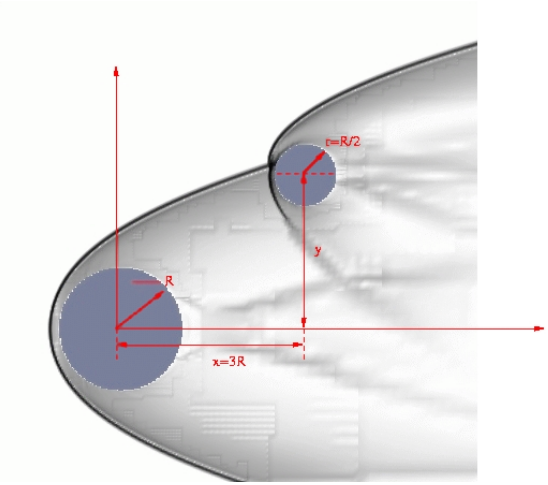
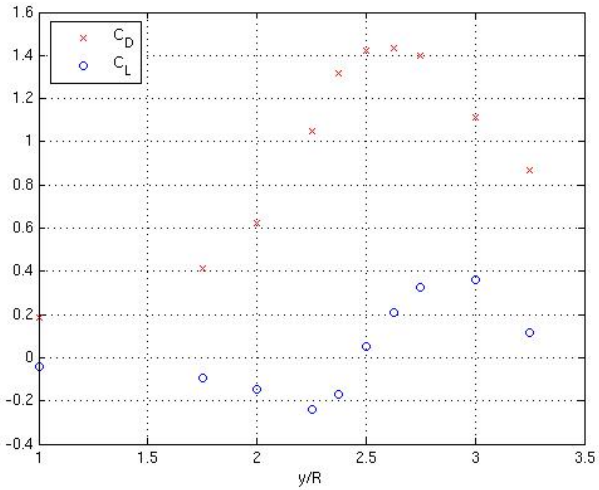
- Spherical bodies, force computation with overlaid lattitude-longitude mesh to obtain drag and lift coefficients $C_{D,L} = \dfrac{2F_{D,L}}{\rho v^2 \pi r^2}$
- inflow $M = 10$, C_D and C_L on secondary sphere, lateral position varied, no motion





Verification and validation

Static force measurements, $M = 10$:
[Laurence et al., 2007]

- Refinement study: $40 \times 40 \times 32$ base grid , up to without AMR up to $\sim 209.7 \cdot 10^6$ cells, largest run $\sim 35,000\,\mathrm{h}$ CPU

$l_{\max}$	C_D	ΔC_D	C_L	ΔC_L
1	1.264		-0.176	
2	1.442	0.178	-0.019	0.157
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- Comparison with experimental results: 3 additional levels, $\sim 2000\,\mathrm{h}$ CPU

	Experimental	Computational
C_D	1.11 ± 0.08	1.01
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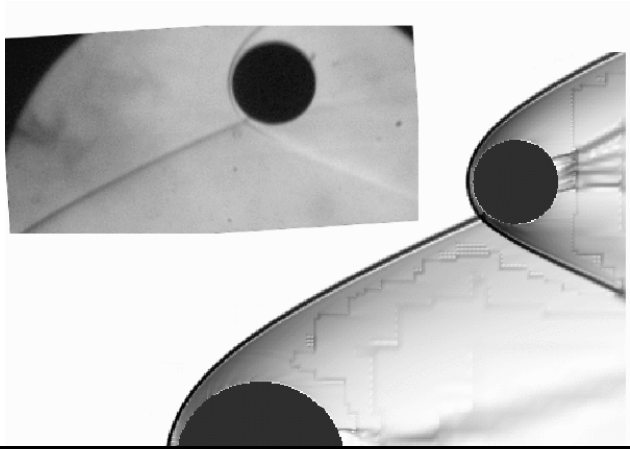

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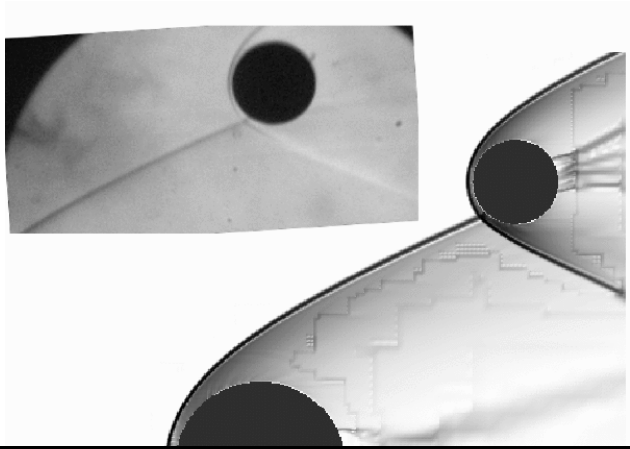
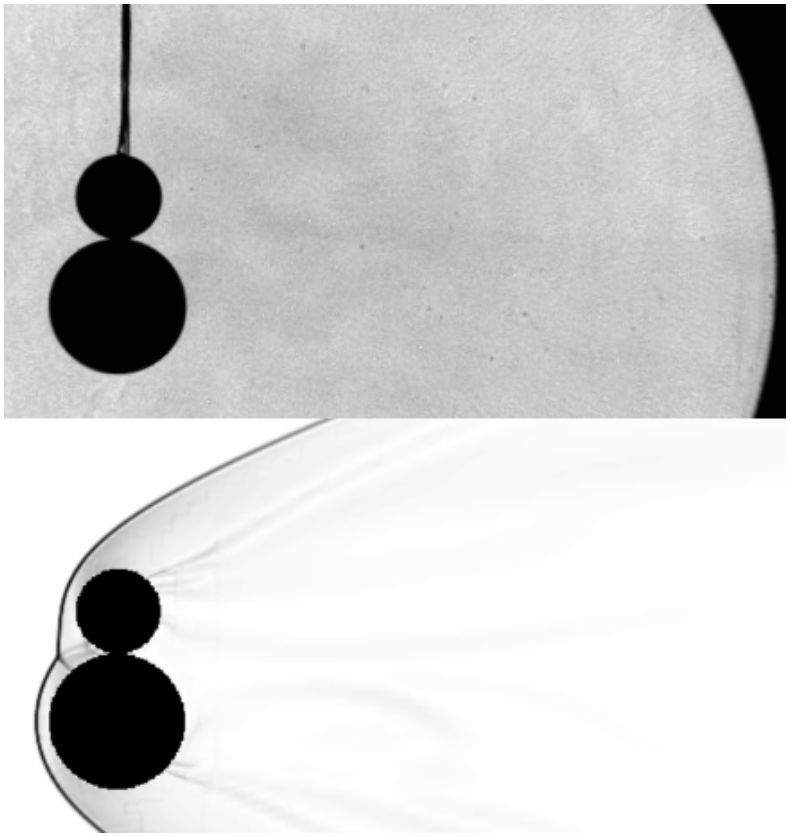


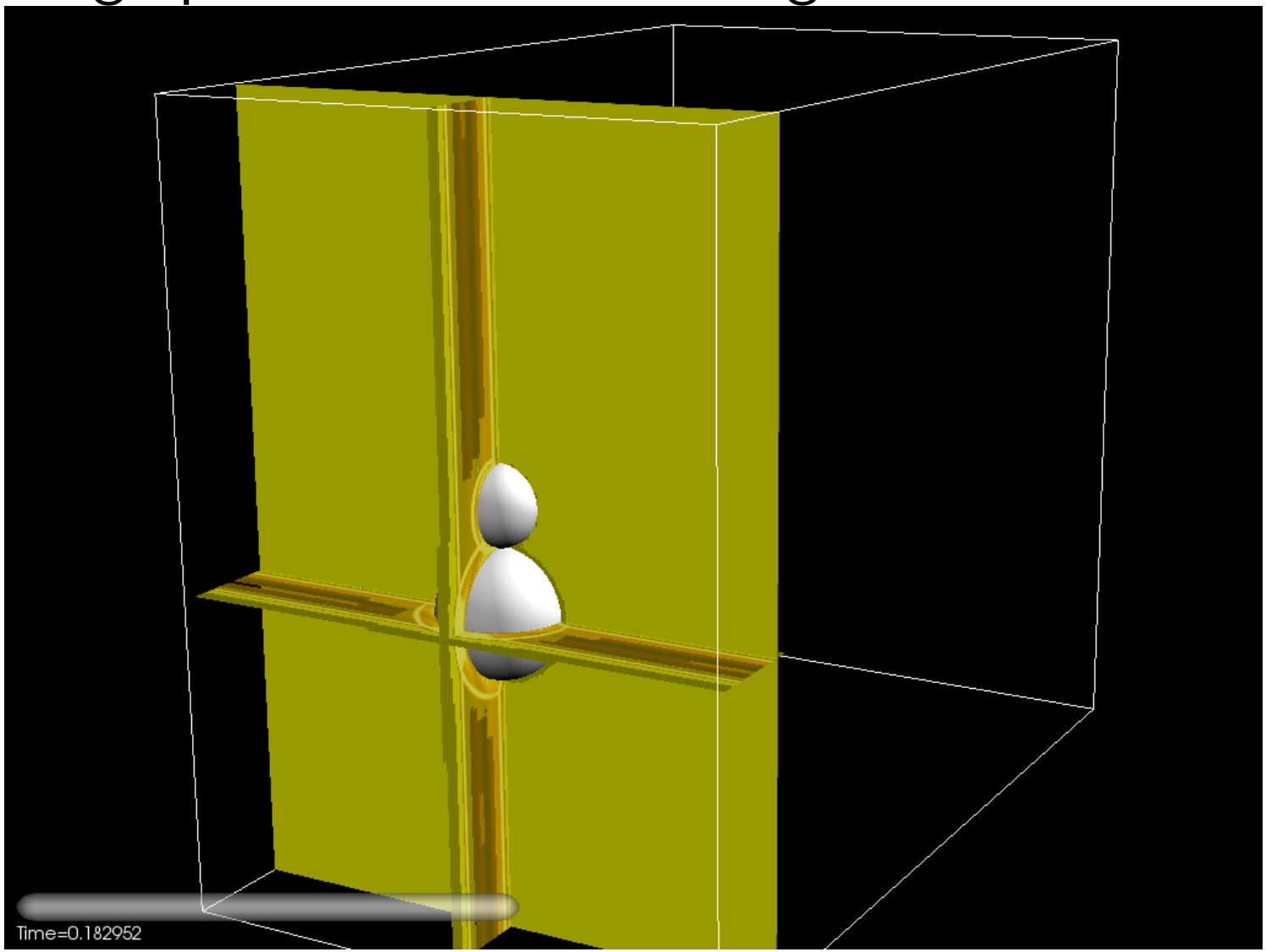
Dynamic motion, $M = 4$:

- Base grid $150 \times 125 \times 90$, two additional levels with $r_{1,2} = 2$
- 24,704 time steps, $36,808\,\mathrm{h}$ CPU on 256 cores IBM BG/P



[Laurence and Deiterding, 2011]

Schlieren graphics on refinement regions



code/amroc/doc/html/apps/clawpack_2applications_2euler_23d_2Spheres_2src_2Problem_8h_source.html

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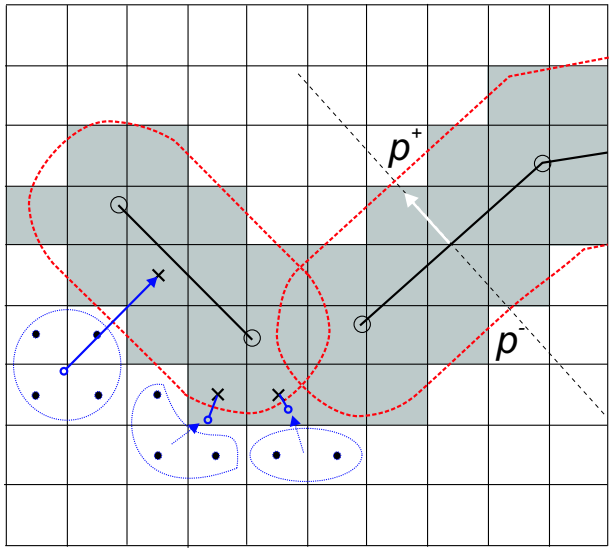
- Treat cells with $0 < \varphi < d$ as ghost fluid cells



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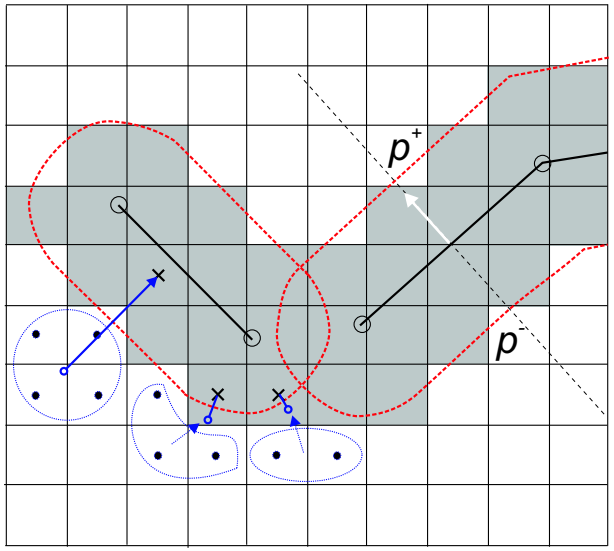

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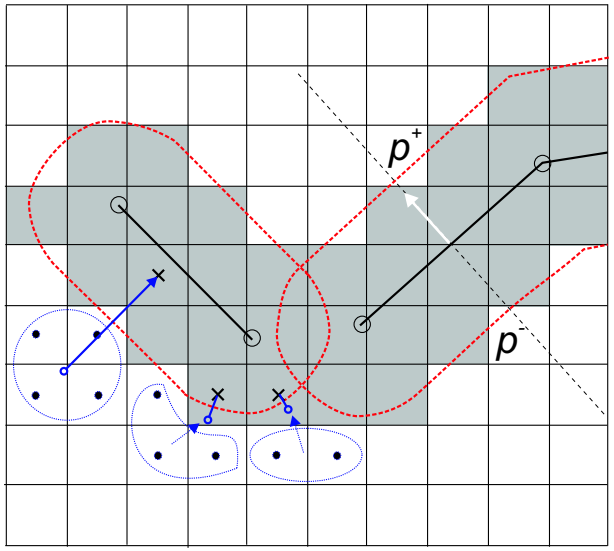

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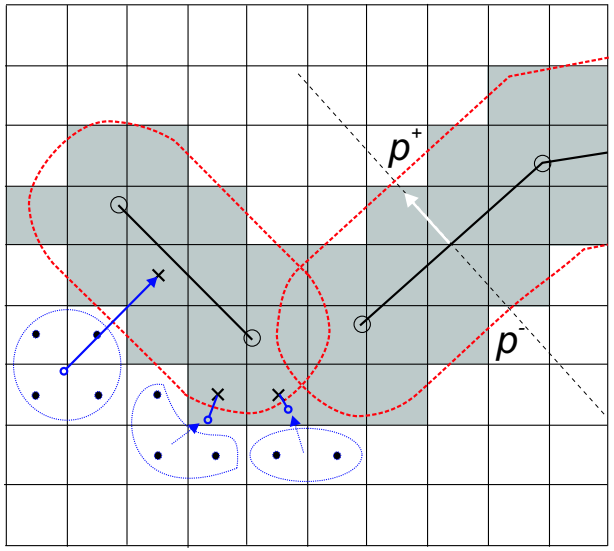

- Leaving φ unmodified ensures correctness of $\nabla\varphi$
- Use face normal in shell element to evaluate in $\Delta p = p^+ - p^-$
- Utilize finite difference solver using the beam equation

$$\rho_s h \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial \bar{x}^4} = p^F$$

to verify FSI algorithms

FSI verification by elastic vibration

- Thin steel plate (thickness $h = 1\,\mathrm{mm}$, length $50\,\mathrm{mm}$), clamped at lower end
- $\rho_s = 7600\,\mathrm{kg/m^3}$, $E = 220\,\mathrm{GPa}$, $I = h^3/12$, $\nu = 0.3$
- Modeled with beam solver (101 points) and thin-shell FEM solver (325 triangles) by F. Cirak

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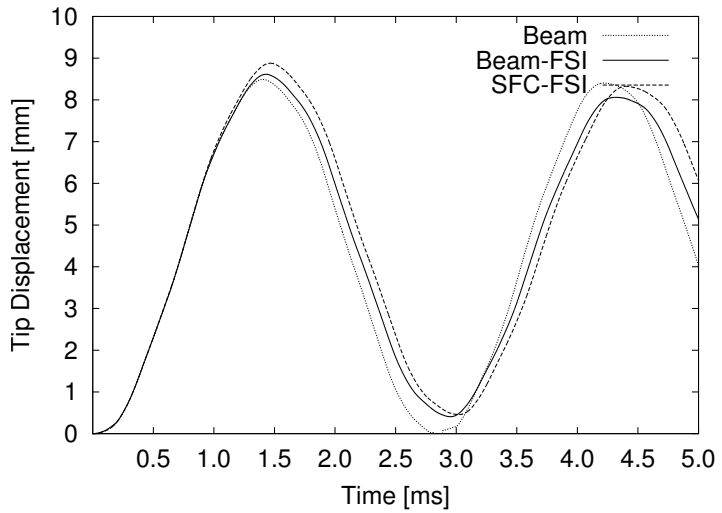
- Left: Coupling verification with constant instantenous loading by $\Delta p = 100\,\mathrm{kPa}$



