

## ***School Location Methodology in Urban Areas of Developing Countries\****

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### **Abstract**

This study is concerned with the location of primary public schools. The public system should have capacity to satisfy all the demand, although students may choose between public or private schools if they can afford the corresponding costs. A number of factors, such as questionable education quality, limited capacity, poor location and social preferences, secure a participation of about 30% to the private school system. The purpose of this study is both the evaluation of the existing public school network and a relocation proposal. The result of the former was the identification of areas with shortage and excess in school offer. The latter suggests school relocation using capacitated and uncapacitated models. ArcView, a software of the GIS family, was employed, allowing the efficient handling of large problems and improving the presentation and evaluation of the findings. This methodology was applied to the primary public school network in the area of Vitoria, a state capital located in the southeast region of Brazil with about 300,000 inhabitants. Finally, the practical use of this study and its importance for planning purposes are discussed.

### **Keywords**

*School Location; P-Median Model; Capacitated Model; Geographic Information Systems.*

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## ***1. INTRODUCTION***

The fast and dramatic changes societies around the world have been experiencing along the past century have imposed a multidimensional role to the educational system. Schools have been attributed, among other functions, that of supporting technological changes and preparing the society for competition in a worldwide scale.

Most countries – if not all – currently considered to be in a developing stage have been subjected to a colonial ruling system and have attained their independence not sufficiently long ago to enjoy the benefits brought by metropolitan development and to overcome prejudices. Colonial exploitation, often associated to slavery, has left an unjust social structure which makes such countries face a much broader spectrum of challenges in order to meet modern development standards and to reach a satisfactory position in the UN's Human Development Index.

In Brazil, the focus of the present study, high birth rates and the explosive urbanizing process observed from the 1950's through the 1980's have amplified the social problems. In addition, many young children are forced to work before receiving proper education. These handicapped conditions accentuate the existing development gap in relation to leading countries. On the other hand, the society, broadly speaking, is certainly aware that heavy investments in public education are the fundamental basis for development and for the improvement of social indicators. As a result of this growing concern with education, significant progress can be noticed in the reduction of the illiterate population, the increase in the number of years spent in school and the larger number of university students, among other indicators. However, serious problems of educational sub-standards remain not only in broad rural areas but also in the less modernized parts of the country as well as in the periphery of metropolitan areas.

Regarding the public school system, one of the consequences of this population explosion observed in the main urban centers of the country was a mismatch between supply and demand. Another consequence was the severe location problem that resulted from the generally anarchic dynamics of population expansion. Forced by circumstances and opportunities, slums and *favelas* have quickly developed totally out of city control. The public school network was found to be inadequate and with poor spatial distribution, with many areas devoid of schools. Presently, given the significant reduction in birth rates and the exhaustion of the internal migration process, combined with

the ongoing urbanization of slum areas, the goal of optimizing the school network is starting to be technically meaningful.

This work is organized as follows. Section 2 reviews the specific literature on school location. Section 3 outlines the proposed methodology, which involves two fairly independent stages: the evaluation of the present location situation and the relocation proposal. This section also briefly reviews exact and heuristic methods for p-median models as applied to uncapacitated and capacitated problems. Section 4 describes the application of the proposed methodology to the capital city of Vitoria, located in the southeast region of Brazil with a population of 300,000 inhabitants. Section 5 presents the conclusions, outlines the importance of this study and discusses the receptivity of local education authorities to the suggestions presented.

## ***2. LITERATURE REVIEW***

For diverse purposes, school location studies are regularly carried out in many countries for both urban and rural areas. For countries facing strong reduction in birth rates, the problem is to choose the schools to be closed. Other countries face the opposite problem of high birth rates associated with population mobility and want to evaluate and eventually improve the quality of the education system as well as the location of the school network. In fact, the struggle for modernization in developing societies requires the expansion of the education system to previously disregarded sectors of society such as the rural population.

Molinero (1988) has examined the location of primary and secondary schools in Southampton originally placed in the center of attraction, or catchment, areas. Reduction in birth rates lead to the problem of closing schools. However, such reduction was not uniform across social classes. The author has pointed out the conflicting views of the education administration, which focuses on costs, and the community, concerned with the quality of the services. Tewari and Jena (1987) have considered the location of high schools in rural India. They have proposed the use of the p-median model and the selection of locations that would maximize the population covered within a maximum service distance of 8 km. Pizzolato and Silva (1997) have examined some portions of the metropolitan area of Rio de Janeiro, where throughout the past 50 years nearly five million people have disorderly settled down, and have also modeled the problem as a p-median model.

Antunes (1994) has studied the center-west region of Portugal and applied mathematical methods. Viegas (1987) has used a network-flow algorithm to select the most economical sites to construct or expand schools in order to keep the maximum access distance to a school at a predetermined value, the reduction of such value requiring more facilities and more costs. Tewari (1992) has examined the access to services and facilities in rural areas of India and the study contrasts the use of operations research models with rural development concepts proposed by USAID in the late 1970's. Armstrong et al (1990) have developed a system for providing decision support to people in charge of making location decisions. Related studies can also be found in Rahman and Smith (1991), Stock (1983), ReVelle and Swain (1970), Banerji and Fisher (1974), Fisher and Rushton (1975), and Beguin et al (1989).

