Chapter 6

LAGRANGEAN/SURROGATE HEURISTICS FOR p-MEDIAN PROBLEMS

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Abstract: ***The p-median problem is the problem of locating p facilities (medians) on a network so as to minimize the sum of all the distances from each demand point to its nearest facility. A successful approach to approximately solve this problem is the use of Lagrangean heuristics, based upon Lagrangean relaxation and subgradient optimization. The Lagrangean/surrogate is an alternative relaxation proposed recently to correct the erratic behavior of subgradient like methods employed to solve the Lagrangean dual. We propose in this paper Lagrangean/surrogate heuristics to p-median problems. Lagrangean and surrogate relaxations are combined relaxing in the surrogate way the assignment constraints in the p-median formulation. Then, the Lagrangean relaxation of the surrogate constraint is obtained and approximately optimized (one-dimensional dual). Lagrangean/surrogate relaxations are very stable (low oscillating) and reach the same good results of Lagrangean (alone) heuristics in less computational times. Two primal heuristics was tested, an interchange heuristic and a location-allocation based heuristic. The paper presents several computational tests which have been conducted with problems from the literature, a set of instances presenting large duality gaps, a set of time consuming instances and a large scale instance.***

1. INTRODUCTION

The search for p-median nodes on a network is a classical location problem. The objective is to locate p facilities (medians) so as to minimize the sum of the distances from each demand point to its nearest facility.

Hakimi (1964), (1965) was the first to formulate the problem for locating a single and multi-medians. He also proposed a simple enumeration procedure to solve the problem. The problem is well known to be NP-hard (Garey and Johnson 1979). Several heuristics have been developed for p-median problems. Some of them are used to obtain good initial solutions or to calculate intermediate solutions on search tree nodes. Teitz and Bart (1968) proposed simple interchange heuristics (see also (Maranzana 1964)). More complete approaches explore a search tree. They appeared in Efroymson and Ray (1966), Jarvinen and Rajala (1972), Neebe (1978), Christofides and Beasley (1982), Galvão and Raggi (1989) and Beasley (1993). The combined used of Lagrangean relaxation and subgradient optimization in a primal-dual viewpoint was found to be a good solution approach to the problem (Christofides and Beasley 1982), (Galvão and Raggi 1989), (Beasley 1993).

Beasley (1993) describes very effective heuristics for a class of location problems. They are called Lagrangean heuristics, and use Lagrangean relaxation and subgradient optimization. At each subgradient iteration, Lagrangean solutions are made primal feasible maintaining the median set and reallocating the non-medians to their nearest median. The reallocation can be improved by interchange heuristics. Lorena and Narciso (1996) introduced relaxation heuristics for generalized assignment problems (GAP), using a generalized subgradient algorithm. The new relaxation presented is a surrogate relaxation that was used before in other applications, such as set covering problems (Lorena and Lopes 1994) and multidimensional knapsack problems (Freville *et al*. 1990). Narciso and Lorena (1999) complemented their work (Lorena and Narciso 1996) considering the combined application of Lagrangean and surrogate relaxation for GAP problems. The new relaxation, called Lagrangean/surrogate, was applied considering three kinds of relaxation constraints, including the Lagrangean decomposition approach (Narciso and Lorena 1999).

The objective of this work is to present Lagrangean/surrogate heuristics for p-median problems. The Lagrangean/surrogate combines the two well-known Lagrangean and surrogate relaxation for the p-median problem. The relaxations are combined relaxing in the surrogate way the assignment constraints in the p-median formulation. Then, the Lagrangean relaxation of the surrogate constraint is obtained and approximately optimized (one-dimensional dual). Previous works have confirmed that

Lagrangean/surrogate relaxations are very stable (low oscillating) and reach the same good results of Lagrangean (alone) heuristics in less computational times (Freville *et al*. 1990), (Lorena and Lopes 1994), (Lorena and Narciso 1996), (Narciso and Lorena 1999). Two primal heuristics are tested to make feasible the intermediate dual solutions, an interchange heuristic used before on Beasley (1993) and a location-allocation heuristic proposed by Cooper (1963) and used before on Taillard (1996). The set of test problems is divided in three, one with small problems presenting large dual gaps, other with the (hard) time consuming instances of the OR-library (Beasley 1990), i.e., the instances for which the number of medians is about 1/3 of the number of nodes, and a large scale instance studied on Taillard (1996).