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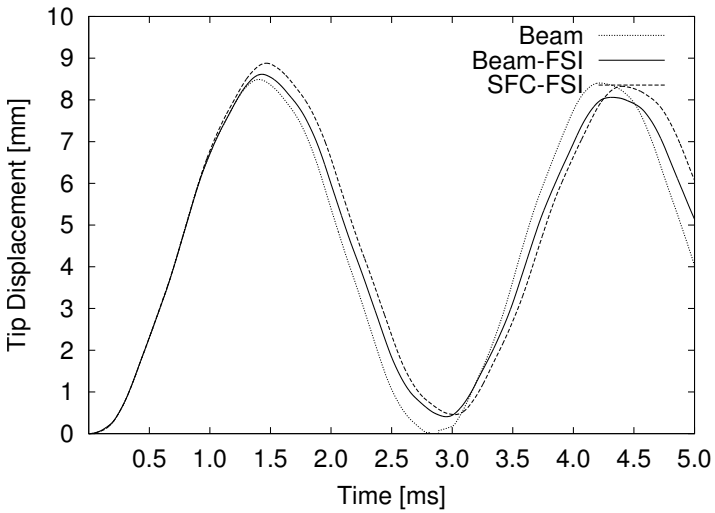
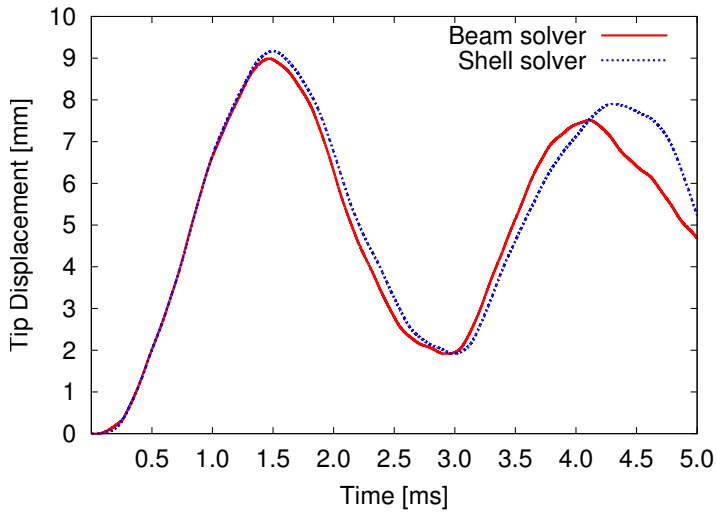
- Right: FSI verification with Mach 1.21 shockwave in air ($\gamma = 1.4$)

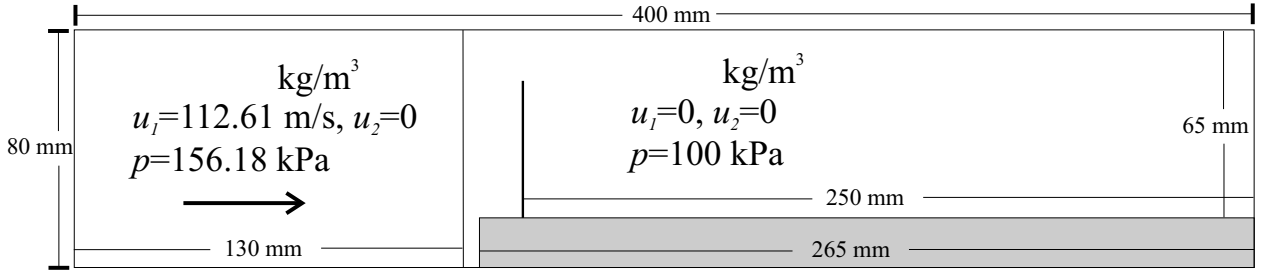




Shock-driven elastic panel motion

Test case suggested by [Giordano et al., 2005]

- Forward facing step geometry, fixed walls everywhere except at inflow



- SAMR base mesh $320 \times 64(\times 2)$, $r_{1,2} = 2$

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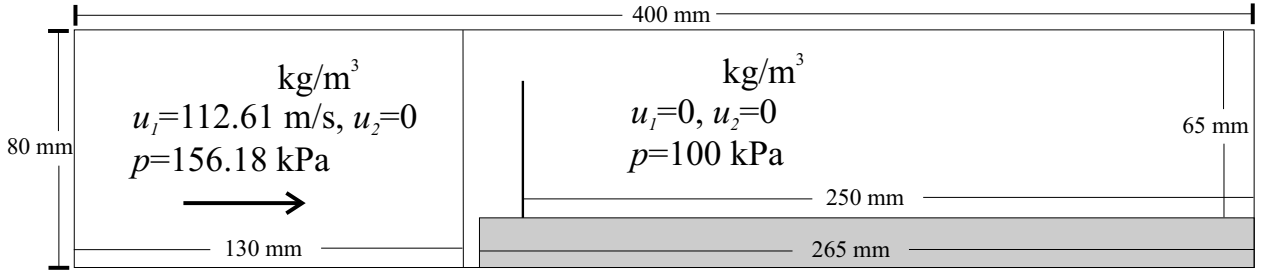
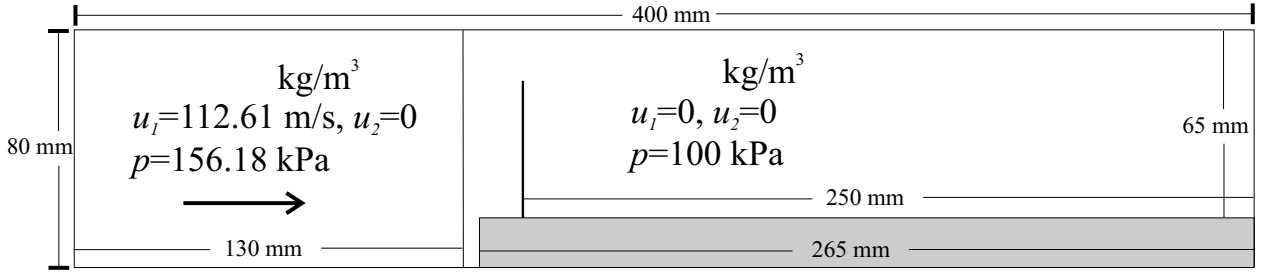


- SAMR base mesh $320 \times 64 (\times 2)$, $r_{1,2} = 2$
- Intel 3.4GHz Xeon dual processors, GB Ethernet interconnect
  - Beam-FSI: 12.25 h CPU on 3 fluid CPU + 1 solid CPU
    `code/doc/html/capps/beam-amroc_2VibratingBeam_2src_2FluidProblem_8h_source.html`,
    `code/doc/html/capps/beam-amroc_2VibratingBeam_2src_2SolidProblem_8h_source.html`
  - FEM-FSI: 322 h CPU on 14 fluid CPU + 2 solid CPU
    `code/doc/html/capps/sfc-amroc_2VibratingPanel_2src_2FluidProblem_8h_source.html`,
    `code/doc/html/capps/VibratingPanel_2src_2ShellManagerSpecific_8h_source.html`

Shock-driven elastic panel motion

Test case suggested by [Giordano et al., 2005]

- Forward facing step geometry, fixed walls everywhere except at inflow



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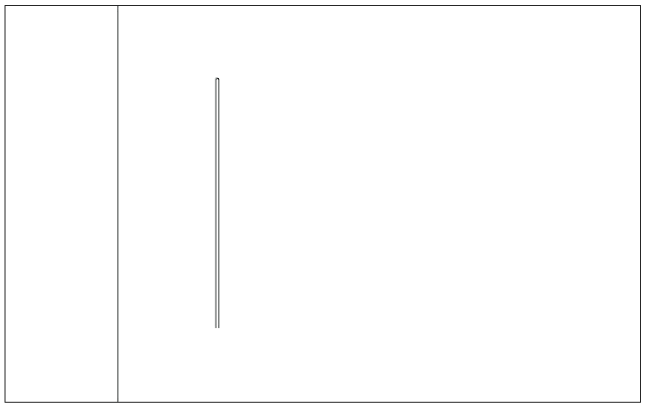

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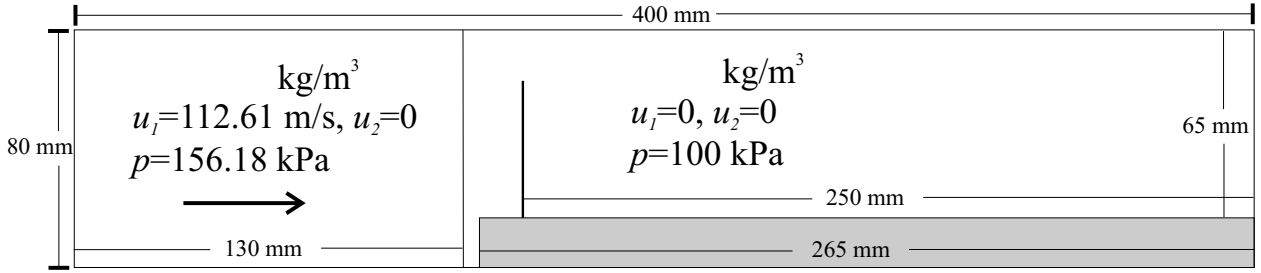

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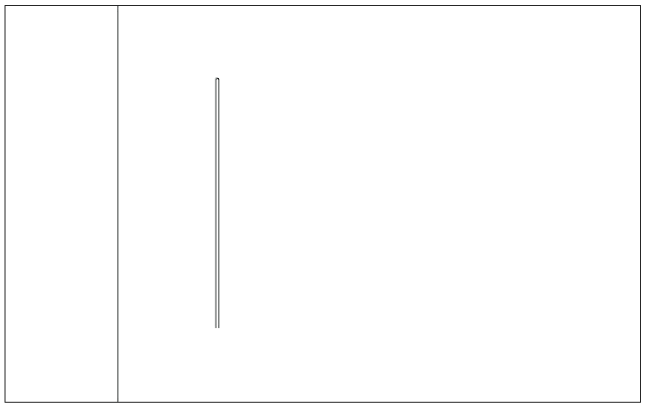

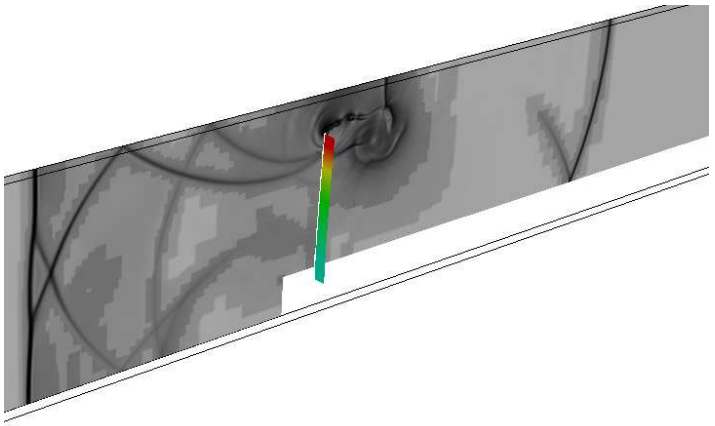

$t = 0.43$ ms after impact

Shock-driven elastic panel motion

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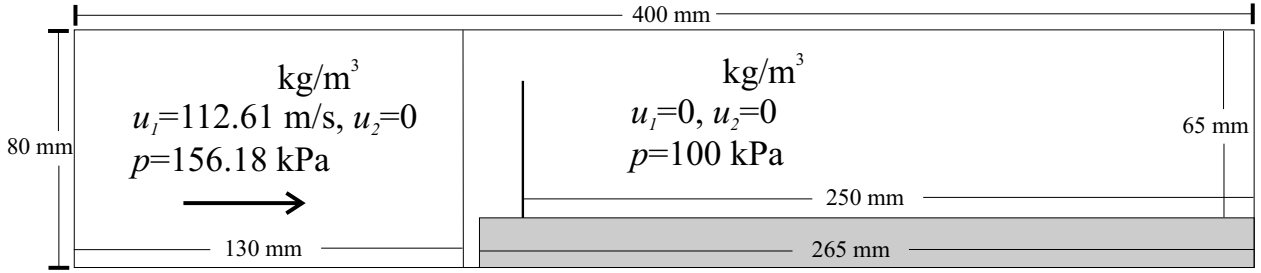

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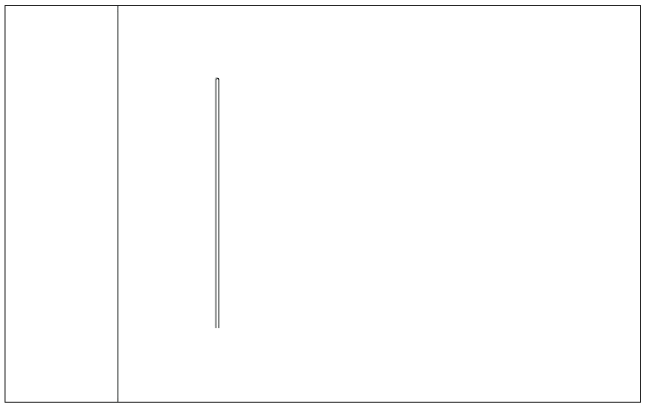

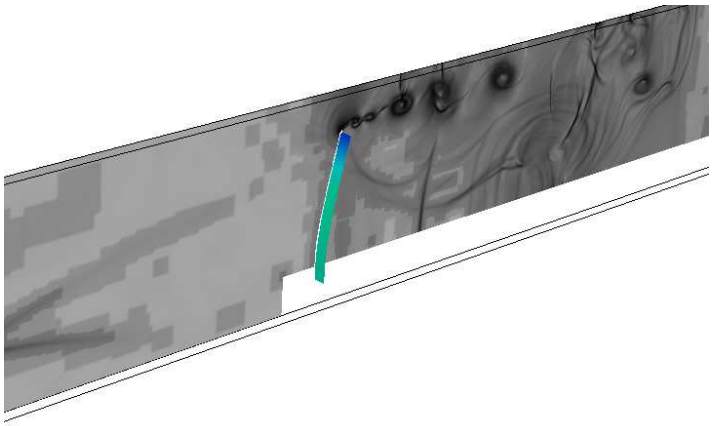

$t = 1.56$ ms after impact

Detonation-driven plastic deformation

Chapman-Jouguet detonation in a tube filled with a stoichiometric ethylene and oxygen ($\mathrm{C_2H_4} + 3\,\mathrm{O_2}$, $295\,\mathrm{K}$) mixture. Euler equations with single exothermic reaction $A \longrightarrow B$

$$\partial_t \rho + \partial_{x_n}(\rho u_n) = 0\ , \quad \partial_t(\rho u_k) + \partial_{x_n}(\rho u_k u_n + \delta_{kn} p) = 0\ , k = 1, \ldots, d$$

$$\partial_t(\rho E) + \partial_{x_n}(u_n(\rho E + p)) = 0\ , \quad \partial_t(Y\rho) + \partial_{x_n}(Y\rho u_n) = \psi$$

with

$$p = (\gamma - 1)(\rho E - \frac{1}{2}\rho u_n u_n - \rho Y q_0) \quad \text{and} \quad \psi = -kY\rho \exp\left(\frac{-E_\mathrm{A}\rho}{p}\right)$$

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modeled with heuristic detonation model by [Mader, 1979]

$V := \rho^{-1}$, $V_0 := \rho_0^{-1}$, $V_{CJ} := \rho_{CJ}$
$Y' := 1 - (V - V_0)/(V_{CJ} - V_0)$
`If` $0 \le Y' \le 1$ `and` $Y > 10^{-8}$ `then`
  `If` $Y < Y'$ `and` $Y' < 0.9$ `then` $Y' := 0$
  `If` $Y' < 0.99$ `then` $p' := (1 - Y')p_{CJ}$
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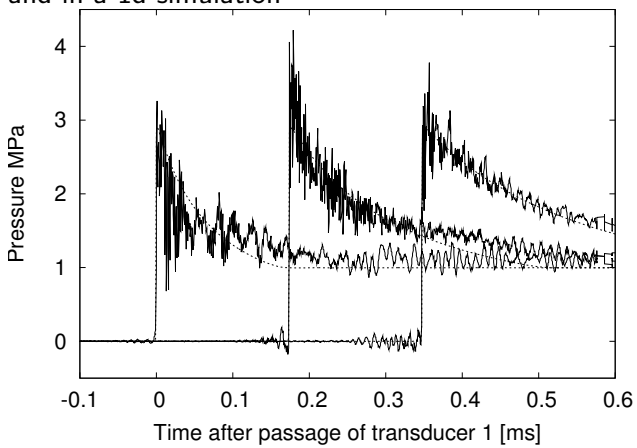
Comparison of the pressure traces in the experiment and in a 1d simulation



Tube with flaps

- Fluid: VanLeer FVS
  - Detonation model with $\gamma = 1.24$, $p_{\mathrm{CJ}} = 3.3\,\mathrm{MPa}$, $D_{\mathrm{CJ}} = 2376\,\mathrm{m/s}$
  - AMR base level: $104 \times 80 \times 242$, $r_{1,2} = 2$, $r_3 = 4$
  - $\sim 4 \cdot 10^7$ cells instead of $7.9 \cdot 10^9$ cells (uniform)
  - Tube and detonation fully refined
  - Thickening of 2D mesh: $0.81\,\mathrm{mm}$ on both sides (real $0.445\,\mathrm{mm}$)

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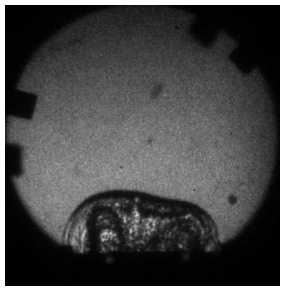
$0.032\,\mathrm{ms}$



$0.030\,\mathrm{ms}$

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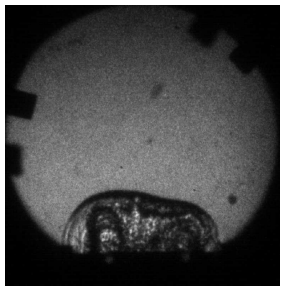
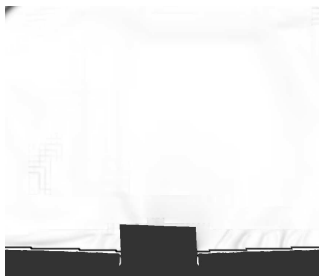
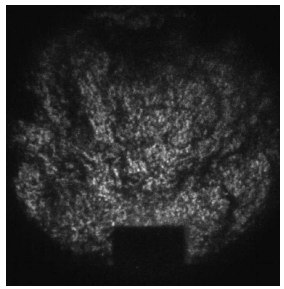
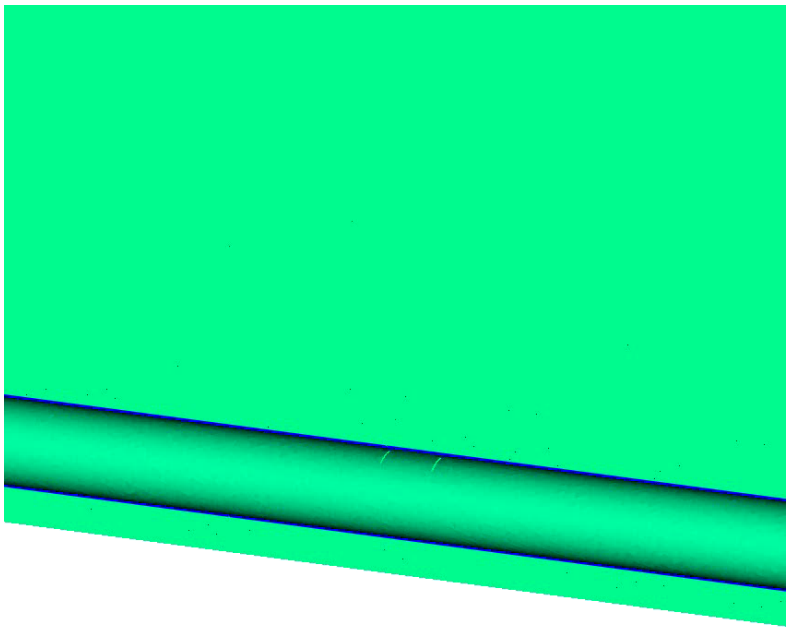