### **3. BASIC METHODOLOGY**

The first step in a school location study requires a map of the region and information on the population, usually distributed as a non-uniform continuum and thus unfit for modeling. The natural discretization of the population is provided by the census tracts, which are small geographic units defined by the Census Bureau. Based on these units, demographic elements are periodically collected including age distribution, family education standards, income levels, etc. For the study discussed here, all of the required digital information including maps and census tract boundaries was at hand.

The second step of the study requires the determination of distances between centroids or points near the geographical center of each census tract. These centroids are assumed to concentrate all pupils living in the corresponding census tract. Such assumption is considered reasonable since census tracts are small and their population tends to be simply scattered around, making the centroid a good descriptor of the average location. Local measurement of distances would provide more reliable data but would greatly increase the clerical work involved. Consequently, Euclidean distances are generally preferable. If the topography of the area is absolutely regular, there is a proportional reduction in all distances. Otherwise, some distortion might be introduced, but it can be subsequently reviewed in the validation analysis.

With the population in the centroids and distances, one can produce an undirected weighted graph  $G = (V, A)$  with  $V = \{1, \dots, n\}$  vertices or nodes, representing the census tracts in the region.

For each node  $i \in V$  a weight  $q_i$  is associated to represent its schooling population demand, and for each pair of nodes  $i$  and  $j$  a symmetric arc  $(i,j)$  is associated whose length  $d_{ij}$  represents the minimum distance between these two nodes. The area of influence of each school can be determined by several methods, such as concentric circles, the Voronoi diagram and the bubble method, among others, but the most popular system is certainly the allocation stage of the  $p$ -median model. Formally, for a given graph  $G = (V, A)$ , the  $p$ -median problem might be seen as the problem of constructing a set of  $p$  trees,  $T = \{T^1, T^2, \dots, T^p\}$ , whose corresponding roots or medians form the set  $P = \{r^1, r^2, \dots, r^p\}$  indicating the locations where a facility either exists or should exist, and each of the other nodes in the graph belongs to the tree whose root is the closest one.

The  $p$ -median is based on several behavioral principles. The basic one is that children prefer the school closest to their residence, assuming that walking is the ordinary way of reaching the school. Obviously, this hypothesis may not apply in urban areas of countries in which bus transportation to schools is broadly employed. Otherwise, such hypothesis is currently observed and is reinforced when schools are equally attractive, have the same quality, similar teaching, recreation spaces, tradition, pupils from the same social stratum, and so on. For rural areas, on the other hand, public transportation tends to be widely used as a contingency to take pupils to distant schools, therefore the proposed methodology is not directly applicable.

Our experience indicates that a school location study should be divided in two stages: (1) evaluation of the distribution of the existing school network and (2) proposal of an optimal location. To these, an additional third stage is proposed here, namely (3) the capacitated model. For the two-stage study, the underlying models are the same, since the second is a  $p$ -median model for which each centroid is a possible median, while the first is a particular case in which the medians are previously defined to match the existing schools and the remaining centroids are allocated to the nearest school. As it often happens, particular cases might also offer quite different formulations, as will be shown in Section 3.1. The additional third stage is applied when new school units are standardized. This stage has been attracting some interest because of the new trend to adopt standard schools of a moderate size, avoiding both the very small and the very large units.

### 3.1 Evaluation of Present Locations

The methodology applied here requires mapping the existing schools and identifying their relevant characteristics such as capacity, current enrollment, services offered, expansion possibilities and so on. In one way or another, large urban areas have a public school network and the evaluation of its spatial distribution and its corresponding capacity is the object of the first stage in this study. For this first stage,  $P = \{r^1, r^2, \dots, r^p\}$  is accordingly known beforehand and subsequently the clusters  $C^k$ , rooted at existing schools, are composed.

After identifying all centroids belonging to each cluster  $C^k$ , their estimated unbalance  $U^k$  may be calculated by the expression:

$$U^k = S^k - \sum_{i \in C^k} q_i$$

where  $S^k$  is the capacity of the school located in the cluster  $C^k$ , while its corresponding schooling population is given by  $\sum_{i \in C^k} q_i$ .

If  $U^k > 0$  there is an excess of school places since the capacity of the school is greater than the population in its area of influence; otherwise, if  $U^k < 0$ , there is an additional need for school capacity. Using a color code, these unbalances can be conveniently represented in a map, which would convey the severeness of the unbalances for each existing school. In general, this first stage of the study tends to be especially important, since the relative importance of the unbalances suggests some degree of priority intervention. At this point the decision maker may introduce in the analysis his/her own evaluation of the potential demand for each specific school. Certainly, when several trees lack sufficient capacity, additional capacity can only be gained by new constructions.

On the other hand, when scarce capacity is locally identified, a simple way to smooth out the unbalances is to displace capacity from an area with excessive capacity to another with insufficient capacity. This is often viable because in many cases with minor investments a school may gain additional classrooms and simply receive teachers and furniture from a school with excessive capacity. Note also that, for every cluster  $C^k$ , a number of additional elements may be calculated such as the average and maximum distances between a centroid and a school, as well as a number of other useful census-provided social indicators.

