Lagrangean/surrogate relaxation and column generation: new bounds and new columns

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Abstract

Column generation and Dantzig-Wolfe decomposition are very successful methods for large-scale linear programming problems. It is also well known the instability of the overall process. Stabilising methods work constraining the norm of the dual vector arising from restricted masters. This work shows how to combine the Lagrangean/surrogate relaxation and column generation aiming a stabilised method.

1. Introduction

This work combines the Lagrangean/surrogate relaxation and the column generation process for linear programming problems. The Lagrangean/surrogate relaxation works controlling the dual variable norm by a multiplier, optimised by dichotomous search [1]. Column generation is a powerful tool for solving large-scale linear programming problems. Such linear programming may arise when the columns in the problem are not known in advance and a complete enumeration of all columns is not an option, or the problem is rewritten using Dantzig-Wolfe decomposition [2].

2. Lagrangean/surrogate and column generation

The optimisation subproblems arising on column generation and Lagrangean/surrogate are similar, except for the Lagrangean/surrogate multiplier. The dichotomous search showed to be able to select productive columns and to accelerate a column generation method applied to p-median like problems. The generated Lagrangean/surrogate bounds are better than the ordinary Lagrangean bounds and also than the Farley's bounds. Some applications are suggested, for vehicle routing, symmetric travelling salesman, generalised assignment and binary cutting problems.

3. Computational tests and conclusion

Computational tests with p-median like problems show that the combined method accelerates the traditional column generation, mainly for a large-scale TSPLIB instance (3038 vertices). It also works well with an admissible reduced pool of columns and showed to be faster than the combined use of Lagrangean/surrogate and a subgradient method. Some open questions are left like the use of the combined approach on branch-and-price methods.

References

[1] Narciso, M.G.; Lorena, L.A.N. *Lagrangean/surrogate Relaxation for Generalized Assignment Problems.* European Journal of Operational Research, 114(1), 165-177, 1999.
[2] Barnhart, C.; Johnson, E.L.; Nemhauser, G.L.; Savelsbergh, M.W.P. and Vance, P.H. *Branch-and-Price: Column Generation for Solving Huge Integer Programs*, Operations Research 46 (1998) 316-329.