0.030 ms


0.212 ms


0.210 ms

Tube with flaps: results



Fluid density and diplacement in y-direction in solid

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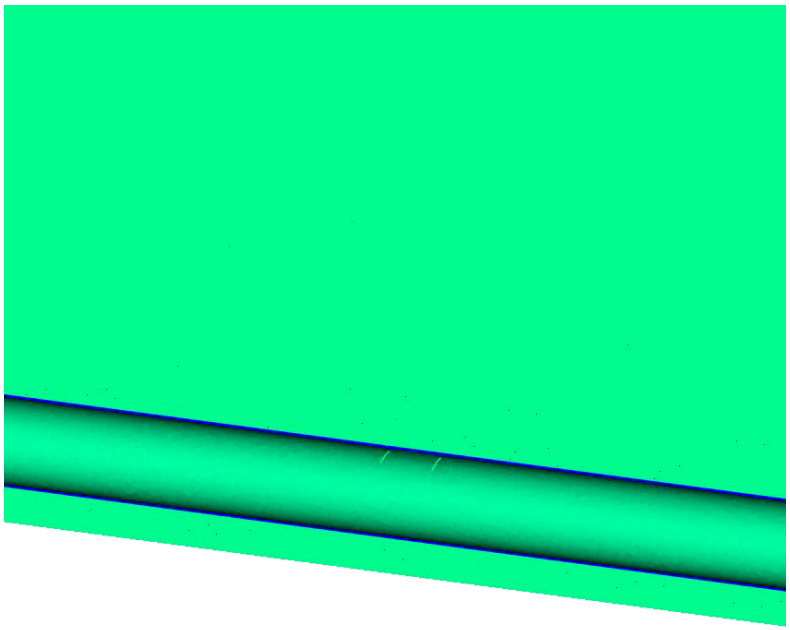

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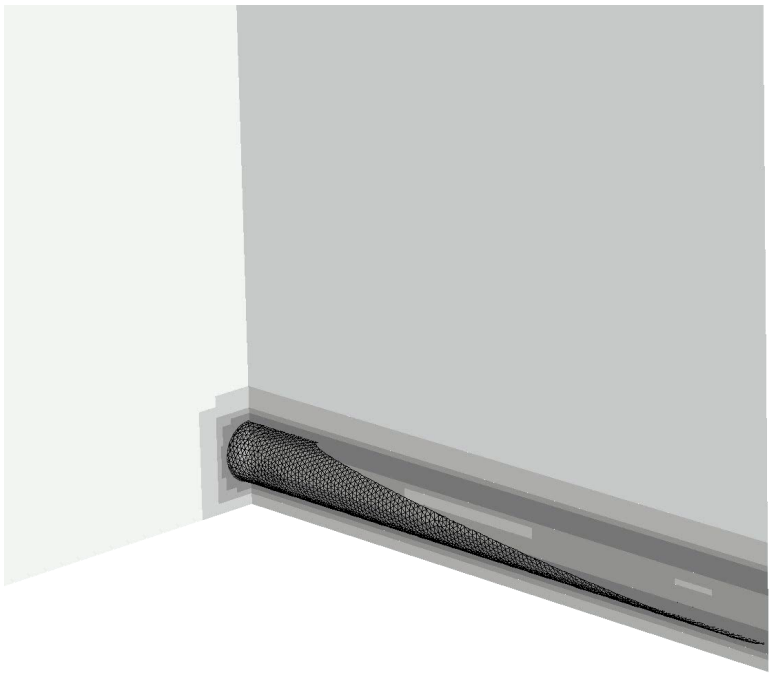
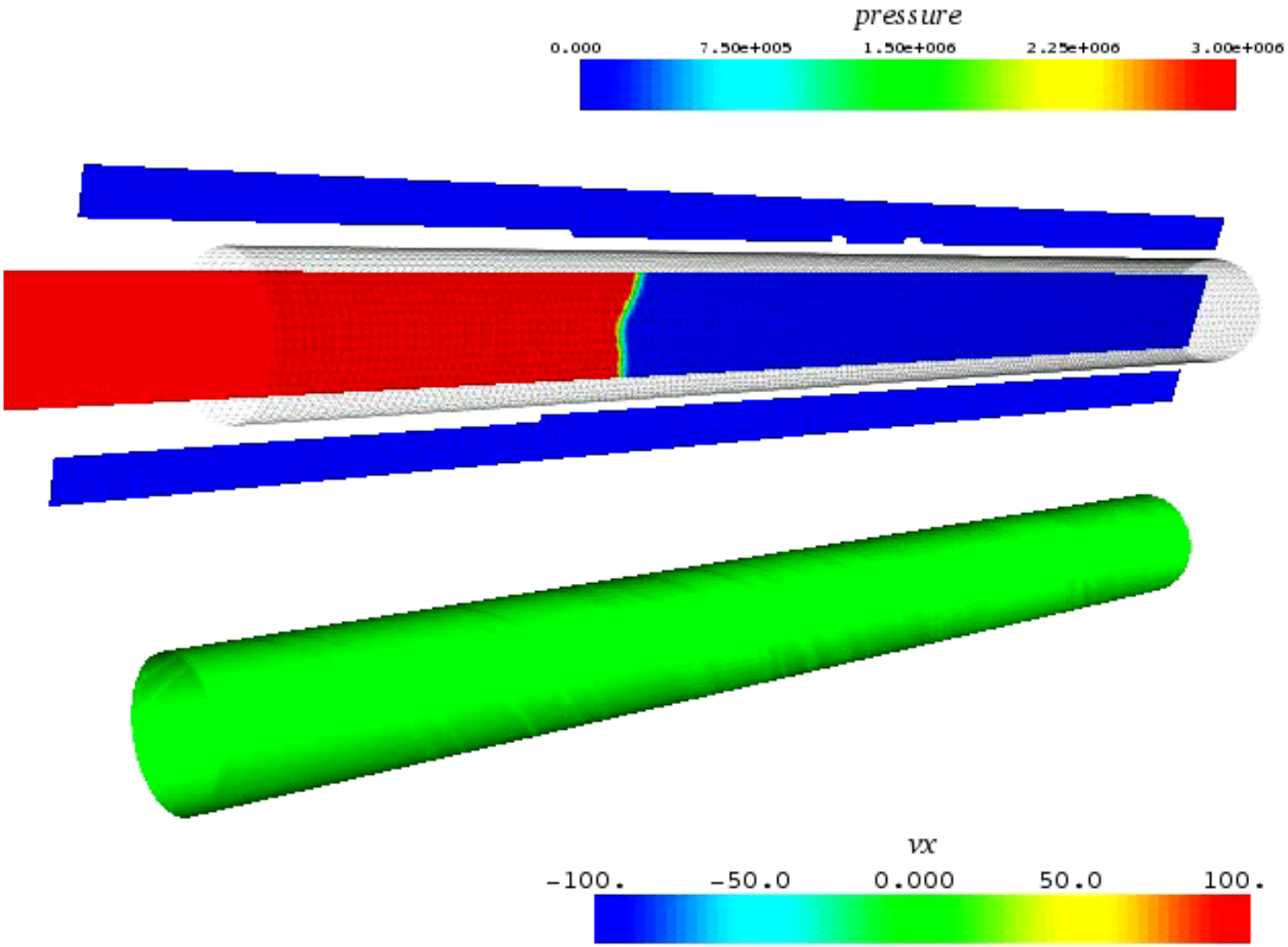


Schlieren plot of fluid density on refinement levels

[Cirak et al., 2007]

code/doc/html/capps/sfc-amroc_2TubeCJBurnFlaps_2src_2FluidProblem_8h_source.html,
code/doc/html/capps/TubeCJBurnFlaps_2src_2ShellManagerSpecific_8h_source.html

Coupled fracture simulation



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code/doc/html/capps/TubeCJBurnFrac_2src_2ShellManagerSpecific_8h_source.html

Underwater explosion modeling

Volume fraction based two-component model with $\sum_{i=1}^{m} \alpha^i = 1$, that defines mixture quantities as

$$\rho = \sum_{i=1}^{m} \alpha^i \rho^i \,, \quad \rho u_n = \sum_{i=1}^{m} \alpha^i \rho^i u_n^i \,, \quad \rho e = \sum_{i=1}^{m} \alpha^i \rho^i e^i$$

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and the overall set of equations [Shyue, 1998]

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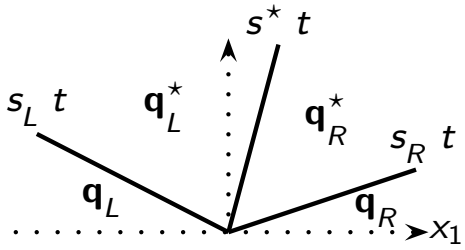
Oscillation free at contacts: [Abgrall and Karni, 2001][Shyue, 2006]

Approximate Riemann solver

Use HLLC approach because of robustness and positivity preservation

$$\mathbf{q}^{HLLC}(x_1,t) = \begin{cases} \mathbf{q}_L\,, & x_1 < s_L\,t, \\ \mathbf{q}_L^\star\,, & s_L\,t \le x_1 < s^\star\,t, \\ \mathbf{q}_R^\star\,, & s^\star\,t \le x_1 \le s_R\,t, \\ \mathbf{q}_R\,, & x_1 > s_R\,t, \end{cases}$$



Wave speed estimates [Davis, 1988] $s_L = \min\{u_{1,L} - c_L, u_{1,R} - c_R\}$,
$s_R = \max\{u_{1,L} + c_L, u_{1,R} + c_R\}$

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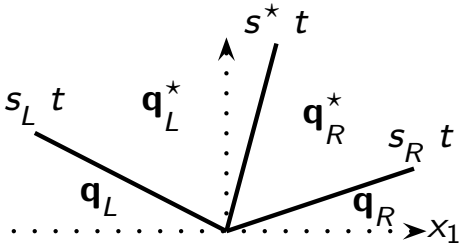

Wave speed estimates [Davis, 1988] $s_L = \min\{u_{1,L} - c_L, u_{1,R} - c_R\}$,
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Unkown state [Toro et al., 1994]

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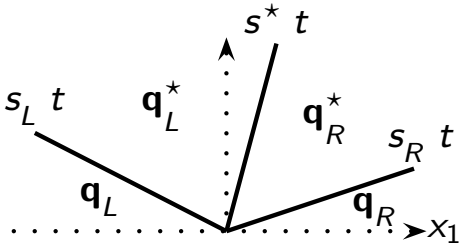

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$$\mathbf{q}_\tau^\star = \left[\eta, \eta s^\star, \eta u_2, \eta\left[\frac{(\rho E)_\tau}{\rho_\tau} + (s^\star - u_{1,\tau})\left(s_\tau + \frac{p_\tau}{\rho_\tau(s_\tau - u_{1,\tau})}\right)\right], \frac{1}{\gamma_\tau - 1}, \frac{\gamma_\tau p_{\infty,\tau}}{\gamma_\tau - 1}\right]^T$$

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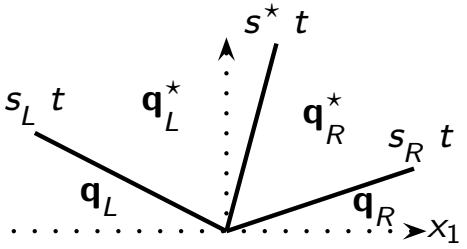

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Evaluate waves as $\mathcal{W}_1 = \mathbf{q}_L^\star - \mathbf{q}_L$, $\mathcal{W}_2 = \mathbf{q}_R^\star - \mathbf{q}_L^\star$, $\mathcal{W}_3 = \mathbf{q}_R - \mathbf{q}_R^\star$ and $\lambda_1 = s_L$, $\lambda_2 = s^\star$, $\lambda_3 = s_R$ to compute the fluctuations $\mathcal{A}^-\Delta = \sum_{\lambda_\nu<0}\lambda_\nu\,\mathcal{W}_\nu$, $\mathcal{A}^+\Delta = \sum_{\lambda_\nu\ge0}\lambda_\nu\,\mathcal{W}_\nu$ for $\nu = \{1,2,3\}$

Overall scheme: Wave Propagation method [Shyue, 2006]

Underwater explosion FSI simulations

- Air: $\gamma^A = 1.4$, $p_\infty^A = 0$, $\rho^A = 1.29\,\mathrm{kg/m^3}$
- Water: $\gamma^W = 7.415$, $p_\infty^W = 296.2\,\mathrm{MPa}$, $\rho^W = 1027\,\mathrm{kg/m^3}$

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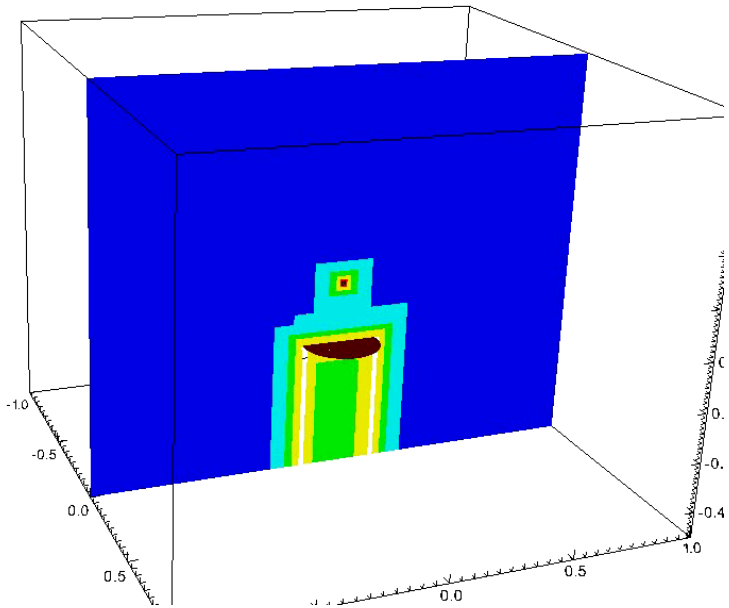
  - Water-filled shocktube $1.3\,\mathrm{m}$ with driver piston [Deshpande et al., 2006]
  - Piston simulated with separate level set, see [Deiterding et al., 2009] for pressure wave
  - $\rho_s = 8920\,\mathrm{kg/m3}$, $E = 130\,\mathrm{GPa}$, $\nu = 0.31$, J2 plasticity model, $\sigma_y = 38.5\,\mathrm{MPa}$, cohesive interface model, max. tensile stress $\sigma_c = 525\,\mathrm{MPa}$

Underwater explosion simulation

- AMR base grid $50 \times 40 \times 50$, $r_{1,2,3} = 2$, $r_4 = 4$, $l_c = 3$, highest level restricted to initial explosion center, 3rd and 4th level to plate vicinity
- Triangular mesh with 8148 elements
- Computations of 1296 coupled time steps to $t_{end} = 1\,\mathrm{ms}$
- 10+2 nodes 3.4 GHz Intel Xeon dual processor, $\sim 130\,\mathrm{h}$ CPU

Maximal deflection $[\mathrm{mm}]$

	Exp.	Sim.
$20\,\mathrm{g}, d = 25\,\mathrm{cm}$	28.83	25.88
$30\,\mathrm{g}, d = 30\,\mathrm{cm}$	30.09	27.31



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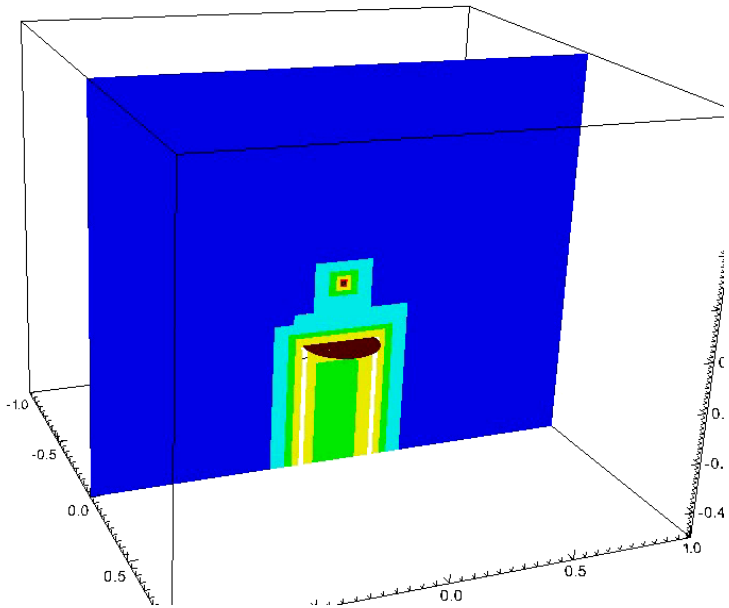