### 3.2 Location Proposal

In the case of the second stage of this study, which is the definition of proposed locations, the medians are precisely the objects of interest. The purpose is to determine the ideal location of the  $p$  medians, or  $p$  schools, in order to minimize the sum of the distances of the population centered in every node to its closest school. This second stage might be useful when schools are poorly located, the unbalances are significant and a drastic intervention might be desirable or necessary, including the relocation of some schools and the construction of others.

Assuming that all vertices in  $V$  can be elected as medians, the  $p$ -median problem is modeled as the following binary integer-programming problem:

$$v(PMP) = \text{Min} \sum_{i=1}^n \sum_{j=1}^n q_i d_{ij} x_{ij}$$

$$(PMP) \quad \text{subject to} \quad \sum_{i=1}^n x_{ij} = 1; \quad i \in V \quad (1)$$

$$\sum_{j=1}^n x_{jj} = p \quad (2)$$

$$x_{ij} \leq x_{jj}; \quad i, j \in V \quad (3)$$

$$x_{ij} \in \{0,1\}; \quad i, j \in V \quad (4)$$

where:

$[d_{ij}]_{n \times n}$  is a symmetric cost (distance) matrix with  $d_{ii} = 0, \forall i$ ;

$[x_{ij}]_{n \times n}$  is the allocation matrix, with  $x_{ij} = 1$  if node  $i$  is allocated to node  $j$  and  $x_{ij} = 0$  otherwise;

$x_{jj} = 1$  if node  $j$  is a median and  $x_{jj} = 0$  otherwise;

$p$  is the number of facilities (medians) to be located;

$q_i$  represents the schooling population of node  $i$ ;

$n$  is the number of nodes in the network, and  $V = \{1, \dots, n\}$ .

Constraints (1) and (3) impose that each node  $i$  is allocated to only one node  $j$ , which must be a median. Constraint (2) determines the exact number of medians  $p$  to be located and (4) gives the integer conditions.

The  $p$ -median problem might be seen as the problem of optimally constructing  $p$  trees,  $T_1, T_2, \dots, T_p$ , whose respective medians are the desired locations and each of the other nodes in the graph belongs to the tree whose median is the closest, in such a way that the total impedance of the system,

$$\sum_{i=1}^n \sum_{j=1}^n q_i d_{ij} x_{ij}, \text{ be minimized.}$$

The early solution methods for the  $p$ -median problem have been of heuristic nature. Heuristic approaches are still popular for several reasons. In particular, heuristics often have a simpler nature, are convenient to handle large real problems, can provide good solutions, are adequate to evaluate alternative solutions and offer the possibility of direct intervention in the coding, making such methods capable of handling restrictions or preferences disregarded in the integer programming model. Among the heuristic methods, the one by Teitz and Bart (1968) is certainly the most acknowledged one, while the one developed by Pizzolato (1994) appears to offer better performance specially when facing problems with large values of  $p$ .

Exact methods for the  $p$ -median problem require the solution of an integer linear programming problem with  $nxn$  variables and  $nxn + n+1$  constraints. Although it is technically viable for large values of  $n$ , hardware and software requirements might make it unfeasible for practical studies.

The approach used in this paper was presented in Senne and Lorena (2000) and employs the Lagrangean/surrogate heuristic proposed in Narciso and Lorena (1999) to approximately solve the problem. The dual bounds are made feasible and improved by means of location-allocation heuristics. The Lagrangean/surrogate relaxation has been recently used as a method to accelerate subgradient like methods, which are often used to optimize the corresponding Lagrangean dual.

A question usually made after the optimal solution is identified is what to do with the existing schools. The conciliation between the optimal solution and the existing network may be done by identifying for every school the proposed tree  $T_k$ ,  $k = 1, \dots, n$ , to which it belongs. Then, let  $Q_k$  be the combined capacity of the schools eventually located in tree  $T_k$ . The unbalance between the capacity



of existing schools and the demand in the proposed tree may be given by the expression  $\mu_j = Q_j - \sum_{i \in C_j} q_i$ .

If  $\mu_j > 0$  there is an excess of school places since the capacity of the schools in the area is larger than the population in the same area of influence; otherwise, if  $\mu_j < 0$ , there is an additional need for greater school capacity. Again, the findings may be presented in a color code so as to emphasize the varying degrees of unbalances. Since this model is recommended when schools are very poorly located, as a result of anarchic population settlement and lack of public investment, the study will point out the areas in which new constructions are necessary and identify schools which could be scaled down or simply abandoned. Such study might be part of the strategic planning for the school network development and be replicated every ten years when a new census is completed.

### 3.3 The Capacitated Model

The *capacitated p-median problem (CPMP)* considers capacities for the service to be offered by each median. The total service demanded by vertices identified by p-median trees cannot exceed their service capacity. This resembles the case of a school with a fixed capacity. This part of the study might be important when the administration is dealing with standardized schools. Empirical studies made in Brazil have shown that the cost per student is higher in both the very small and the very large schools. Therefore, if a standard school of about 700 students is to be adopted – and this seems to be a tendency – this part of the study might be relevant for education authorities.