In section two we present the Lagrangean/surrogate relaxation for p-median problems and a summary of the theory to explain their possibly good behavior. Section three details the subgradient heuristic. The computational tests that have been conducted with problems from the literature are presented in the next section. We conclude confirming that the Lagrangean/surrogate heuristic is able to obtain good results for a number of different p-median instances.

2. THE LAGRANGEAN/SURROGATE RELAXATION

The p-median problem considered in this paper is modeled as the following binary integer programming problem:

$$v(P) = \min \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij}$$

$$\text{(P)} \qquad \text{subject to} \quad \sum_{i=1}^{n} x_{ij} = 1; \ j \in N \qquad (1)$$

$$\sum_{i=1}^{n} x_{ii} = p \qquad (2)$$

$$x_{ij} \le x_{ii}; \ i, j \in N \qquad (3)$$

$$x_{ij} \in \{0,1\}; \ i, j \in N \qquad (4)$$

where:

$[d_{ij}]_{n \times n}$ is a symmetric cost (distance) matrix, with $d_{ii} = 0, \forall i$;

$[x_{ij}]_{n \times n}$ is the allocation matrix, with $x_{ij} = 1$ if node i is allocated to node j, and $x_{ij} = 0$, otherwise; $x_{ii} = 1$ if node i is a median and $x_{ii} = 0$, otherwise;

p is the number of facilities (medians) to be located;
n is the number of nodes in the network, and N = {1, ..., n}.

Constraints (1) and (3) ensure that each node j is allocated to only one node i, which must be a median. Constraint (2) determines the exact number of medians to be located (p), and (4) gives the integer conditions.

We use here relaxation heuristics to approximately solve problem (P). A general description for Lagrangean/surrogate relaxation appeared in Narciso and Lorena work (1999). The surrogate and Lagrangean/surrogate relaxation are presented as follows.

As proposed by Glover (1968), for a given $\lambda \in R^n_+$, a surrogate relaxation of (P) can be defined by:

$$v(SP^{\lambda}) = \min \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij}$$

$$(SP^{\lambda}) \qquad \text{subject to} \quad \sum_{j=1}^{n} \sum_{i=1}^{n} \lambda_j x_{ij} = \sum_{j=1}^{n} \lambda_j \qquad (5)$$

and (2), (3) and (4).

The optimal value $v(SP^{\lambda})$ is less than or equal to v(P), and its best value can result in a surrogate dual $\max_{\lambda \geq 0} v(SP^{\lambda})$. The surrogate function *s*: $R^n_+ \rightarrow R$, $(\lambda, v(SP^{\lambda}))$ has some properties that make it difficult to find a dual solution. Some methods proposed in the literature find the approximate solution of the surrogate dual, such as that of Dyer (1980) and Karwan and Rardin (1979). Note here that problem (SP^{λ}) can not be easily solved, as it is an integer linear problem with no special structure to explore. See (Parker and Rardin 1988) for a book describing Lagrangean and surrogate relaxations.

Due to the difficulties with relaxation (SP^{λ}) we proposed to relax again the problem, now in the Lagrangean way. For a given t ≥ 0, constraint (5) is relaxed, and the *Lagrangean/surrogate* relaxation is given by:

$$v(L_t SP^{\lambda}) = \min \sum_{j=1}^{n} \sum_{i=1}^{n} (d_{ij} - t\lambda_j) x_{ij} + t \sum_{j=1}^{n} \lambda_j$$

$$(L_t SP^{\lambda}) \qquad \text{subject to (2), (3) and (4).}$$

For given t ≥ 0 and $\lambda \in R^n_+$, $v(L_t SP^{\lambda}) \leq v(SP^{\lambda}) \leq v(P)$. $(L_t SP^{\lambda})$ is solved considering implicitly constraint (2) and decomposing for index i, obtaining the following n problems:

$$\min \sum_{j=1}^{n} (d_{ij} - t\lambda_j) x_{ij}$$

subject to (3) and (4).