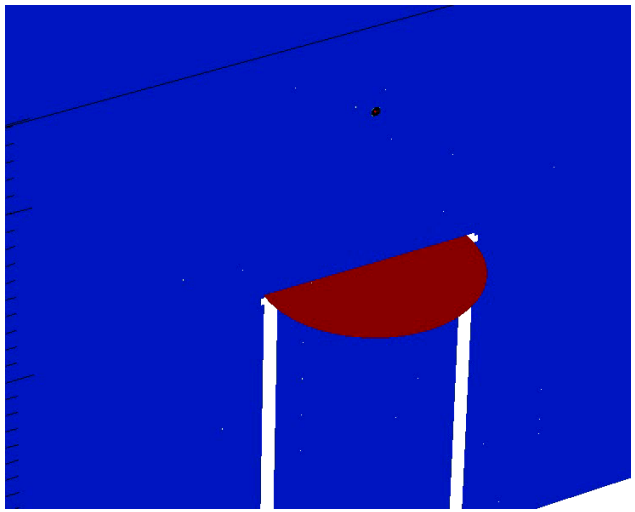
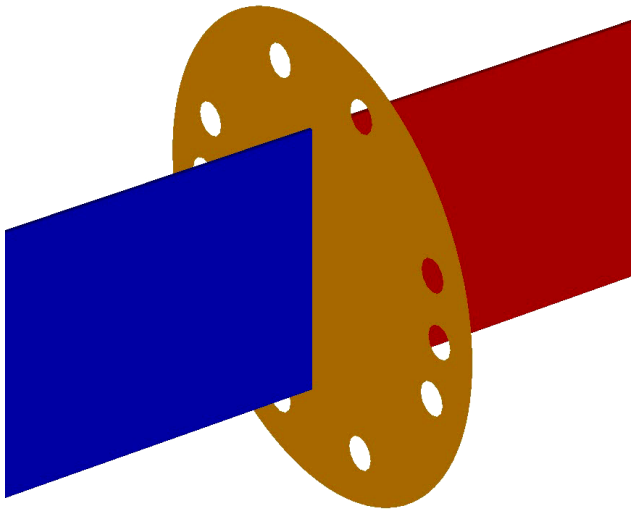


Plate in underwater shocktube

- AMR base mesh $374 \times 20 \times 20$, $r_{1,2} = 2$, $l_c = 2$, solid mesh: 8896 triangles
- $\sim$ 1250 coupled time steps to $t_{end} = 1\,\mathrm{ms}$
- 6+6 nodes 3.4 GHz Intel Xeon dual processor, $\sim 800\,\mathrm{h}$ CPU

`code/doc/html/capps/sfc-amroc_2WaterBlastFracture_2src_2FluidProblem_8h_source.html`,
`code/doc/html/capps/WaterBlastFracture_2src_2ShellManagerSpecific_8h_source.html`



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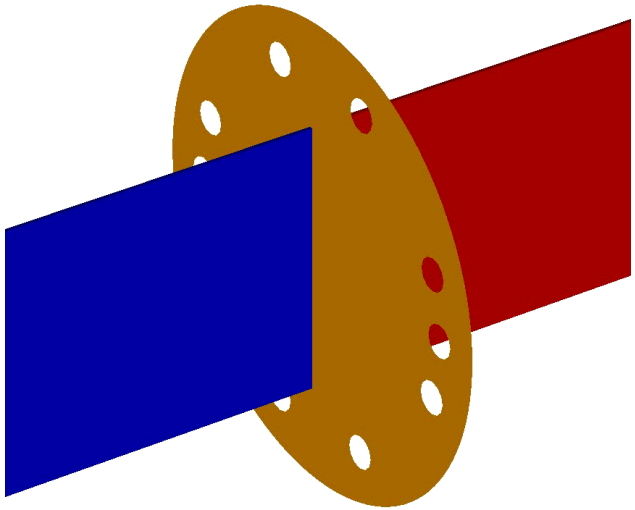

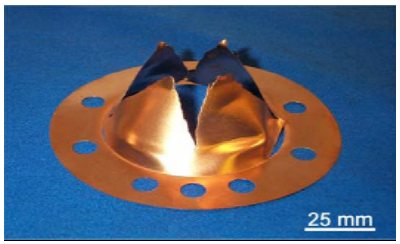

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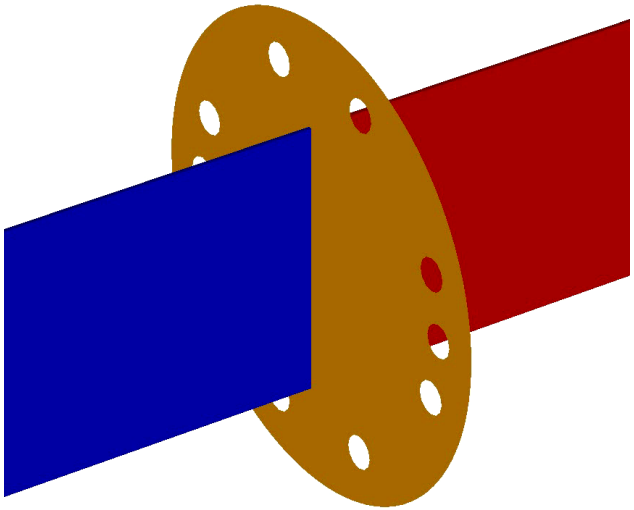

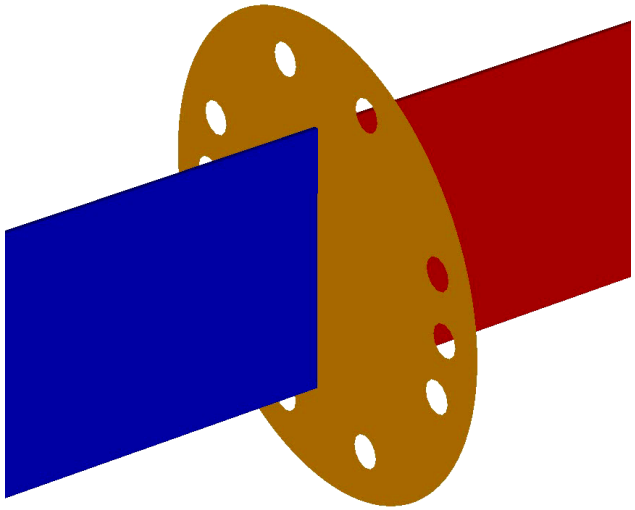

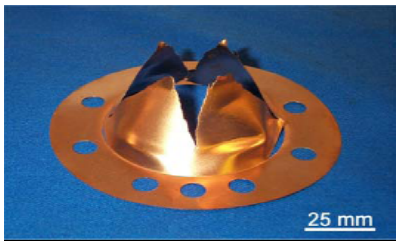

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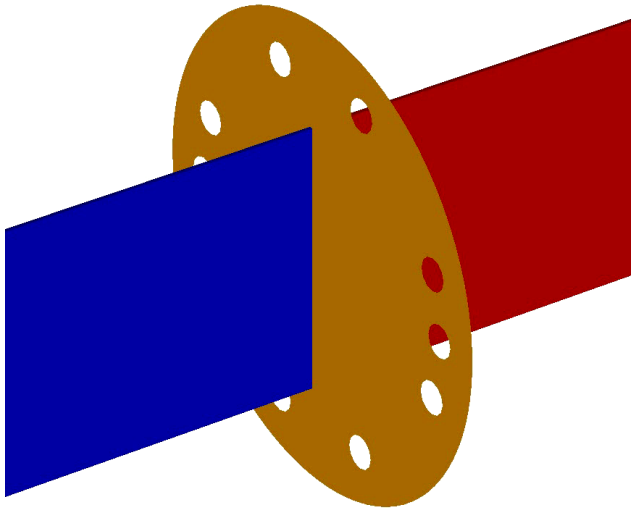

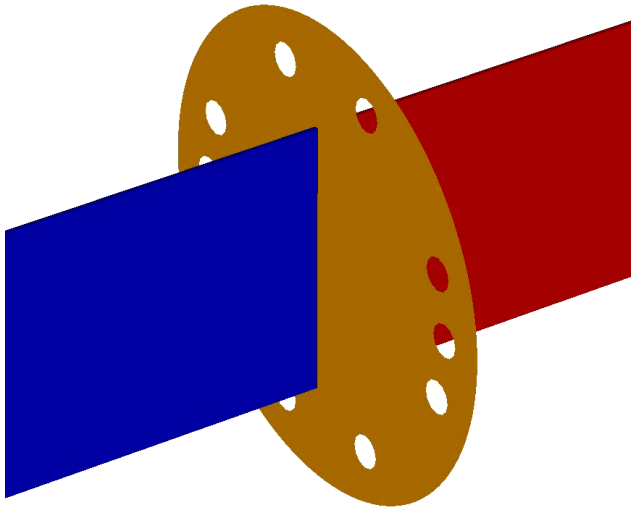

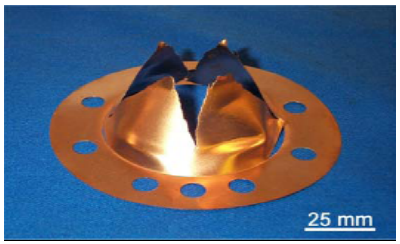

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Blast explosion in a multistory building

- $20\,\mathrm{m} \times 40\,\mathrm{m} \times 25\,\mathrm{m}$ seven-story building similar to [Luccioni et al., 2004]
- Spherical energy deposition $\equiv 400\,\mathrm{kg}$ TNT, $r = 0.5\,\mathrm{m}$ in lobby of building
- SAMR: $80 \times 120 \times 90$ base level, three additional levels $r_{1,2} = 2$, $l_{\mathrm{fsi}} = 1$, $k = 1$
- Simulation with ground: $1,070$ coupled time steps, $830\,\mathrm{h}$ CPU ($\sim 25.9\,\mathrm{h}$ wall time) on 31+1 cores
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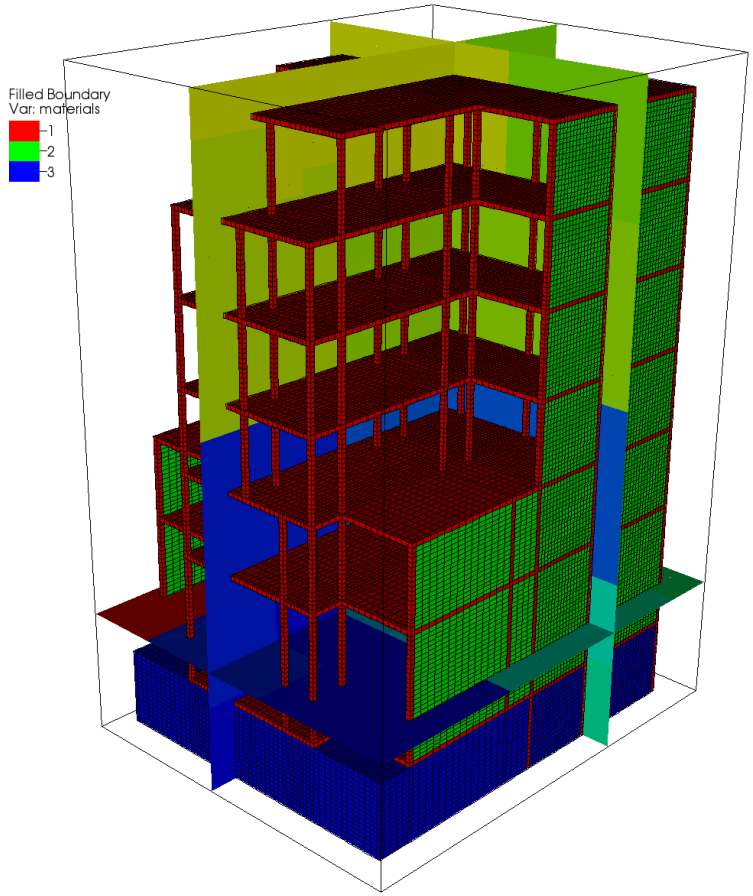

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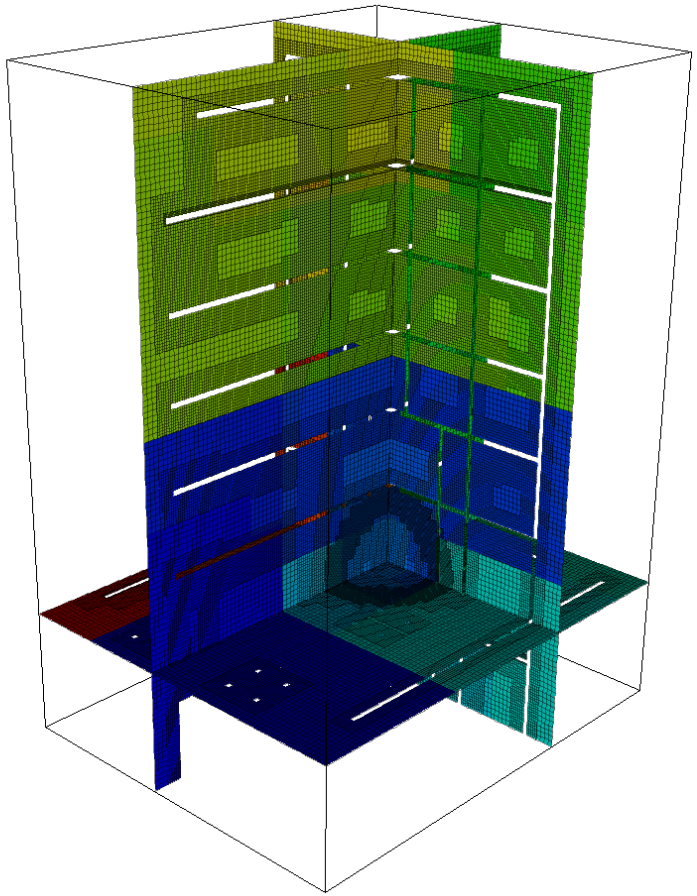

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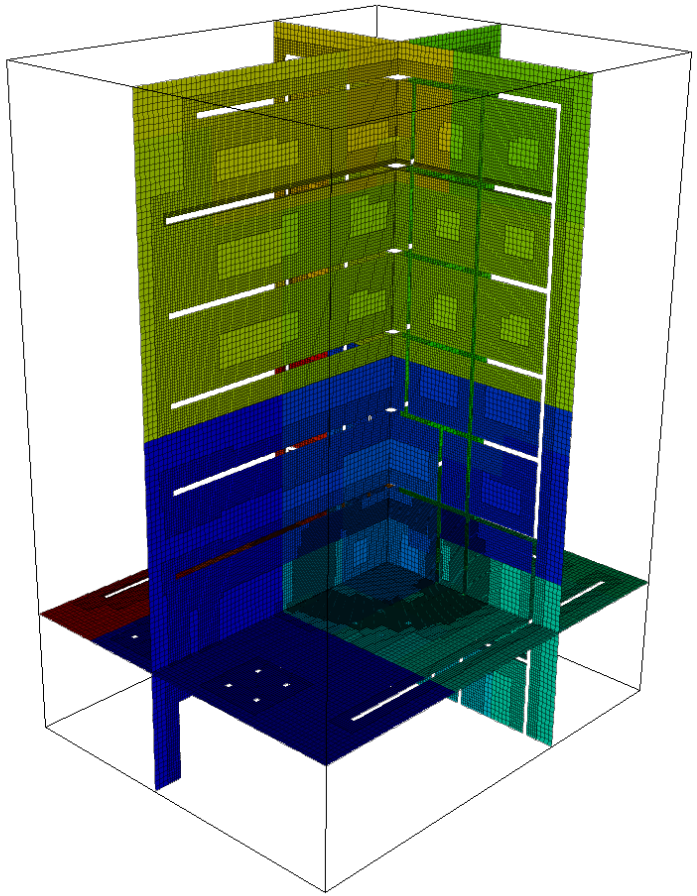

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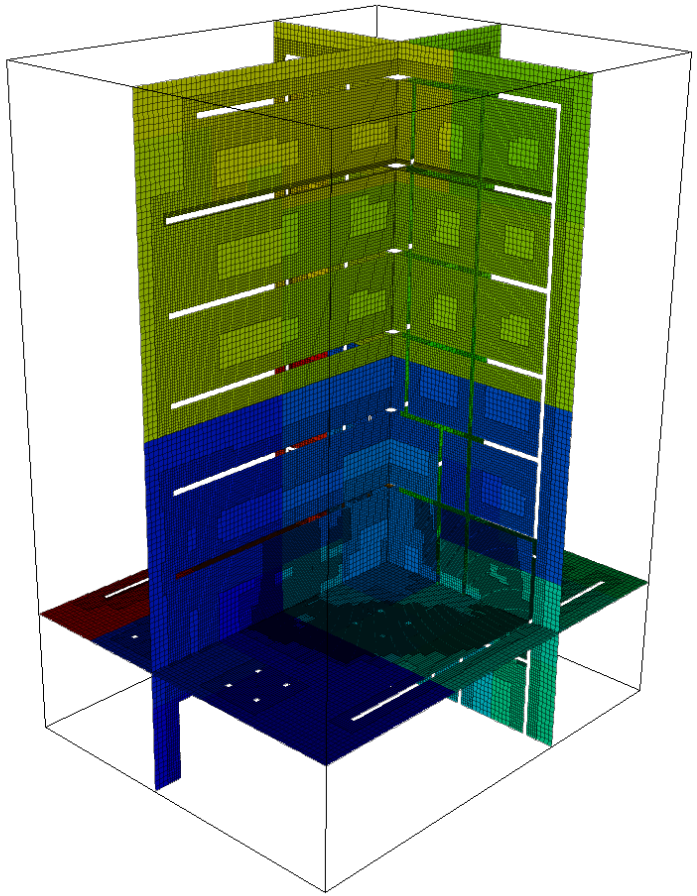