Apparently, problem CPMP has not been so intensively studied as the classical p-median problem. An extensive bibliography of related problems and a set of test problems were presented in Osman and Christofides (1994). For the CPMP considered in this paper the set of constraints (3) in the PMP model is replaced by the capacity constraints:

$$\sum_{i \in V} q_i x_{ij} \leq Q_j x_{jj} ; j \in V \quad (5)$$

For the sake of completeness, the Lagrangean/surrogate application to CPMP by Lorena and Senne (2003) is summarized as follows. For a given  $\lambda \in R_+^n$  and  $t \geq 0$ , the Lagrangean/surrogate relaxation of CPMP is given by:

$$(L_tCPMP^I) \quad v(L_tCPMP^I) = \text{Min} \sum_{i \in V} \sum_{j \in V} (d_{ij} - tI_i)x_{ij} + t \sum_{i \in V} I_i \quad (6)$$

subject to (2), (4) and (5).

Problem  $(L_tCPMP^I)$  is solved by implicitly considering constraint (2) and decomposing for index  $j$ . As a result, the following  $n$  0-1 knapsack-like problems are obtained:

$$v(knap_j) = \text{Min} \sum_{i \in V} (d_{ij} - tI_i)x_{ij} \quad (7)$$

subject to (4) and (5).

Each problem has real objective function coefficients and is solved by means of an adapted version of the Horowitz and Sahni code (see Martello and Toth (1990)). Let  $J$  be the index set of the  $p$  smallest  $v(knap_j), j \in V$ . The Lagrangean/surrogate value is given by:

$$v(L_tCPMP^I) = \sum_{j \in J} v(knap_j) + t \sum_{i \in V} I_i \quad (8)$$

The interesting feature of relaxation  $(L_tCPMP^I)$  is that for  $t = 1$  we have the usual Lagrangean relaxation with the multiplier  $\lambda$ . To find an approximate best Lagrangean/surrogate multiplier  $t^*$  we have used the dichotomous search procedure *SH* described in Senne and Lorena (2000). A subgradient algorithm is used to optimize the dual and the Lagrangean/surrogate solutions are made feasible and improved by local search heuristics as shown in Lorena and Senne (2003).

#### 4. THE CASE OF VITORIA

Vitoria is a city with approximately 300,000 people, according to the 1996 population census, and it is the capital of the State of Espirito Santo, located in the southeast region of Brazil. Similarly to other state capitals, it has experienced dramatic demographic expansion in the last few decades.

According to the same census, there were 41,678 students between 7 and 14 years old. In addition, the metropolitan area of Vitoria is composed by a number of less developed counties which

very likely offer much poorer education services. This education quality gap drives a sizable but unknown number of students to public schools at the state capital, imposing an extra demand upon the system.

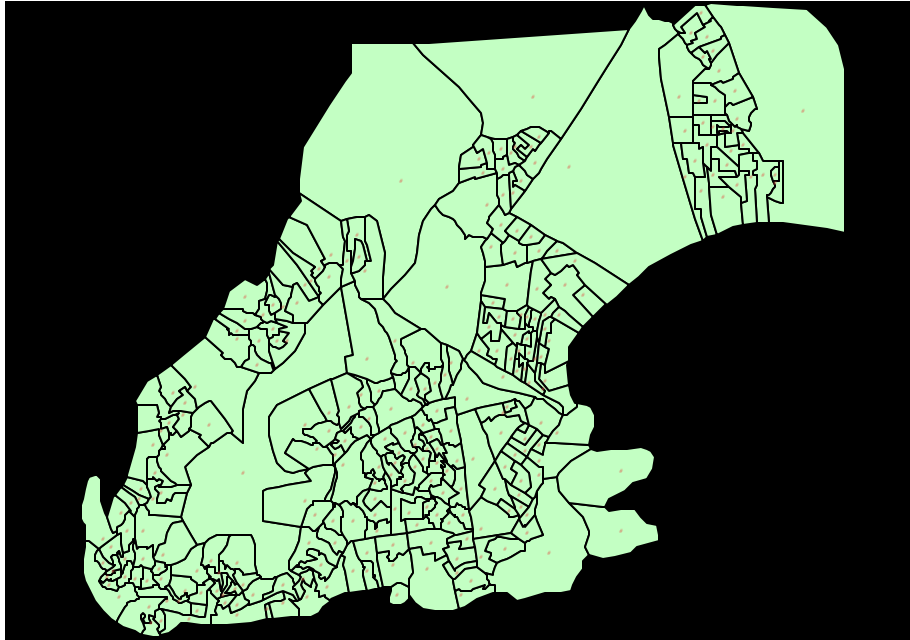
The first level of education is composed by eight years of study that officially starts at age 7 and finishes at age 14, but only nearly half of the population follows the official prescription. A few perform better than the rule, starting at 6 and finishing at 13. Due to several malfunctions and inefficiencies, such as late depart, repetition, dropout and so on, as well as cultural backwardness, a large number of children finish at higher ages and many do not finish the first level, while some never enter the system.