Each problem is easily solved by letting:

$$\beta_i = \sum_{j=1}^{n} \{ \min (0, d_{ij} - t\lambda_j) \} \qquad (6)$$

and choosing I as the index set of the p smallest β_i (here constraint (2) is considered implicitly). Then, a solution x_{ij}^{λ} to problem $(L_t SP^{\lambda})$ is:

$$x_{ii}^{\lambda} = \begin{cases} 1, & \text{if } i \in I \\ 0, & \text{otherwise} \end{cases}$$

and for all $i \neq j$:

$$x_{ij}^{\lambda} = \begin{cases} 1, & \text{if } i \in I \text{ and } d_{ij} - t\lambda_j < 0 \\ 0, & \text{otherwise} \end{cases}$$

The Lagrangean/surrogate solution is given by:

$$v(L_t SP^{\lambda}) = \sum_{i=1}^{n} \beta_i x_{ii} + t \sum_{j=1}^{n} \lambda_j$$

The interesting characteristic of relaxation $(L_t SP^{\lambda})$, is that for t = 1 we have the usual Lagrangean relaxation using the multiplier λ. For a fixed multiplier λ, the best value for t can be found by solving a Lagrangean dual:

$$(D_t^{\lambda}) \qquad v(D_t^{\lambda}) = \max_{t \geq 0} v(L_t SP^{\lambda}).$$

It is immediate that $v(SP^{\lambda}) \geq v(D_t^{\lambda}) \geq v(L_1 SP^{\lambda})$. It is well known that the Lagrangean function *l*: $R^{+} \rightarrow R$, $(t, v(L_t SP^{\lambda}))$ is concave and piecewise linear (Parker and Rardin 1988). The best Lagrangean/surrogate relaxation value gives an improved bound to the usual Lagrangean relaxation. An exact solution to (D_t^{λ}) may be obtained by a search over different values of t (see Minoux (1975) and Handler and Zang (1980)). However, in general, we have an interval of values $t_0 \leq t \leq t_1$ (with t_0=1 or t_1=1) which also produces improved bounds (see Figure 1, for the case t_1=1).

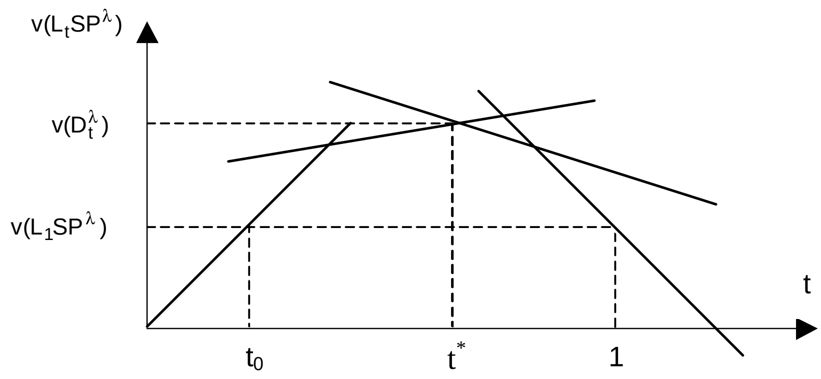

Figure 1. Lagrangean/surrogate bounds

So, in order to obtain an improved bound to the usual Lagrangean relaxation it is not necessary to find the best value t*, as is enough to find a value T such as $t_0 \leq T \leq t_1$. To find the approximate best Lagrangean/surrogate multiplier T we have used the following search procedure:

Search Heuristic (SH)

Let
- s be the initial step size;
- k be the number of iterations;
- kmax be the maximum number of iterations;
- t_0 be the initial value of Lagrangean/surrogate multiplier;
- t be the current value of Lagrangean/surrogate multiplier;
- T be the value of Lagrangean/surrogate multiplier;
- z be the maximum value of $(L_t SP^{\lambda})$;

Set
- k = 0;
- z = 0;
- $t = t_0$;
- T = t;
- $t^+ = t^- =$ undefined;

Repeat
- k = k + 1;
- Solve $(L_t SP^{\lambda})$ obtaining x^{λ}

If ($v(L_t SP^{\lambda}) > z$) then

z = $v(L_t SP^{\lambda})$;

T = t;

Calculate $\mu^{\lambda} = \sum_{j=1}^{n} \lambda_j \left(1 - \sum_{i=1}^{n} x_{ij}^{\lambda}\right)$ (μ^{λ} is the slope of the Lagrangean/surrogate function);

If ($\mu^{\lambda} < 0$) then

$t^{-} = t$;

$z^{-} = z$;

If (t^{+} is undefined) then

t = t + s;

Else

Try to improve the current multiplier solving $(L_{(z^{+}t^{+} + z^{-}t^{-})/(z^{+}+z^{-})} SP^{\lambda})$, updating T if necessary and Stop.