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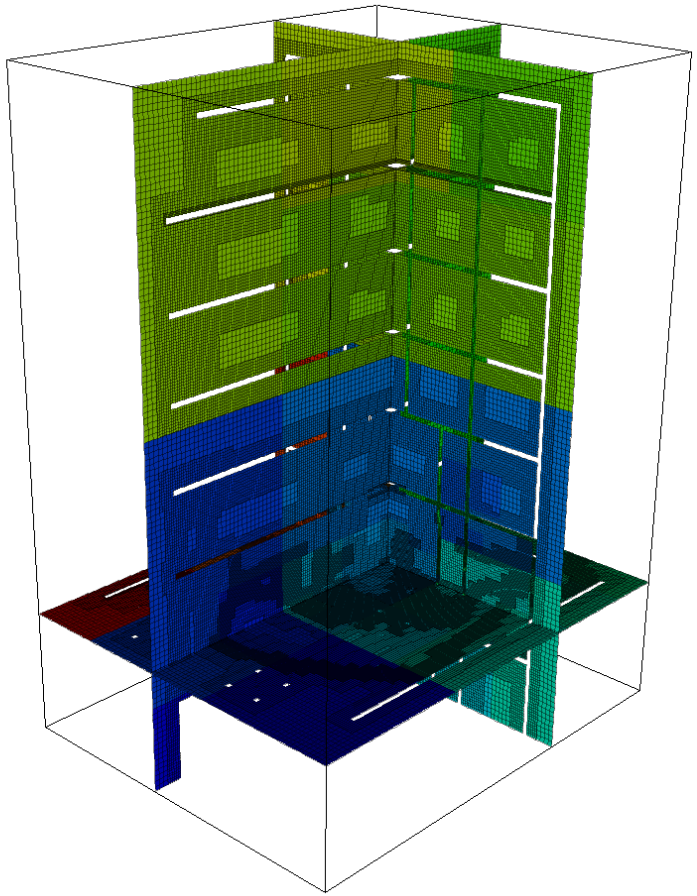

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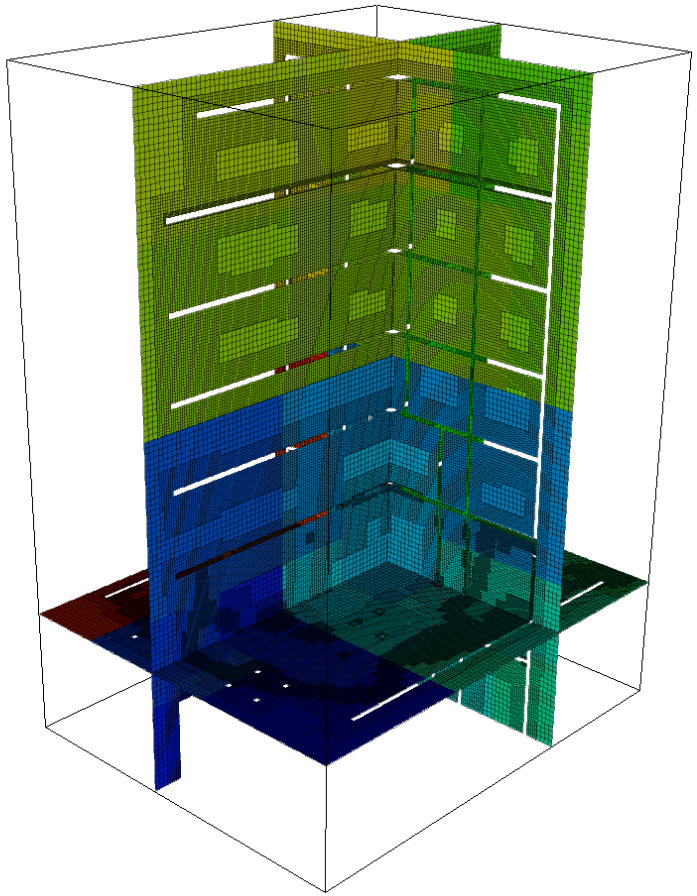

	ρ_s [kg/m^3]	σ_0 [MPa]	E_T [GPa]	β	K [GPa]	G [GPa]	$\bar{\epsilon}^p$	p_f [MPa]
Columns	2010	50	11.2	1.0	21.72	4.67	0.02	-30
Walls	2010	25	11.2	1.0	6.22	4.67	0.01	-15

Blast explosion in a multistory building

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- Simulation with ground: $1,070$ coupled time steps, $830\,\mathrm{h}$ CPU ($\sim 25.9\,\mathrm{h}$ wall time) on 31+1 cores
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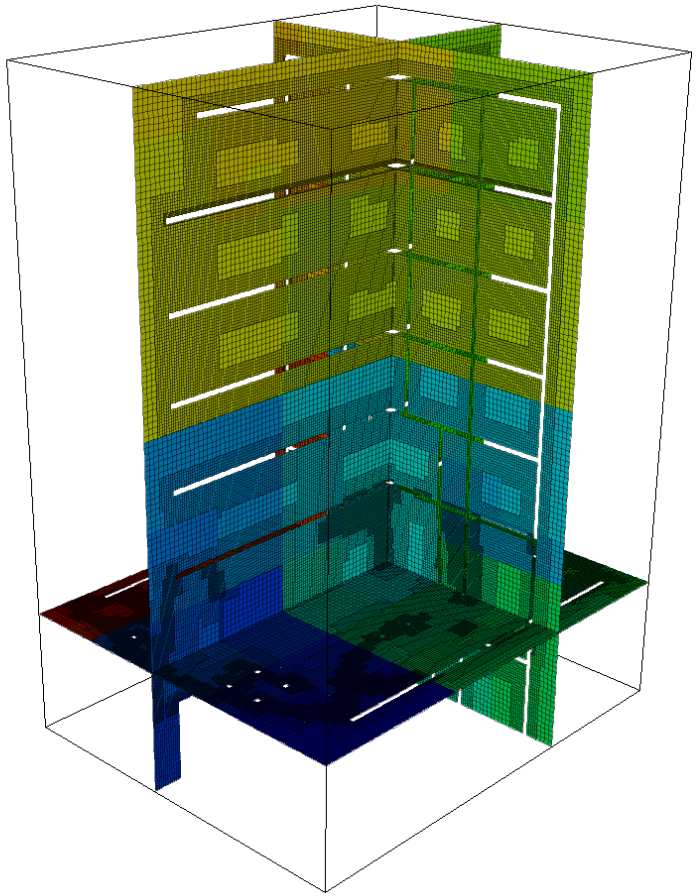

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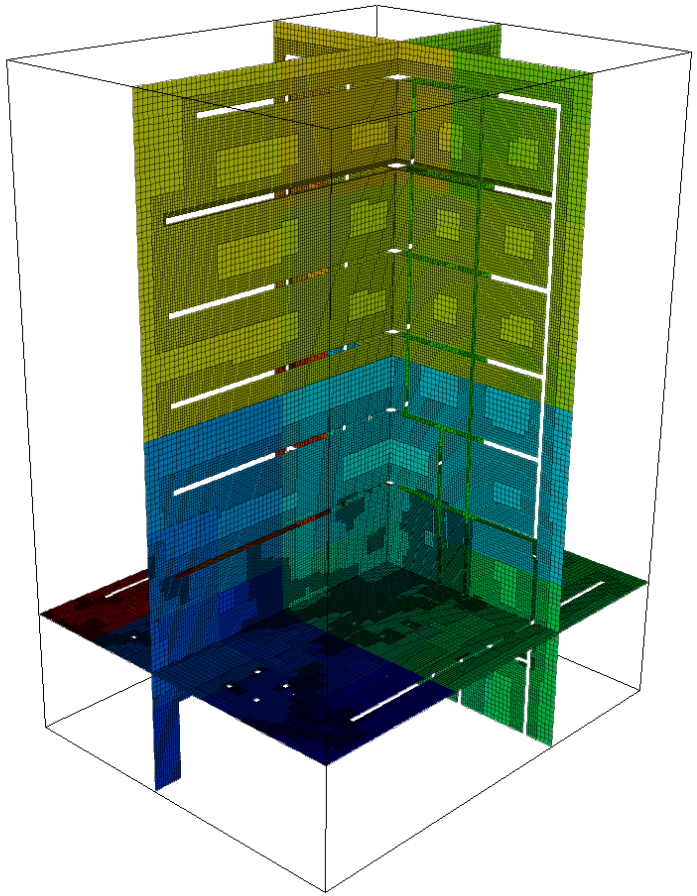

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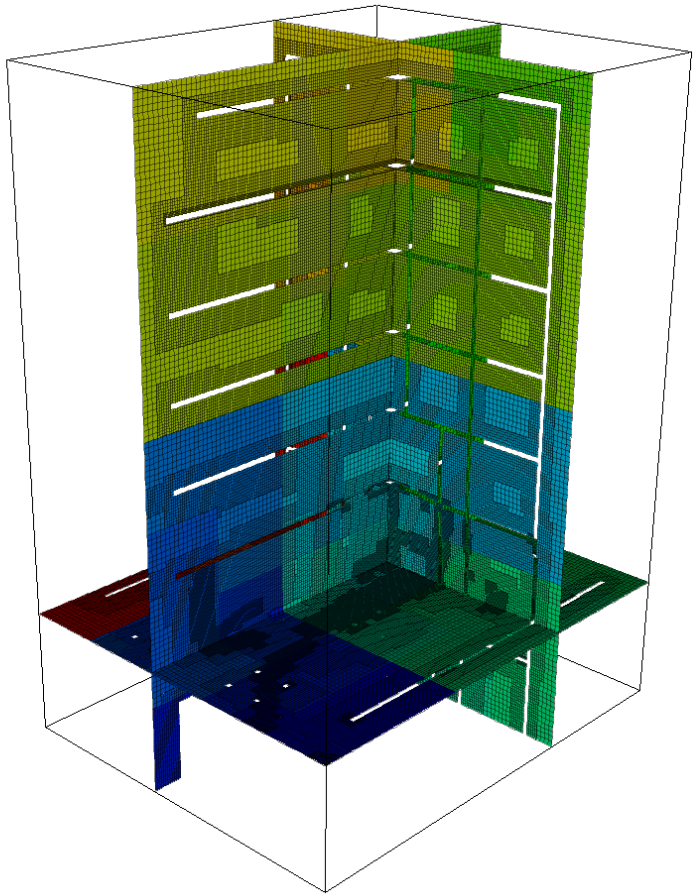

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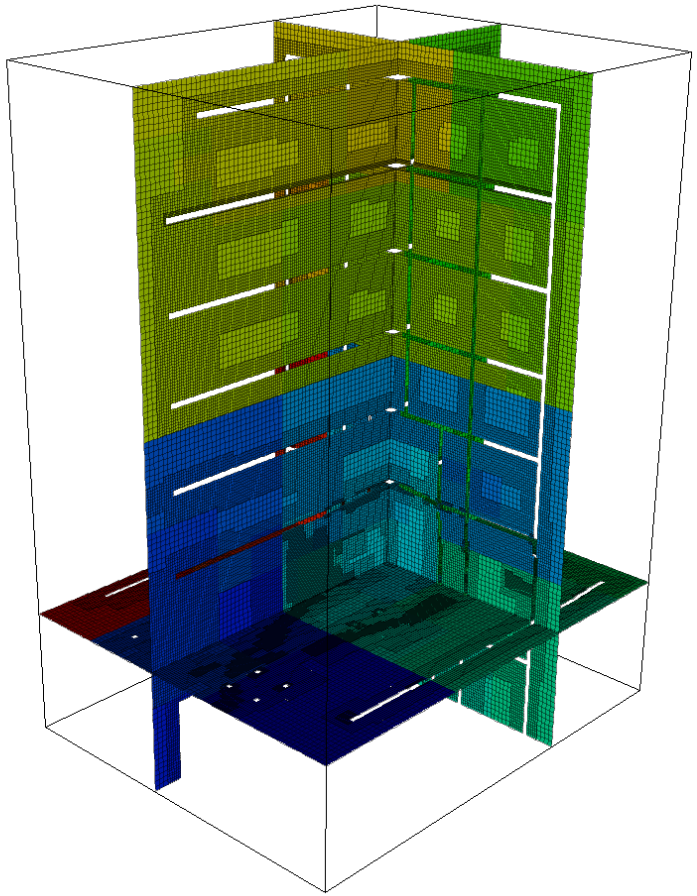

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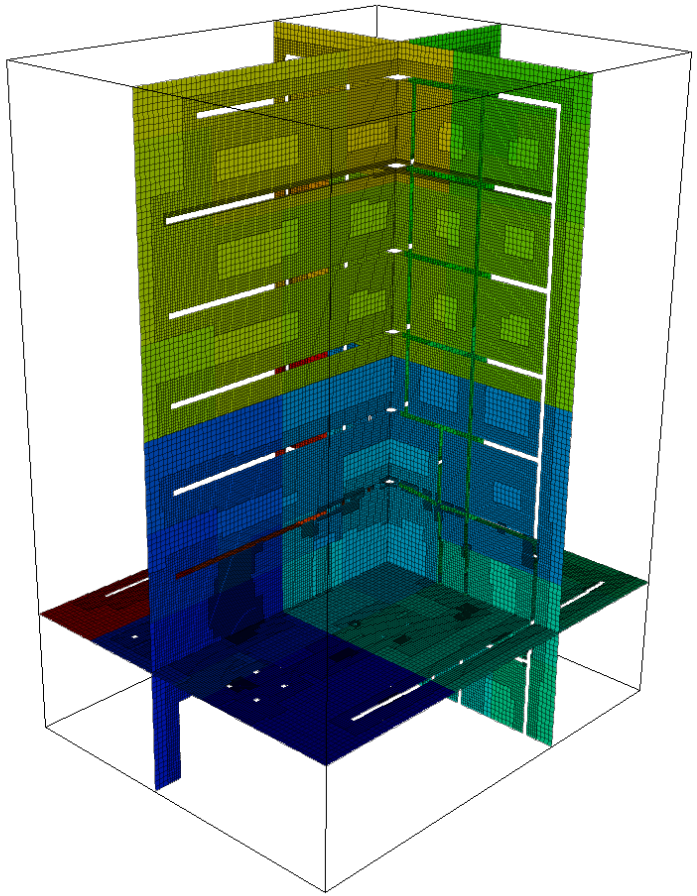

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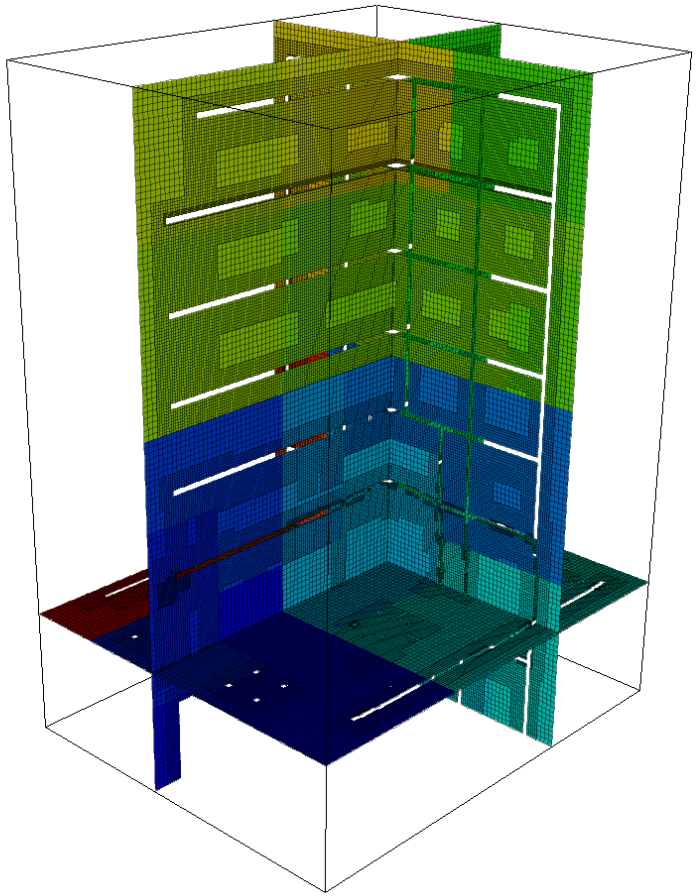

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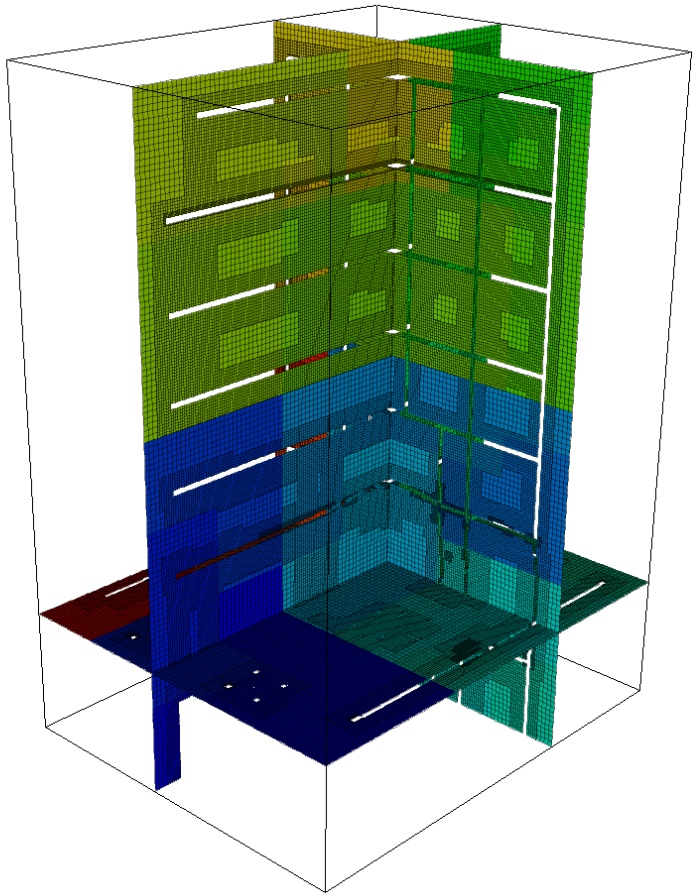