As a consequence, forecasting the effective demand for the public system is not an easy task. Analytically, the demand is given by the existing student population multiplied by a factor of 0.97, to account for those that never join the system or go to special schools, plus those older than 14, up to 18 years old, who are still in the regular first level of education, minus those around 30% that prefer private schools, minus those that finish the first level before being 14 years old, and minus those that have prematurely abandoned the system. The net result is that the number of enrolled first-level students is a bit larger than the 7-14 year-old population. For the exploratory capacitated study shown below, we have adopted an estimated net addition of 5% to the existing population of 41,678 to represent the demand for the public system. Certainly, as education quality improves, the flow of students will be faster and the effective demand will decrease.

There are 271 census tracts in Vitoria, as shown in Figure 1. The vertices of the associated graph are the centroids of the corresponding census tracts. Since the area being studied is highly urbanized, the tracts are generally small and the centroids are located in each geographical center. Therefore, each centroid is supposed to concentrate all of the school population in the corresponding census tract. The very large tracts correspond to uninhabited areas such as the city airport, industrial sites, non occupied hills, penitentiaries, etc.

The next three subsections examine the application of each of the three complementary aspects of the present study.

**Fig. 1 - Census tracts in Vitoria – 1996 Census**



#### **4.1 Evaluation of the Present Location**

Current school positions and the attraction area of each school are shown in Figure 2. There are 51 public schools in the city with different sizes and capacities, varying degrees of construction projects, in some cases specific pedagogical concepts according to the population served, and other specificities. Therefore, homogeneity seems to be a distant prospect but is certainly a desirable one, to which a technical study as this shall contribute. The web-like construction defines each cluster, associating for each centroid its closest school. Therefore, for each existing school, the figure indicates the demand and its origin. For the evaluation of the present location, several indicators must be extracted such as capacity and demand for each school, the resulting unbalances, the maximum distance covered by one student, etc.

Fig. 2 – Current school positions and areas of attraction of each school

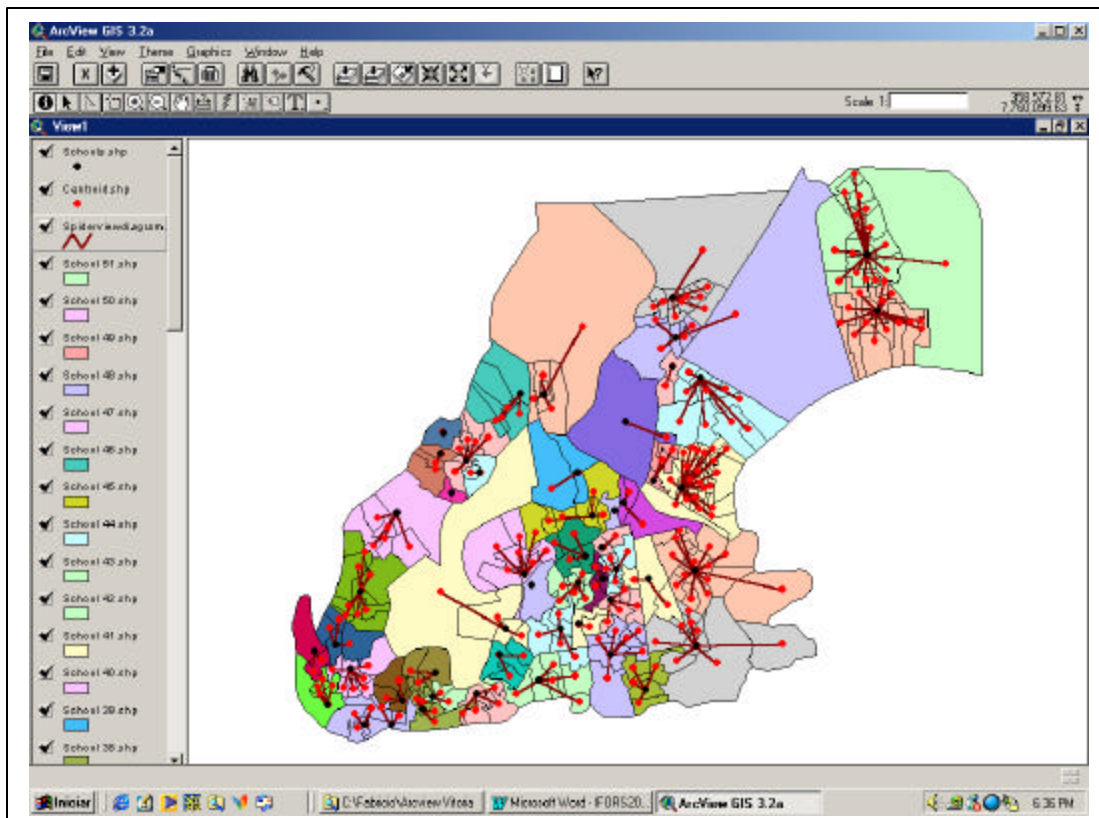


Figure 3 illustrates the most important information, which is the unbalance of the current school distribution. The table to the left of the image has five columns identifying, respectively, the school, the number of census tracts in its area of influence, the corresponding population, the school's capacity, and the excess or shortage of school places. The first line of the table, for example, informs that school number 17 serves 15 census tracts according to the shortest distance, whose total schooling population is 2,566. Since the capacity of the school is 997 students, there is a shortage of 1,569 places.