End_If

Else

$t^{+} = t$;

$z^{+} = z$;

If (t^{-} is undefined) then

t = t – s;

Else

Try to improve the current multiplier solving $(L_{(z^{+}t^{+} + z^{-}t^{-})/(z^{+}+z^{-})} SP^{\lambda})$, updating T if necessary and Stop.

End_If

End_If

Else

Try to improve the current multiplier solving $(L_{t-s/2} SP^{\lambda})$, updating T if necessary and Stop.

End_If

Until (k < kmax).

3. THE SUBGRADIENT HEURISTIC

The following general subgradient algorithm is used as a base to the relaxation heuristics proposed in this work. In this algorithm, C = $\{ i \mid x_{ii} = 1 \}$ is the set of nodes already fixed as medians.

Subgradient Heuristic (SubG)

Given $\lambda \geq 0$, $\lambda \neq 0$;
Set lb = $-\infty$, ub = $+\infty$, C = $\varnothing$;
Repeat
  Solve relaxation ($L_T SP^{\lambda}$) obtaining x^{λ} and $v(L_T SP^{\lambda})$;
  Obtain a feasible solution x_f and their value v_f using x_f;
  Update lb = max [lb, $v(L_T SP^{\lambda})$];
  Update ub = min [ub, v_f];
  Fix $x_{ii} = 1$ if $v(L_T SP^{\lambda} \mid x_{ii} = 0) \geq ub$, $i \in N - C$;
  Update the set C accordingly;
  Set $g_j^{\lambda} = 1 - \sum_{i=1}^{n} x_{ij}^{\lambda}$, $j \in N$;
  Update the step size θ;
  Set $\lambda_j = \max \{ 0, \lambda_j + \theta g_j^{\lambda} \}$, $j \in N$;
Until (stopping tests).

In this algorithm, T is the approximately best value for t* obtained by the procedure SH described in section two. SH results in a multiplier T that is used in the Lagrangean/surrogate relaxation. However, if the search procedure SH produces the same multiplier T for n_consec consecutive iterations of SubG, then the next Lagrangean/surrogate relaxations will use this fixed value T as the multiplier and the search is no more performed. In this work we have used the following parameter values in SH: [s = 0.5, t_0 = 0.0, kmax = 10, n_consec = 5]. The initial λ used is $\lambda_j = \min_{i \in N} \{d_{ij}\}$, $j \in N$. The step sizes used are: $\theta = \pi\,(ub - lb) / \| g^{\lambda} \|^2$. The control of parameter π is the Held and Karp (1971) classical control. It makes $0 \leq \pi \leq 2$, beginning with $\pi = 2$ and halving π whenever lb does not increase for 30 successive iterations. The stopping tests used are:
a) $\pi \leq 0.005$;
b) ub - lb < 1;
c) $\| g^{\lambda} \|^2 = 0$
d) Every median was fixed.

Solution x^{λ} is not necessarily feasible to (P), but the set I identifies median nodes that can be used to produce feasible solutions to (P). The non-median nodes are reallocated to their nearest medians producing the initial x_f as:

$$x_{f\,ii}^{\lambda} = \begin{cases} 1, & \text{if } i \in I \\ 0, & \text{otherwise} \end{cases}$$

and for all $i \neq k$:

$$x_{f\,ik}^{\lambda} = \begin{cases} 1, & \text{if } i \in I \text{ and } k = \text{index of } \min_{i \in I}\{d_{ij}\} \\ 0, & \text{otherwise} \end{cases}$$

and $v_f = \sum_{j=1}^{n} \left(\min_{i \in I} d_{ij} \right)$.

Solutions x_f are calculated at each iteration of SubG while π is not halved, but it produces poor upper bounds. It can be improved by two additional heuristics. A location-allocation heuristic based in the works of Cooper (1963) and Taillard (1996) is used whenever lb improves. Besides, the interchange heuristic suggested by Beasley (1993) is used when π is updated to $\pi/2$.

Considering that in expression (6) the β_i $(i \in N)$ are sorted in ascending order, the interchange heuristic applies the following procedure:

Interchange Heuristic (IH)

Set

$U = \sum_{j=1}^{n} (\min_{i \in I} d_{ij})$, corresponding to the solution x^{λ} associated with the current maximum lower bound lb.

m = p/10;

For j = p+1 to p+m do
  For i = 1 to p; $i \notin C$ do
    Interchange β_i with β_j, updating I accordingly;
    $v_f = \sum_{j=1}^{n} (\min_{i \in I} d_{ij})$
    If $v_f < U$ then
      $U = v_f$
    Else
      interchange β_i with β_j and update I
    End_If
  End_For
End_For

If U < ub then
ub = U
End_If.