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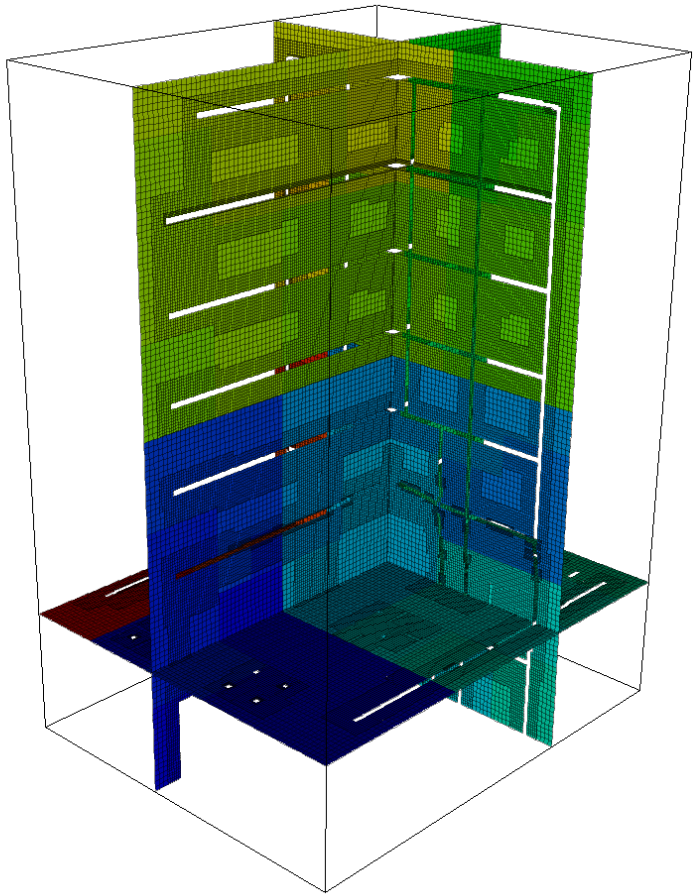

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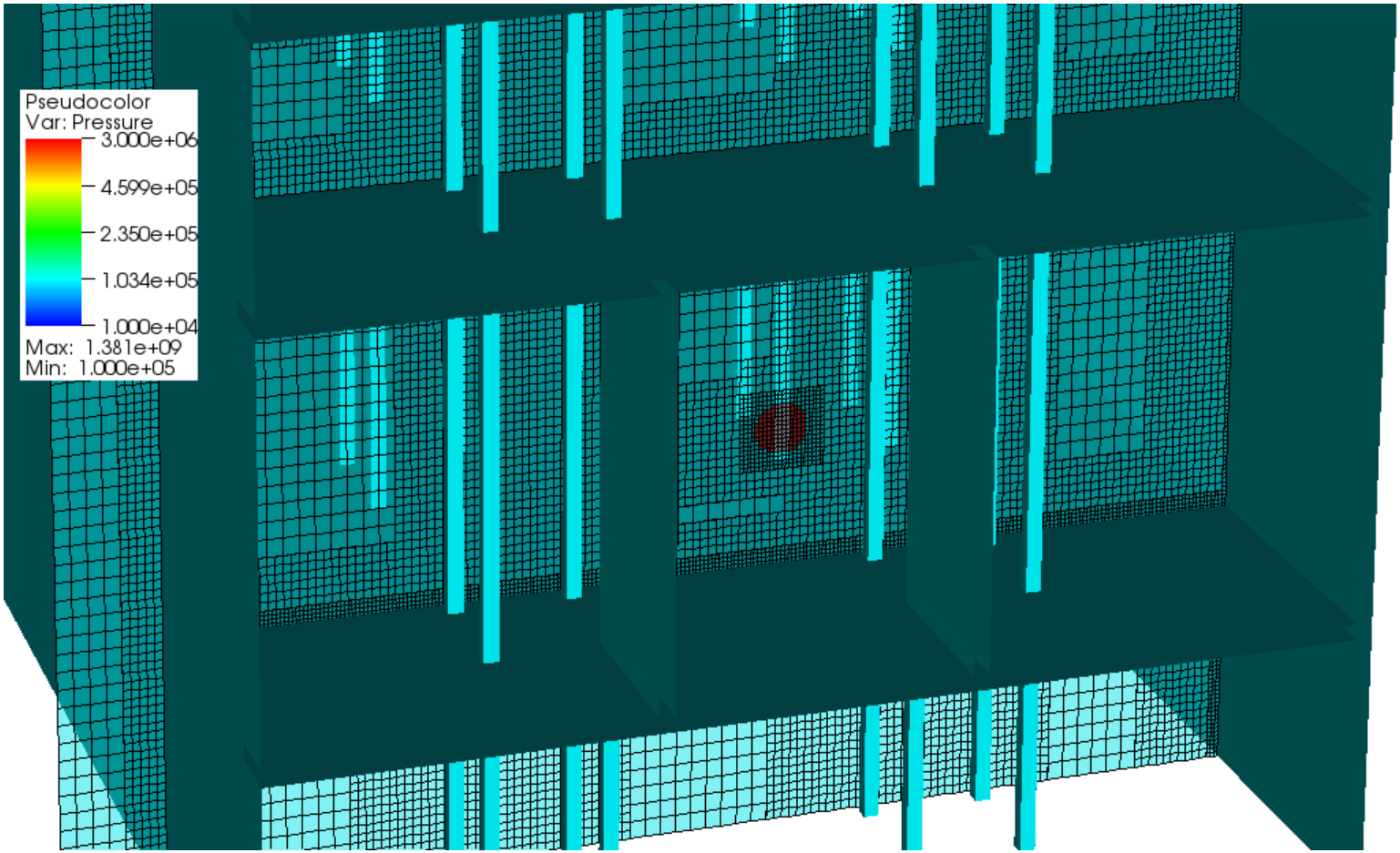
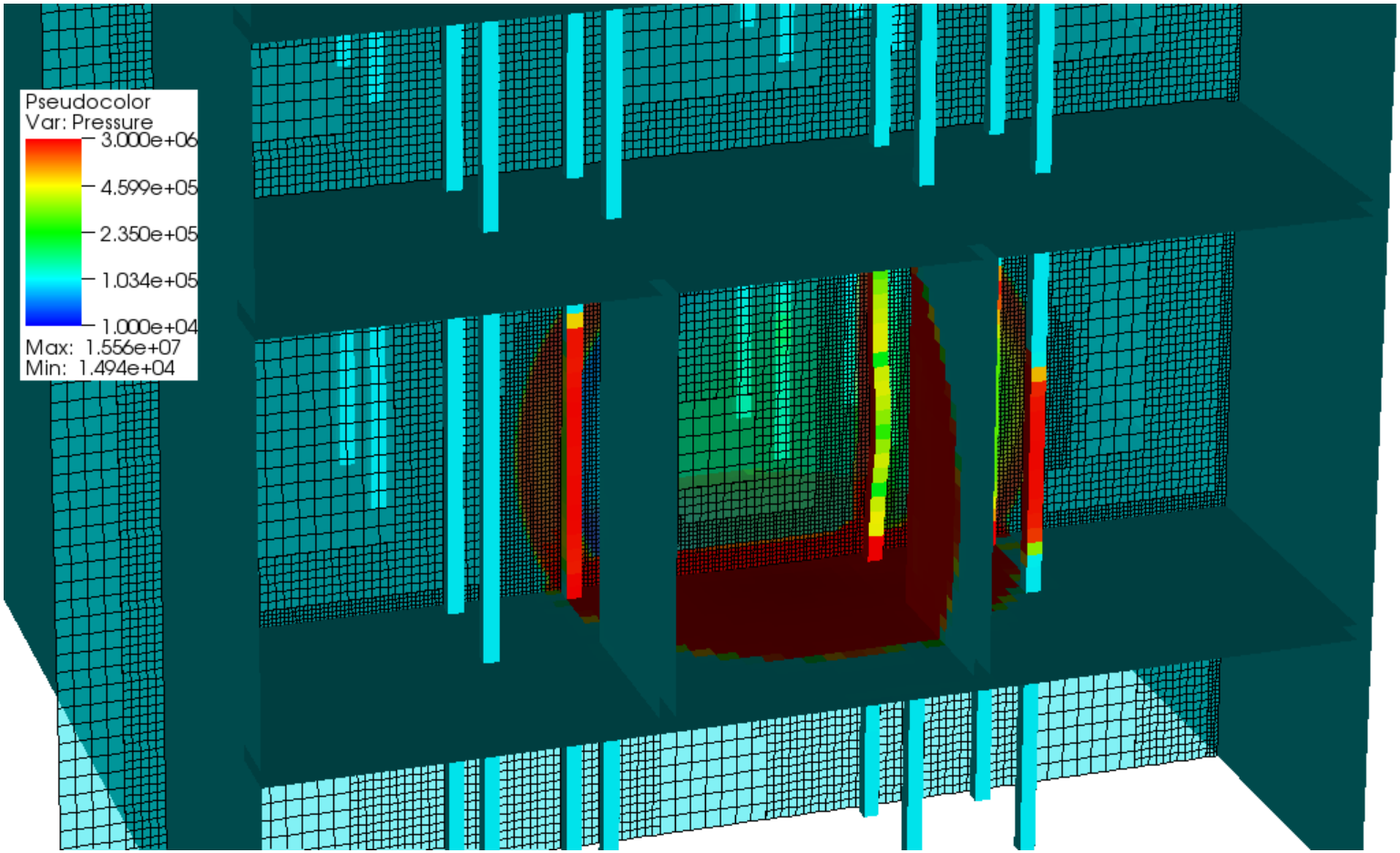
Blast explosion in a multistory building – II