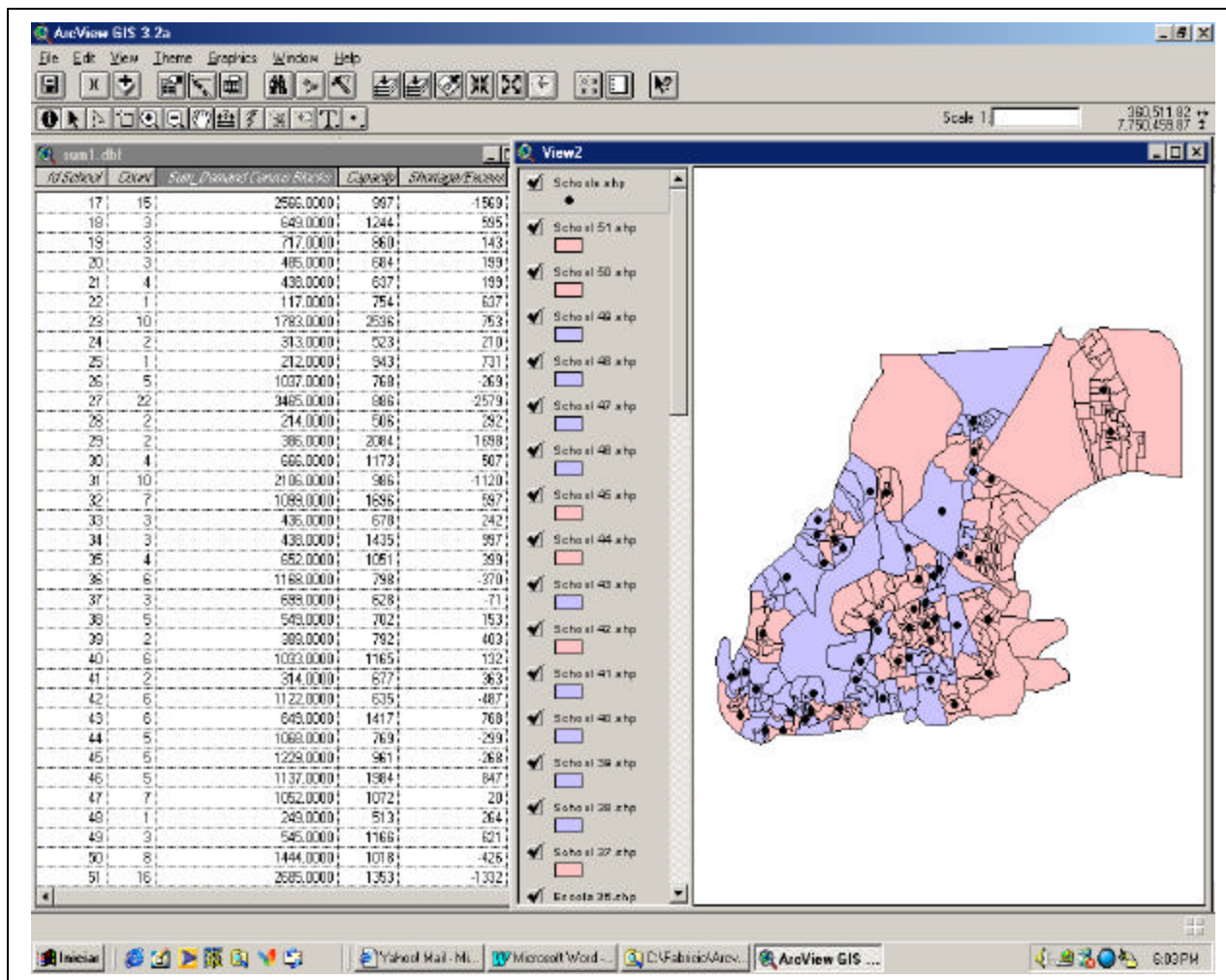
To the right of Figure 3, the blue color in the map indicates the areas with excess of school places and the pink color indicates the areas with insufficient school places. Not surprisingly, the pink color covers areas of the city of more recent occupation and subject to a greater demographic

expansion. Certainly, the merit of this display is the immediate quantification of the unbalances associated with a pictorial representation. The predominance of the pink color suggests that new constructions are a clear requirement. To some extent, however, the less expensive solution of transferring capacities among schools should also be considered.

For comparison purposes, the total impedance of the current solution calculated by the expression  $\sum_{i=1}^n \sum_{j=1}^n q_i d_{ij} x_{ij}$  is 99,229 m. Under the stated assumptions, his number represents

students times distances to their nearest school.

**Fig. 3 – Unbalances for the current solution**



## **4.2 Relocation Proposal: The Uncapacitated Model**

With the distance matrix and the weights, the second stage of the study can be performed, which is the relocation proposal. Assuming that every centroid might be a median, problem PMP is solved using the Lagrangean/surrogate heuristic mentioned in Section 3.2. On a Pentium III machine the program required less than 1 second of CPU. The model produced a feasible solution of value 70,252 and a dual Lagrangean/surrogate bound of 69,356, but the corresponding figure is not reproduced here. Considering the duality gap, this solution is quite reasonable. Comparing this solution with the present location (99,229), the sum of distances to be covered by the entire group of students is reduced by 28,977 meters.