The location-allocation heuristic (LAH) is based on the observation that after the definition of x_f, exactly p clusters can be identified, C_1, C_2, ..., C_p, corresponding to the p medians and their allocated non-medians. Solution x_f can be improved searching for a new median inside each cluster, swapping the current median by a non-median and reallocating. If the set I changes we recalculate $v_f = \sum_{j=1}^{n} (\min_{i \in I} d_{ij})$, and if the new solution is better, we can repeat the reallocation process inside the new clusters, and all the process until no more improvements are reached.

4. COMPUTATIONAL TESTS

The Lagrangean/surrogate heuristics discussed above were programmed in C and run on a Sun Ultra30 workstation (compiled using gcc compiler with -O2 optimization option).

An initial set of instances used for the tests is drawn from OR-Library (Beasley 1990), and can be considered easy problems for Lagrangean approaches in the sense of duality gaps. The gaps can be almost all closed (Beasley 1993). The second set of instances was obtained from the work of Galvão and ReVelle (1996), and although small (n = 100 and n = 150), the instances present duality gaps larger than 1% for some values of p (number of medians). They can be considered hard instances for Lagrangean approaches in the sense of duality gaps. The final instance is the Pcb3038 instance in the TSPLIB, compiled by Reinelt (1998).

For this work the objective is to show that Lagrangean/surrogate heuristics have good performance in reduced computational times. The first set of instances are the time consuming instances of the OR-Library. The instances (n = 700, p = 233), (n = 800, p = 267) and (n = 900, p = 300) were not considered in the OR-Library and their optimal values were obtained running the Lagrangean/surrogate heuristic searching for the optimality condition ub - lb < 1. The second set of instances (Galvão and ReVelle 1996) is included to show the behavior of the heuristics on problems presenting intrinsic large duality gaps for some p values. These two first set of instances were randomly generated and present different characteristics in comparison with the Pcb3038 instance, which correspond to spatially distributed points.

In order to show that Lagrangean/surrogate heuristics reach the same good results of Lagrangean heuristics and to assess the efectiveness of the proposed procedures, we have collected the results obtained by the usual Lagrangean heuristic as well. The results are reported in the tables below, where the results for Lagrangean (alone) heuristic are shown enclosed in brackets. In these tables, all the computer times shown exclude the time needed to setup the problem. Each table contains:

a) the optimal (or the best known) solution for the instance;
b) gap_ub = (100 * [ub - optimal] / optimal), is the percentage deviation from optimal (or the best known) to the best feasible solution value found by the corresponding heuristic procedure;
c) gap_lb = (100 * [optimal - lb] / optimal), is the percentage deviation from optimal (or the best known) to the best relaxation value found by the corresponding heuristic procedure;
d) nLr = the number of Lagrangean relaxations solved. It is important to observe that for Lagrangean heuristic the number of Lagrangean relaxations is also the number of subgradient iterations. For Lagrangean/surrogate heuristic however, the number of Lagrangean relaxations solved includes the relaxations solved by the procedure SH discussed in Section 2 and, therefore, it is greater than the corresponding number of subgradient iterations.
e) the total computational time (in seconds).

The results of Table 1 show that almost all gaps are closed confirming that the OR-Library instances can be considered easy instances also to the Lagrangean/surrogate approach. In order to compare the computational behavior of the usual Lagrangean and Lagrangean/surrogate heuristics we have plotted (see Figure 2) the values of $v(L_tSP^{\lambda})$ obtained at each iteration from these heuristics for the problem n = 600 and p = 200. We can observe that the sequence of Lagrangean/surrogate relaxations is more stable than the corresponding Lagrangean ones. The local searches in SH at the first iterations of SubG accelerated the overall convergence of the Lagrangean/surrogate, although without loss of quality in duality bounds.