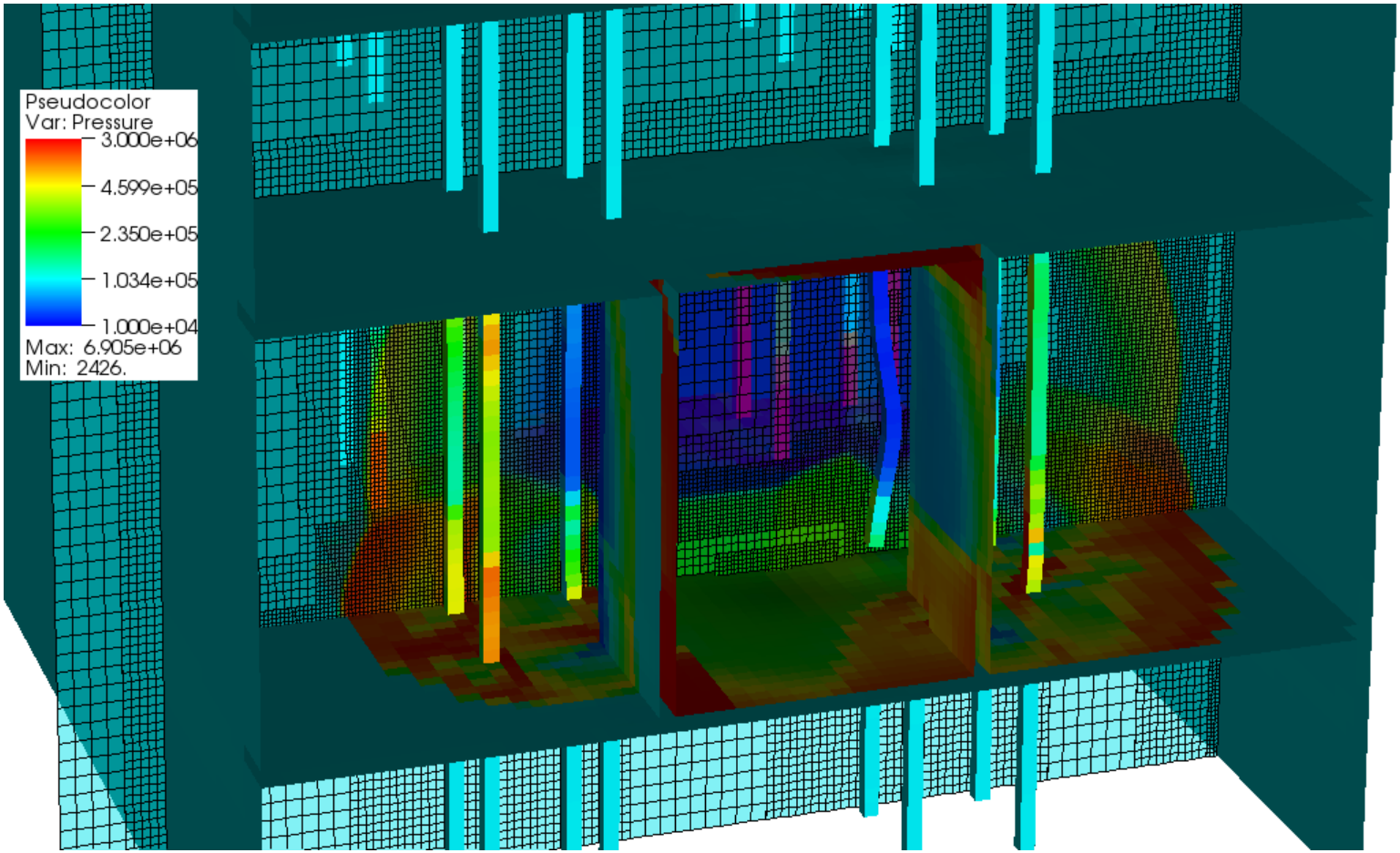


$t = 0$

Blast explosion in a multistory building – II

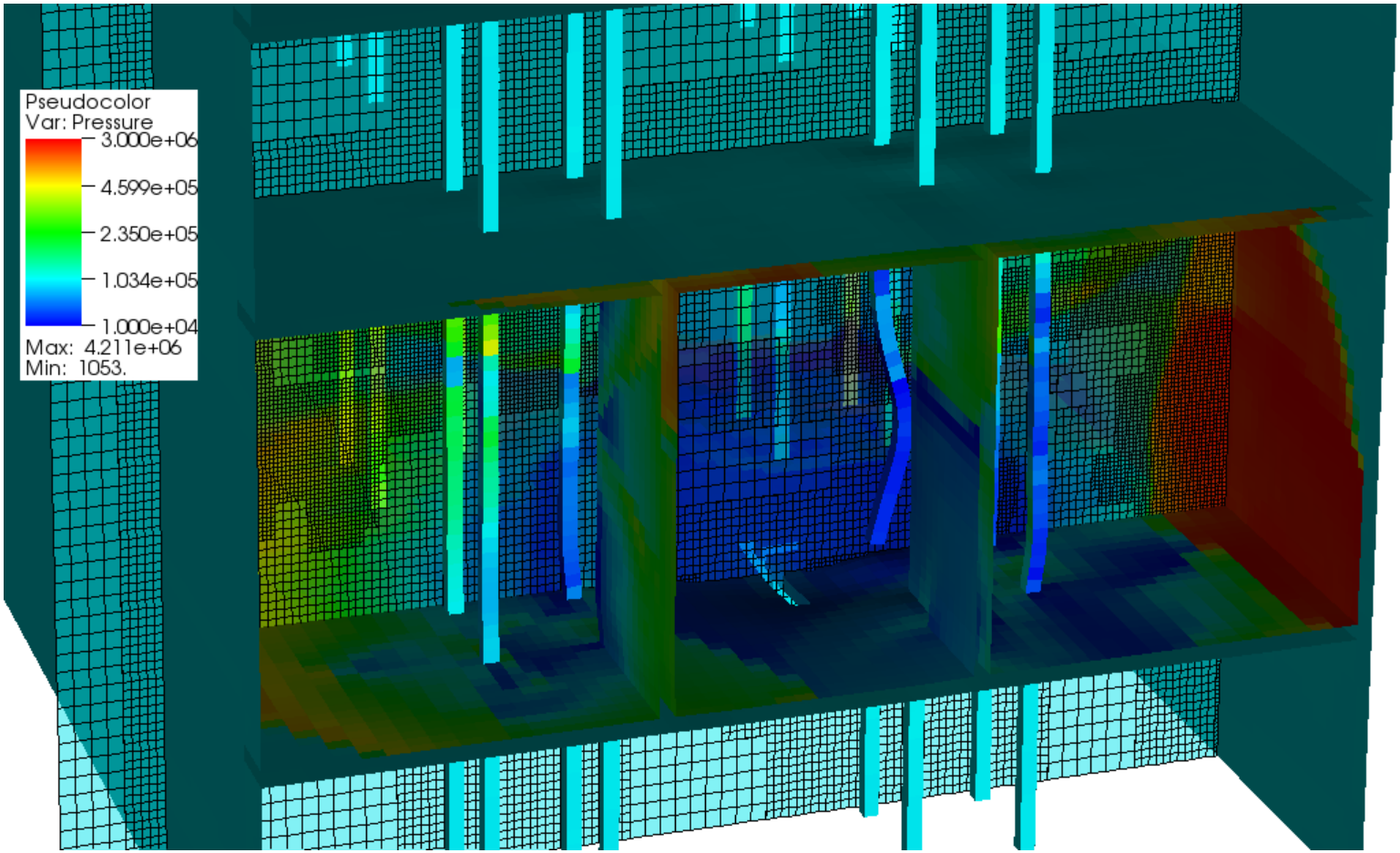


Blast explosion in a multistory building – II

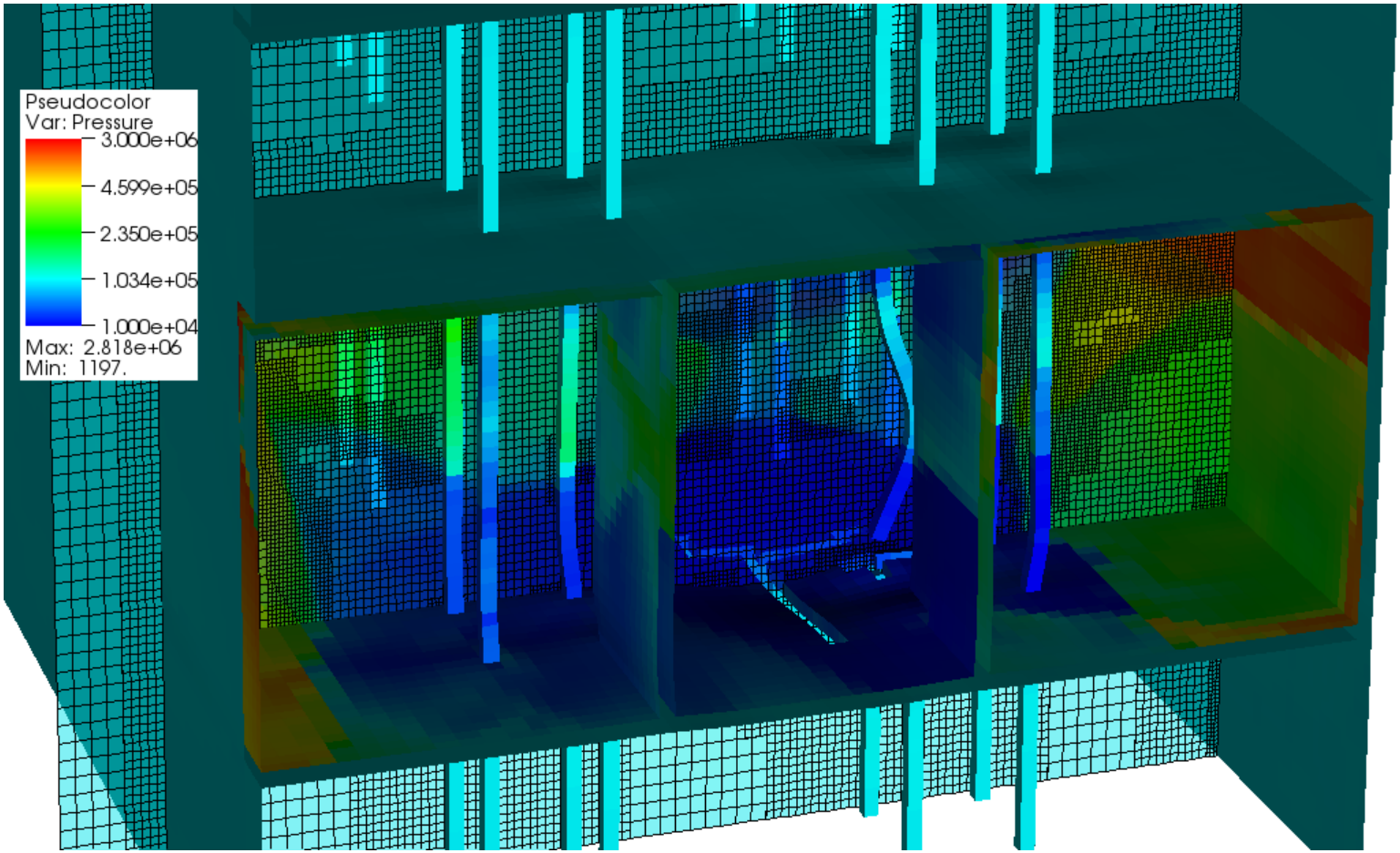


$t = 6.1\,\mathrm{ms}$

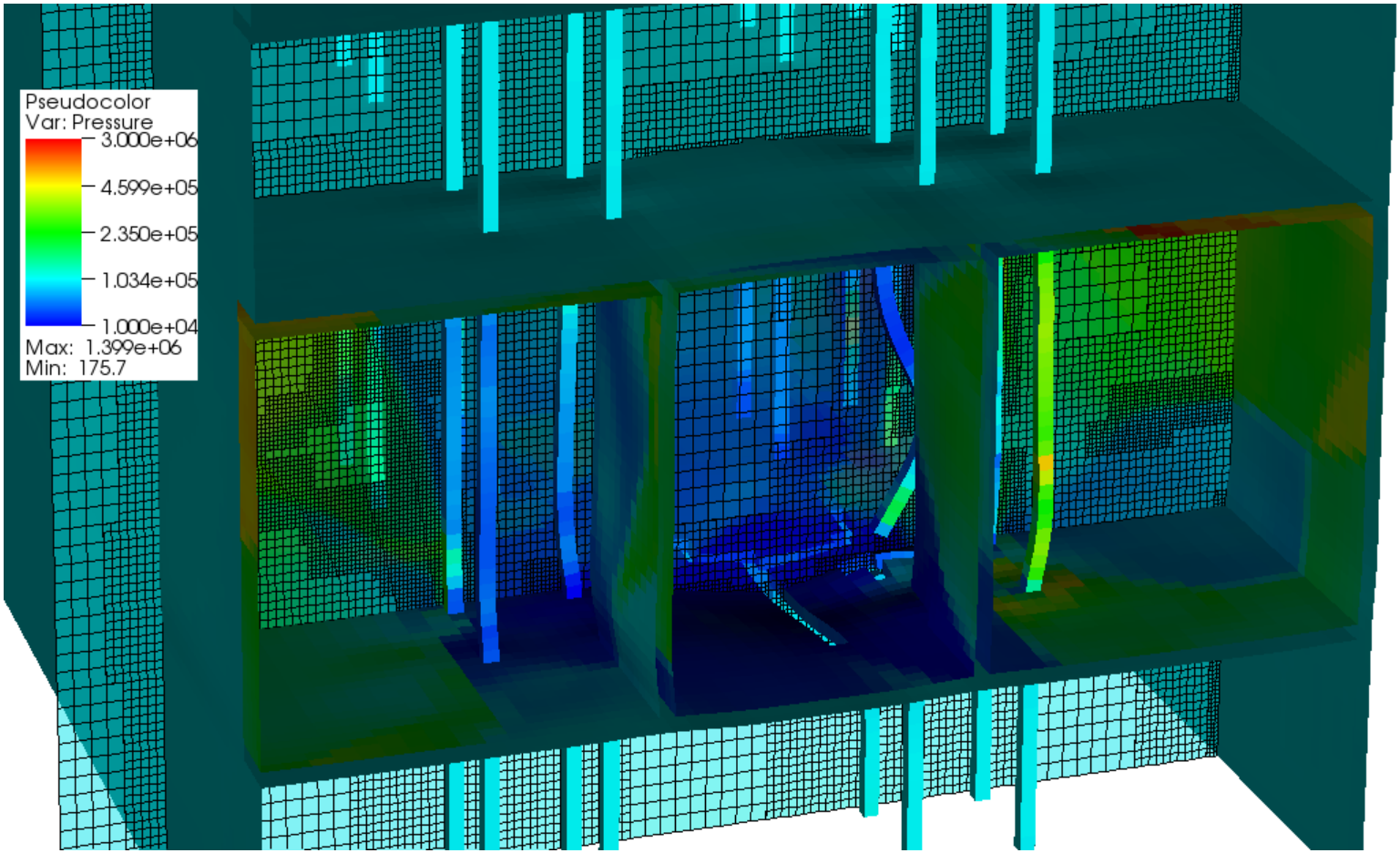
Blast explosion in a multistory building – II



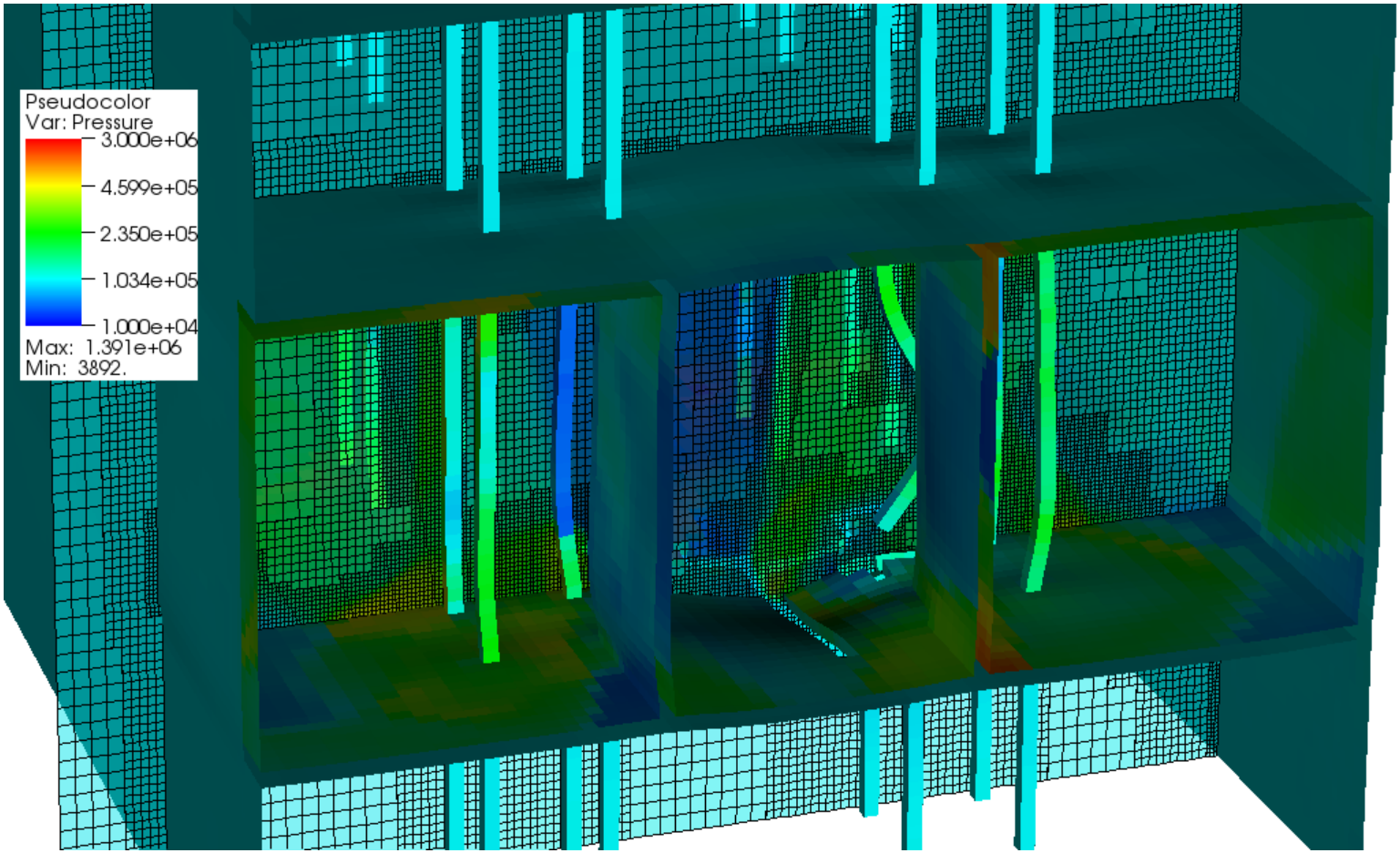
Blast explosion in a multistory building – II



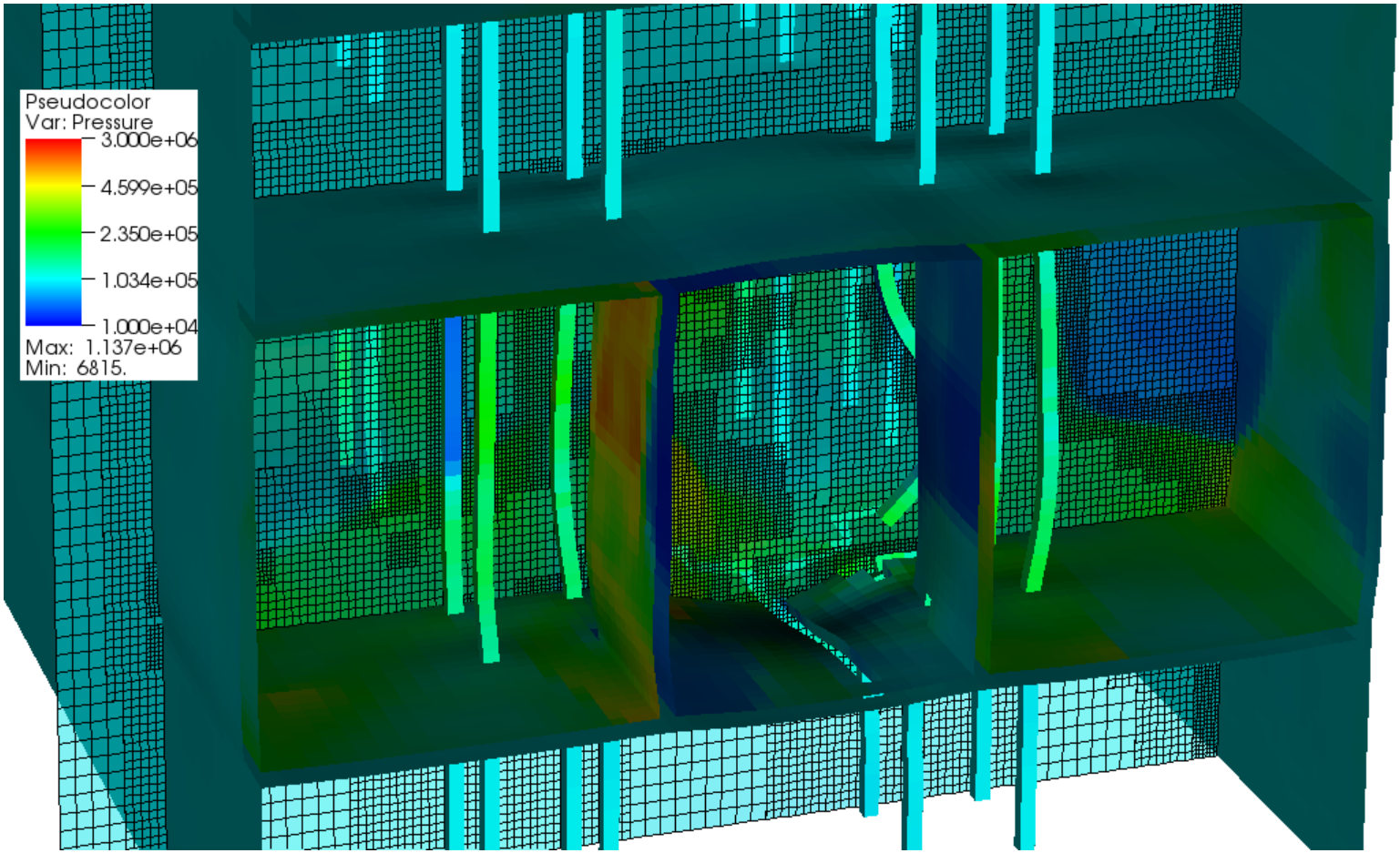
Blast explosion in a multistory building – II



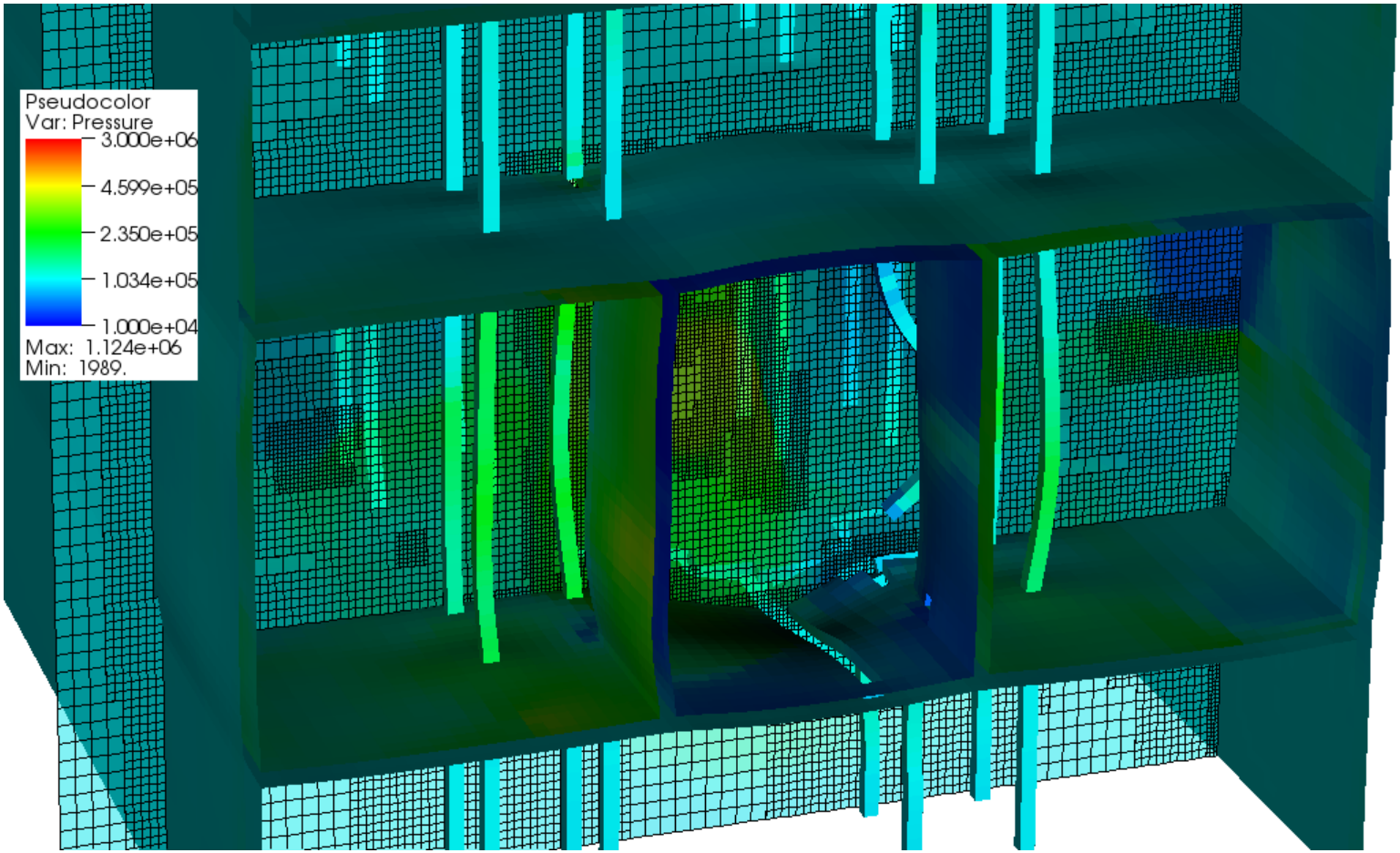
Blast explosion in a multistory building – II