## **4.3 The Capacitated Model**

Although the p-median model proposes a relocation policy, the unbalance derived from the lack of capacity of several schools is not explicitly considered. The study proceeds by running the Lagrangean/surrogate heuristic for the capacitated p-median model and obtaining the feasible solution 85,083 and the Lagrangean/surrogate bound 77,024 (for 685 seconds of CPU at the same machine). As expected, the distance to be covered by students increases. Figure 4 shows the solution considering standard schools of 859 students, where  $(859 = 41,678 \text{ students} \times (1+5\%) / 51 \text{ schools})$ .

It can be conjectured that although the capacitated model increases the distances covered it is more realistic, which is especially important when the education board adopts standard models of school architecture with fixed capacity. This proposal might lead to a clear improvement in attendance satisfaction and other educational settings. It is interesting to note that for some regions, such the one enhanced in Figure 5, more schools are clearly needed. Compared with the current solution shown in Figure 2 (99,229), the sum of distances to be covered is reduced by 14,146 meters.

Figure 4: Capacitated solution

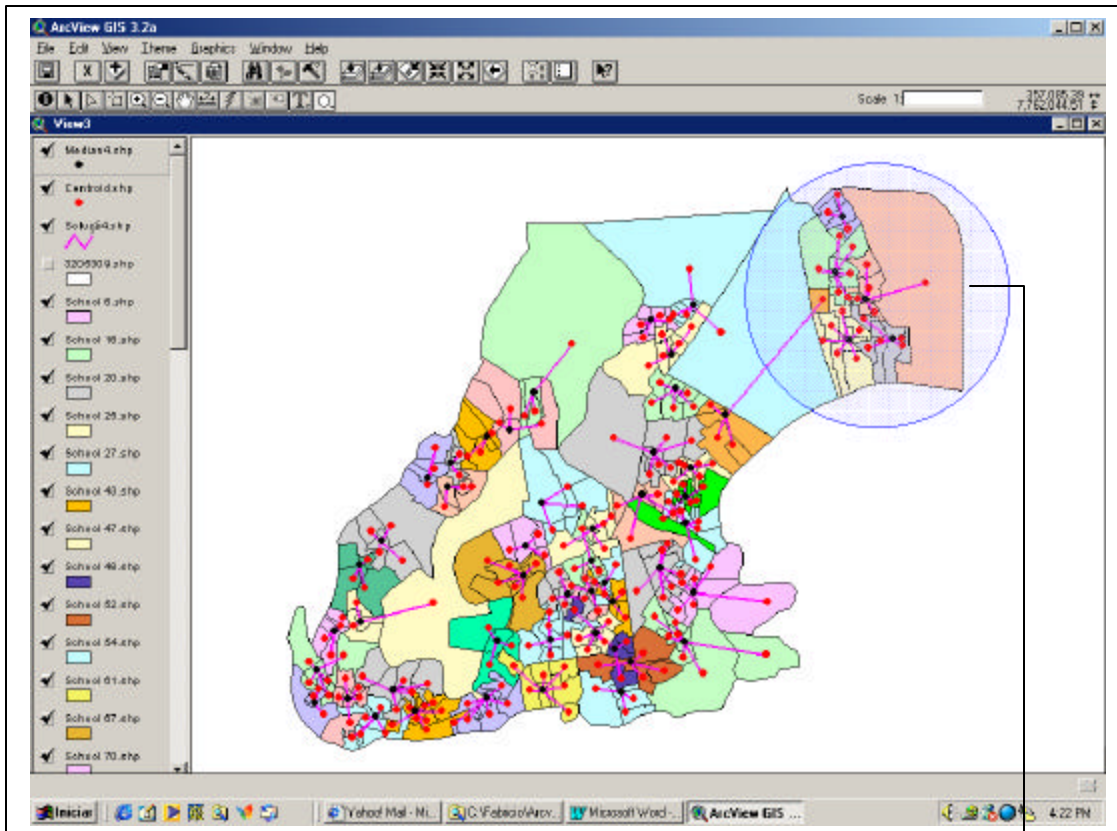
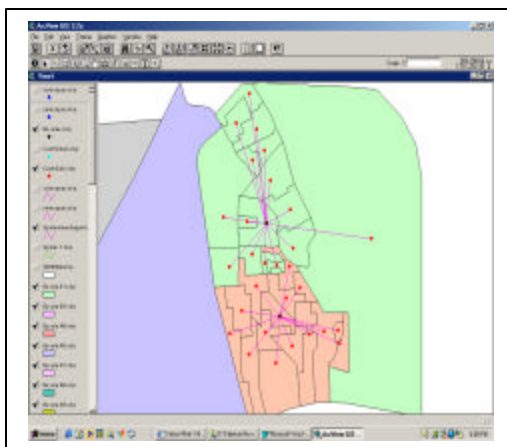
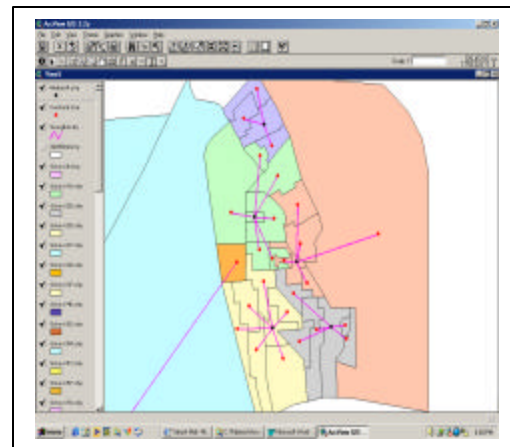


Figure 5: More schools to some regions



Current school locations  
(see fig. 2)



Capacitated solution  
locations



## 5. CONCLUSIONS

The present research has described a methodology to study the spatial distribution of primary schools in urban areas. The modeling employed assumes that students walk to their school, which is predominantly the case, and therefore tend to prefer the one closest to their home.