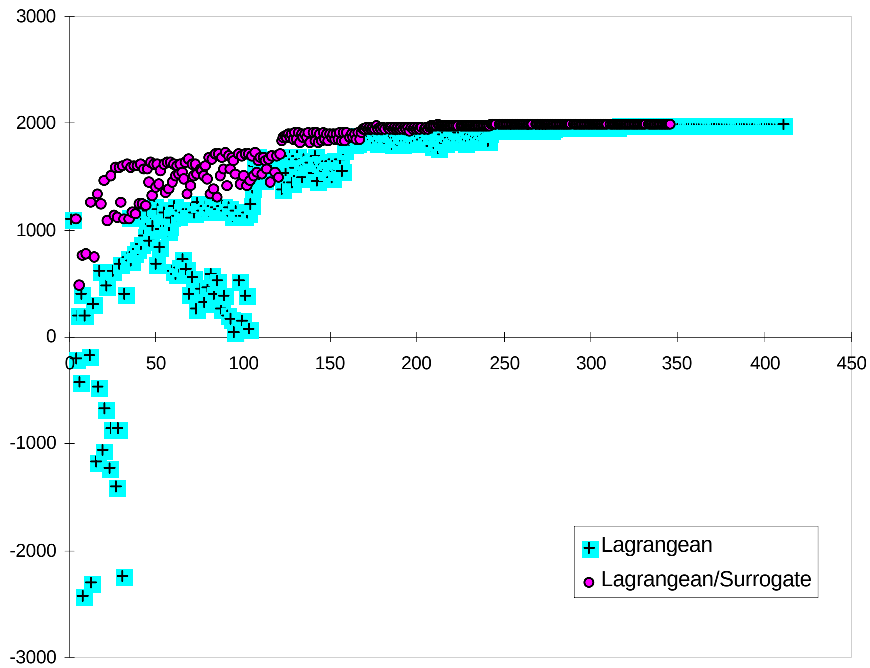

Figure 2. Typical computational behavior

The results obtained for the data set from Galvão and ReVelle (1996), which presents intrinsic duality gap instances, are shown in Table 2. Table 3 reports the results on the Pcb3038 instances (n = 3038). For this last data set the location-allocation heuristic was particularly important at the primal feasibility phase, since the interchange heuristic proved ineffective in this case.

Table 1 - Computational results for OR Library (Beasley 1990) instances

n	p	optimal solution	gap_ub		gap_lb		nLr	Total time	
100	33	1355	–	(–)	–	(–)	237 (226)	0.58	(0.66)
200	67	1255	–	(–)	–	(–)	274 (253)	4.00	(4.00)
300	100	1729	–	(–)	–	(–)	252 (316)	16.78	(20.37)
400	133	1789	–	(–)	–	(–)	244 (348)	51.80	(60.35)
500	167	1828	–	(–)	–	(–)	272 (312)	127.60	(171.20)
600	200	1989	–	(–)	–	(–)	286 (306)	257.02	(302.16)
700	233	1847	–	(–)	–	(–)	239 (240)	482.97	(488.09)
800	267	2026	–	(–)	–	(–)	367 (310)	1374.74	(1387.71)
900	300	2106	0.047	(0.047)	0.004	(0.001)	446 (391)	3058.65	(3212.46)

Table 2 - Computational results for Galvão and ReVelle (1996) instances

n	p	optimal solution	gap_ub		gap_lb		nLr		Total time	
100	5	5703	–	(–)	0.342	(0.447)	496	(485)	0.43	(0.42)
	10	4426	2.643	(2.553)	3.725	(3.730)	500	(472)	0.53	(0.50)
	15	3893	0.745	(0.745)	0.894	(0.899)	444	(433)	0.65	(0.64)
	20	3565	–	(0.084)	0.084	(0.083)	432	(443)	0.81	(0.83)
	25	3291	–	(–)	0.059	(0.060)	426	(394)	0.99	(0.94)
	30	3032	0.066	(0.066)	0.063	(0.060)	444	(443)	1.18	(1.20)
	40	2542	–	(–)	–	(–)	196	(194)	0.59	(0.62)
	50	2083	–	(–)	–	(–)	166	(184)	0.44	(0.69)
150	5	10839	–	(–)	1.404	(1.401)	482	(489)	0.85	(0.86)
	10	8729	0.642	(0.252)	3.163	(3.151)	529	(484)	1.14	(1.06)
	15	7390	1.353	(0.731)	4.895	(4.900)	602	(588)	1.52	(1.51)
	20	6454	3.424	(3.595)	2.967	(2.969)	460	(509)	1.60	(1.66)
	25	5875	1.498	(2.060)	1.010	(1.013)	458	(455)	1.92	(1.92)
	30	5495	1.201	(0.564)	0.209	(0.209)	425	(502)	2.28	(2.41)
	40	4907	0.061	(0.143)	0.068	(0.071)	418	(399)	3.08	(3.02)
	50	4374	–	(–)	0.063	(0.070)	456	(413)	3.93	(3.84)