Blast explosion in a multistory building – II

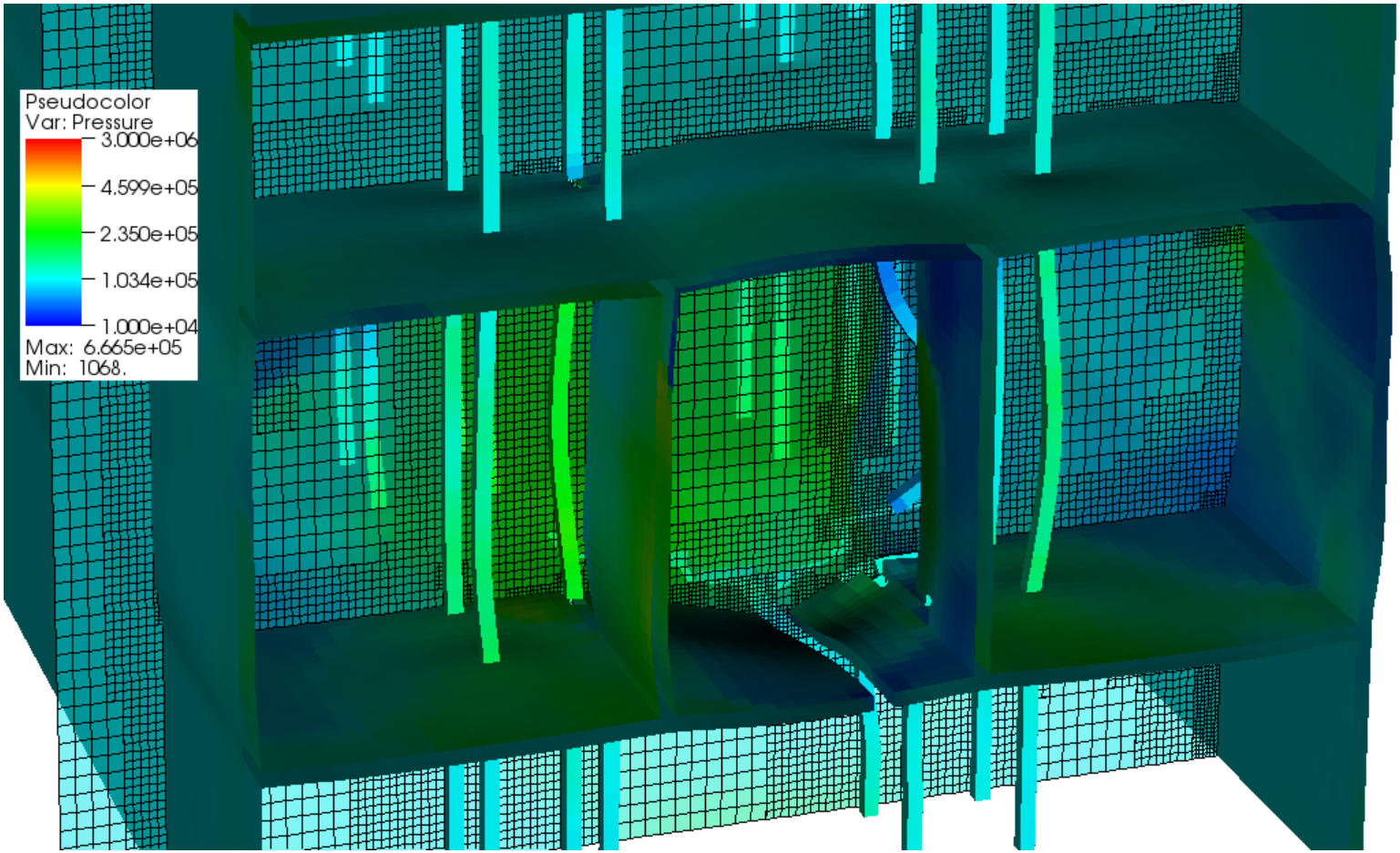


Blast explosion in a multistory building – II

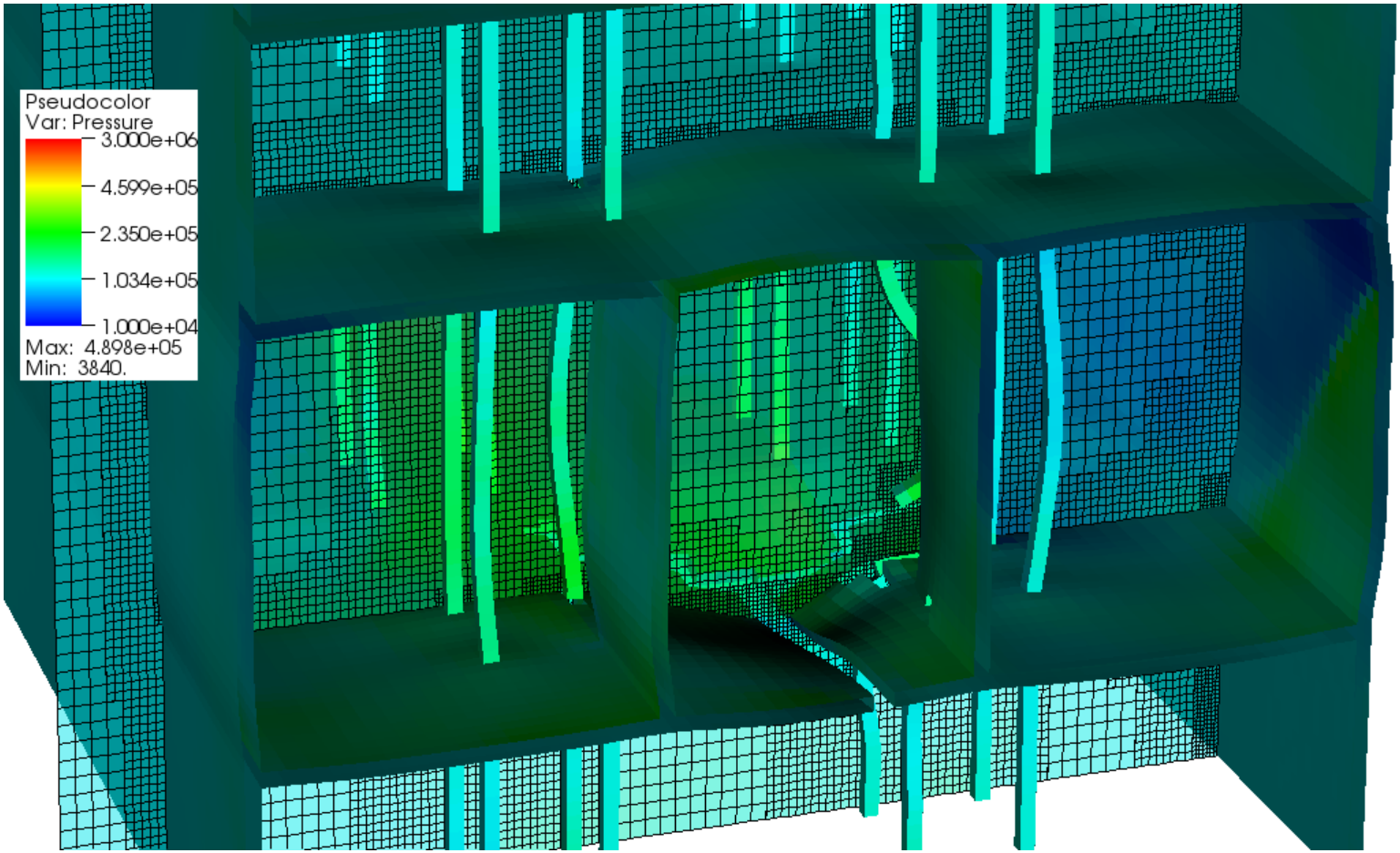


$t = 29.2\,\mathrm{ms}$

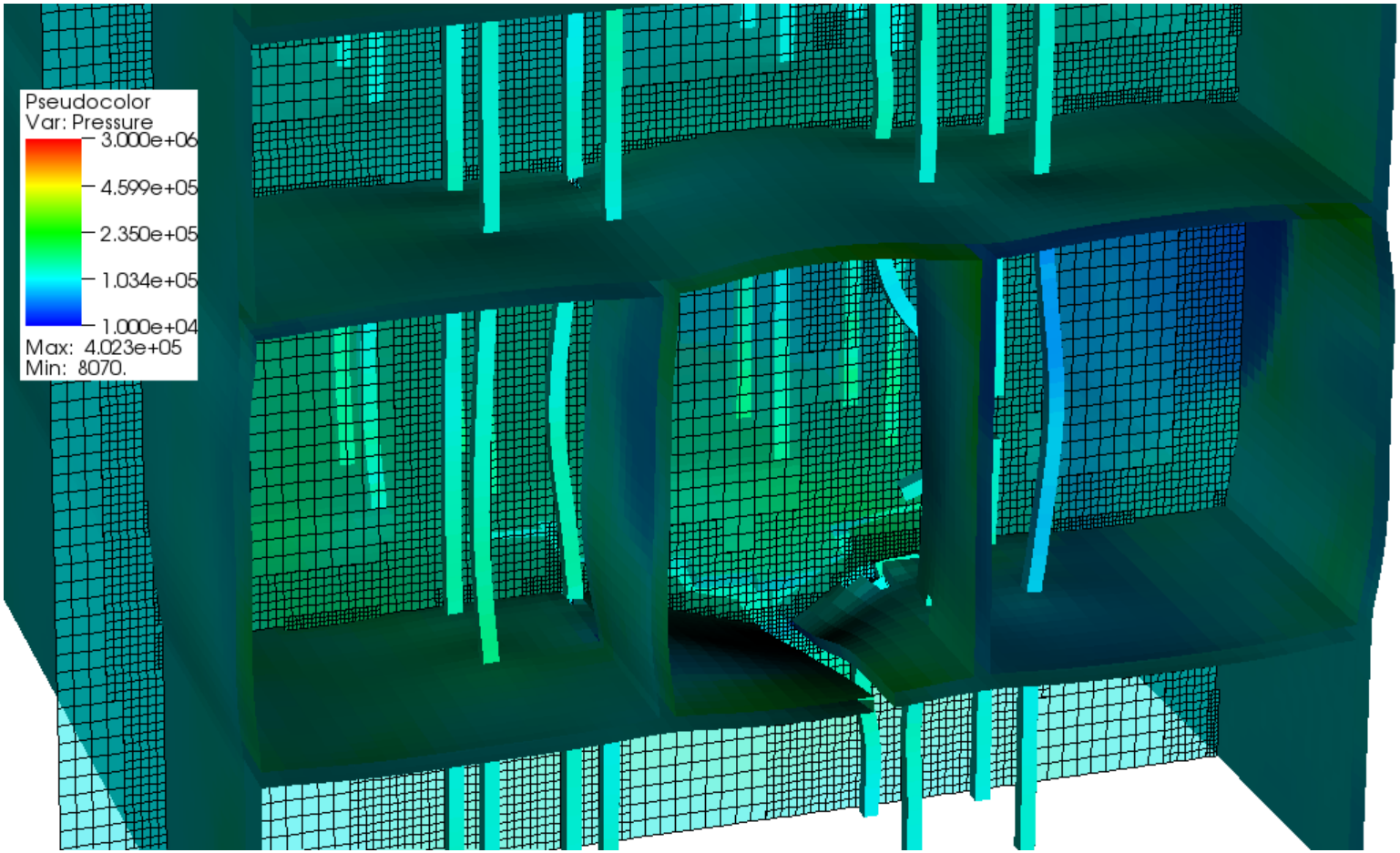
Blast explosion in a multistory building – II



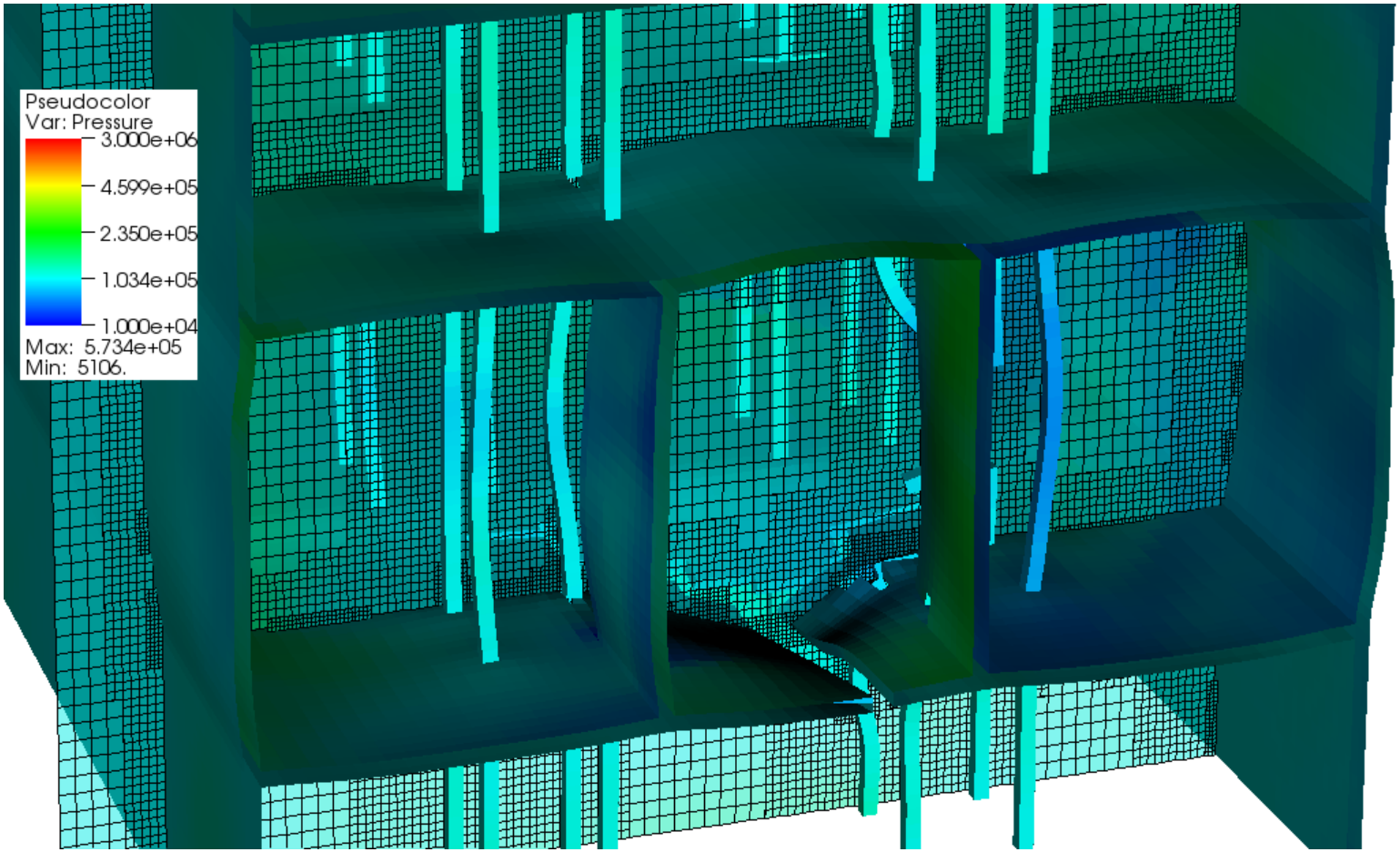
Blast explosion in a multistory building – II



Blast explosion in a multistory building – II



Blast explosion in a multistory building – II



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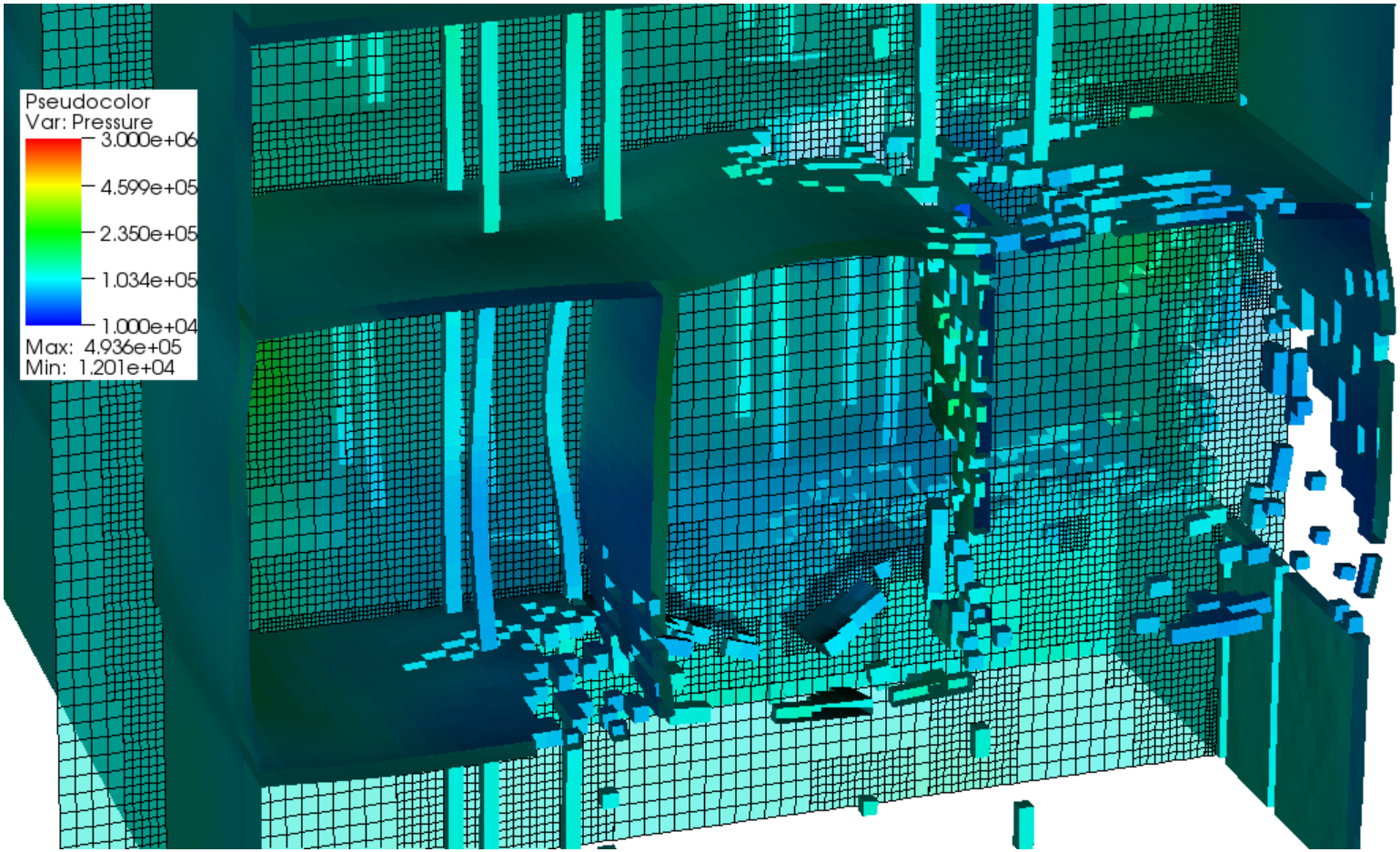

$t = 48.7\,\mathrm{ms}$

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