The study had three complementary but fairly independent purposes. One was to evaluate the existing location of the schools in order to identify areas lacking or exceeding in school places. Given the intense population mobility, high birth rates, disorderly urbanization and so forth, such network evaluation is recommended to be periodically carried out. The consequence of this study was the immediate identification of the areas where the unbalances are more significant. Depending on the numerical results, corrective measures may range from capacity management to new constructions. By capacity management we understand that with simple measures the capacity of a school may be increased. Such measures include expanding existing buildings, adapting unused spaces to function as classrooms, establishing an additional shift and accepting more students per classroom. On the other hand, capacity reduction is much simpler and inexpensive, with the transfer of teachers, chairs, tables and blackboards to areas in need of expansion.

The second purpose was to provide a new relocation proposal. This part of the study is important when severe unbalance is identified or when several new schools have to be built. The ideal locations were derived from the uncapacitated  $p$ -median model. For practical application, the proposed locations have to be reconciled with the existing schools. The idea is to assume that the proposed locations define micro regions or catchment areas in which one or more schools may already exist. Therefore, the ideal location is important for long-term planning but should be reconciled with the existing network.

The third purpose was the capacitated solution. According to common sense, a school is a capacitated facility, although, as already noted, to some extent such capacity is flexible. However, new pedagogy, architecture and cost studies are leading to the adoption of standardized schools. Therefore, if the tendency is to adopt a standard school pattern, the capacitated model should be of great interest.

For the practical application described, basic information such as maps, census tracts and population was obtained from the Brazilian Institute of Geography and Statistics, IBGE, responsible for census research. School data were collected at the State and Municipal Education Boards. The

data were treated using ArcView, a software of the Geographic Information System (GIS) family. This software simplifies the location of schools and centroids, the determination of distance data and the pictorial representation of several types of information. For the relocation part of this study, scripts were prepared to pass the data to computer codes written in C and to return the results for visualization. The codes implemented the Lagrangean/surrogate heuristics for the PMP and CPMP models. The distance matrix was also externally calculated, as the C compiler is quite efficient compared to the native GIS development language.

The reaction of education authorities of Victoria was recorded in a document which outlines the impacts they see in the study. Basically, these are: (1) it is a technical study which will be used to ratify or modify internal reports regarding school network planning; (2) the study offers precise suggestions for the management process, particularly capacity supply; (3) the data collected by the study represents a contribution to their existing but incipient data base; (4) the study shall launch cross initiatives with other sectors of the administration, such as the city development planning, towards an integration of services to the community and better education quality; and (5) their interest in a renewal of this study would be in five years from now rather than ten, as proposed by the study.

## **RELEVANCE OF THE WORK TO OR IN DEVELOPMENT**

This paper has reported a school location problem in which OR has played a determinant role. Although the case focuses on Brazil, similar challenges are faced by most if not all developing countries. In particular, along the past half or full century, popular descriptors of these countries include political independence, population growth, illiteracy reduction, urbanization, social consciousness, respect to human rights and so forth.

As a natural consequence, public services earlier designed to provide services to a minority have been expanded to serve the entire society, including the rural population. However, insufficient budget to satisfy all of the demands creates a relative scarcity of the services. Therefore, every type of public service operates under stress and lies far behind the societies' expectations. In order to satisfy pressure groups the easiest solution is the case by case management. Such approach, however, tends to maintain the public budget directed to more powerful and wealthier sectors, concomitantly reducing investments elsewhere.

At this point, OR techniques might provide rational and broad supporting tools for decision makers. In the authors' opinion, the ultimate decision level is political, but decisions regarding social issues based upon purely political reasons might be poor and subsequently subject to contestation. What is mostly desirable is the use of a rational analysis framework such as those provided by OR tools. With such analytical support, the political level might endorse or even overrule the so-called optimal solutions indicated by such instruments and the OR analyst can certainly understand that optimal solutions might be contested by possible imponderables not explicitly incorporated in the modeling. On the other hand, given the existence of an analytical reference, the adoption of a divergent solution requires explicit justification, imposing to the political decision maker the role of the devil's advocate. In fact, decisions that affect the entire society require a dialogue between the political establishment and technical evaluations, which claim to develop independent models. In this way, politicians and OR specialists may converge to take better decisions.

The above reflections have influenced the study just presented. In large urban settings, the school location problem is a combinatorial kind of problem. Any local decision is certainly myopic in the sense of regarding parts, but not the whole picture. The favorable reception of this study not only indicates the appropriateness of the OR models involved but also the determination of local authorities to pursue a balanced solution which apparently only OR seems to provide.

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