Table 3 - Computational results for Pcb3038 instances (Reinelt 1998)

p	best known solution	gap_ub		gap_lb		nLr		Total time	
100	352704.86	2.858	(3.311)	0.098	(0.108)	494	(431)	661.76	(599.01)
150	281193.96	3.916	(4.304)	0.090	(0.086)	448	(446)	658.49	(669.10)
200	238432.02	2.726	(3.770)	0.105	(0.115)	456	(365)	712.98	(592.07)
250	209241.25	2.306	(2.360)	0.060	(0.062)	434	(444)	715.88	(742.98)
300	187723.46	1.305	(2.508)	0.056	(0.059)	411	(421)	719.04	(737.76)
350	170973.34	2.067	(2.093)	0.050	(0.055)	398	(352)	731.50	(640.66)
400	157030.46	1.630	(1.433)	0.012	(0.015)	404	(385)	919.79	(738.71)
450	145422.94	1.612	(2.341)	0.056	(0.059)	355	(435)	745.86	(847.33)
500	135467.85	2.344	(2.131)	0.040	(0.042)	333	(366)	684.82	(738.19)

In order to avoid the effect of spurious stop tests and to show that the Lagrangean/surrogate sequences are more stable and faster than their Lagrangean counterpart we have collected the computational times necessary to reach some percentage deviations from optimal of lower bound as found by Lagrangean/surrogate heuristic (LSH) and Lagrangean heuristic (LH), for each instance. The results are reported in the tables below, which show the ratios (computational time for LSH) / (computational time for LH) for OR Library instances and Pcb3038 instances (the most time consuming instances), and the average ratios.

Table 4 - Ratios (Time for LSH) / (Time for LH) for OR Library instances

		Percentage deviations from optimal of lower bound				
n	p	5%	4%	3%	2%	1%
100	33	0.78	0.78	0.78	0.75	0.80
200	67	0.64	0.64	0.66	0.74	0.74
300	100	0.57	0.58	0.57	0.40	0.71
400	133	0.34	0.66	0.66	0.66	0.74
500	167	0.54	0.54	0.37	0.70	0.70
600	200	0.53	0.68	0.68	0.68	0.75
700	233	0.52	0.67	0.67	0.67	0.67
800	267	0.68	0.68	0.68	0.68	0.75
900	300	1.02	1.01	1.01	0.68	1.00
Average ratio = 0.68						

Table 5 - Ratios (Time for LSH) / (Time for LH) for Pcb3038 instances

		Percentage deviations from optimal of lower bound				
n	p	5%	4%	3%	2%	1%
3038	100	0.70	0.82	1.02	1.07	1.14
	150	0.75	0.95	1.34	1.12	1.09
	200	0.71	0.83	1.04	1.01	1.00
	250	0.78	0.79	0.85	0.85	0.78
	300	0.61	0.75	0.84	0.74	0.95
	350	0.63	0.69	0.82	0.83	0.85
	400	0.84	1.14	1.06	1.06	1.13
	450	0.59	0.74	0.74	0.77	0.81
	500	0.75	0.78	0.78	0.81	0.71
Average ratio = 0.87						

5. CONCLUSION

This work considers Lagrangean/surrogate heuristics for p-median problems. The Lagrangean/surrogate approach was able to generate as good approximate solutions as the obtained by the traditional Lagrangean approach. However, the combination of relaxations in Lagrangean/surrogate heuristic seems to be interesting to reduce the computational times, mainly for large instances of p-median problems.

For the same initial multiplier, the Lagrangean/surrogate relaxation explores different subgradient directions than the Lagrangean (alone) counterpart. The local optimization on SH corrects wrong step sizes while maintain convergence conditions for the subgradient method. Other subgradient methods considered applicable on Lagrangean relaxation context could be improved by the Lagrangean/surrogate approach.

The use of interchange or location-allocation heuristics proved to be useful for the primal feasibility of intermediate dual solutions. The approach used here has been shown flexible and fast for large-scale real data obtained using Geographical Information Systems. We hope that this feature can be explored for even large-scale problems to produce high quality approximate solutions at reasonable computational cost.

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