LOCAL SEARCH HEURISTICS FOR CAPACITATED P-MEDIAN PROBLEMS

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Abstract

The heuristics implement a search on p-median clusters identified on a Lagrangean/surrogate optimization process. These heuristics are based on location-allocation procedures that swap medians and vertices inside the clusters, reallocate vertices, and iterate until no more improvement is reached. Computational results consider instances from the literature and real data obtained using a Geographical Information System.

Key words: Location problems, Capacitated p-median problems, Clustering, Lagrangean relaxation, Subgradient method.

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1. Introduction

Clustering problems generally appear in classification of data for some purpose like storage and retrieval or data analysis. Any clustering algorithm will attempt to determine some inherent or natural grouping in the data, using "distance" or "similarity" measures between individual data (Spath [17]). In this paper we examine local search heuristics to a clustering problem in graphs, namely, the *capacitated p-median problem (CPMP)*.

The search for *p-median* vertices on a network (graph) is a classical location problem. The objective is locate *p* facilities (medians) so as to minimize the sum of the distances from each demand vertex to its nearest facility. The *CPMP* considers capacities for the service to be given by each median. The total service demanded by vertices identified by p-median clusters can not exceed their service capacity.

Apparently problem *CPMP* was not so intensively studied as the classical p-median problem. Similar problems appeared in Bramel and Simchi-Levi [1], Klein and Aronson [6], Mulvey and Beck [13] and Osman and Christofides [15]. An extensive bibliography of related problems, and also a set of test problems are presented in [15]. They used variations of simulated annealing and tabu search to obtain good approximated solutions to the problem.

The Lagrangean/surrogate relaxation has been used recently to accelerate subgradient like methods, which are often used to optimize the corresponding Lagrangean dual problem as in Lorena and Lopes [8], Lorena and Narciso [9], Lorena and Senne [10], and Narciso and Lorena [14]. In this paper the Lagrangean/surrogate relaxation is combined with location-allocation heuristics, proposed by Cooper [2] and used before on Taillard [18] and Senne and Lorena [16].

The paper is organized in the following sections. Section 2 describes the Lagrangean/surrogate approach to the *CPMP*. Section 3 describes the local search heuristics, and section 4 presents computational results for a set of classical instances and a set of real data collected at the central area of a Brazilian 500,000 inhabitants city.

2. The Lagrangean/Surrogate Approach

The *CPMP* considered in this paper is modeled as the following binary integer programming problem:

$$v(CPMP) = \textbf{\textit{Min}} \sum_{i \in N} \sum_{j \in M} d_{ij} x_{ij} \qquad (1)$$

(CPMP) subject to

$$\sum_{j \in M} x_{ij} = 1 \;;\; i \in N \qquad (2)$$

$$\sum_{j \in M} y_j = p \qquad (3)$$

$$\sum_{i \in N} q_i x_{ij} \le Q_j y_j \;;\; j \in M \qquad (4)$$

$$y_j \in \{0,1\};\; x_{ij} \in \{0,1\};\; i \in N,\; j \in M \qquad (5)$$

where:

$N = \{1,...,n\}$ is the index set of entities to allocate and $M = \{1,...,m\}$ is the index set of possible medians, where p medians will be located;

q_i is the demand of each entity and Q_j the capacity of each possible median;

$[d_{ij}]_{n \times m}$ is a distance matrix;

$[x_{ij}]_{n\times m}$ is the allocation matrix, with $x_{ij}=1$ if entity i is allocated to median j, and $x_{ij}=0$, otherwise; $y_i = 1$ if median i is selected and $y_i = 0$, otherwise.

Constraints (2) and (3) enforce that each entity is allocated to only one median. Constraint (4) impose that a total median capacity must be respected, and (5) gives the integer conditions.

We use here a *Lagrangean/surrogate* relaxation to approximately solve the *CPMP*. The Lagrangean/surrogate approach is a successful substitute to the ordinary Lagrangean relaxation, that obtains similar bounds with less computational efforts. A general description for Lagrangean/surrogate relaxation appeared in [10] and [14]. The Lagrangean/surrogate relaxation is presented as follows.

For a given $\lambda \in R^m_+$ and $t \geq 0$ the Lagrangean/surrogate relaxation of *CPMP* is given by:

$$v(L_tCPMP^{\lambda}) = \boldsymbol{Min} \sum_{i\in N} \sum_{j\in M} (d_{ij} - t\, l_i) x_{ij} + t \sum_{i\in N} l_i \qquad (6)$$

(L_tCPMP^{l}) subject to (3), (4) and (5).

Problem (L_tCPMP^{λ}) is solved considering implicitly constraint (3) and decomposing for index j, obtaining the following m 0-1 knapsack problems:

$$v(knap_j) = \boldsymbol{Min} \sum_{i\in N} (d_{ij} - t\, l_j) x_{ij} \qquad (7)$$

subject to (4) and (5).

Each problem is solved using the Horowitz and Sahni code (see Martello and Toth [12]). Let *J* be the index set of the p smallest v($knap_j$), j $\in$ M (here constraint (3) is considered implicitly). The Lagrangean/surrogate value is given by:

$$v(L_tCPMP^{\lambda}) = \sum_{j \in J} v(knap_j) + t \sum_{i \in N} l_i \; . \qquad (8)$$

The interesting characteristic of relaxation (L_tCPMP^{λ}) is that, for t = 1 we have the usual Lagrangean relaxation using the multiplier λ. For a fixed multiplier λ, the best value for t can be found by solving a Lagrangean dual $v(D_t^{\lambda}) = \underset{t \geq 0}{Max}\ v(L_tCPMP^{\lambda})$.

The best Lagrangean/surrogate relaxation value gives an improved bound to the usual Lagrangean relaxation. To find an approximated best Lagrangean/surrogate multiplier T we have used the search procedure SH described in [16]. The following general subgradient algorithm is used as a base to the local search relaxation heuristics proposed in this work:

Subgradient Heuristic (SubG)

Given $\lambda \geq 0$, $\lambda \neq 0$;

Set lb = $-\infty$, ub = $+\infty$;

Repeat

Solve relaxation ($L_T CPMP^l$) obtaining x^{λ} and v($L_T CPMP^l$);

Obtain a feasible solution x^f and their value v_f using x^f (see section 3);

Update lb = max [lb, v($L_T CPMP^l$)];

Update ub = min [ub, v_f];

Set $g_i^l = 1 - \sum_{j \in M} x_{ij}^l$, i ∈ N;

Update the step size θ;

Set $\lambda_i = \max \{ 0, \lambda_i + \theta g_i^l \}$, i ∈ N;

Until (stopping tests).

In this algorithm, T is an approximately optimal value for t obtained by the procedure SH. The multiplier T is updated for each iteration of SubG. However, if the procedure SH produces the same multiplier T for a number of consecutive iterations of SubG, then the next Lagrangean/surrogate relaxations will use this fixed value T as the multiplier and the search SH is no more performed.

The initial λ used is $l_i = \min_{j \in M} \{d_{ij}\}$, i ∈ N. The step sizes used are: $\theta = \pi\,(ub - lb) / \| g^{\lambda} \|^2$.

The control of parameter π is the Held and Karp [5] classical control. It makes $0 \le \pi \le 2$, beginning with $\pi = 2$ and halving π whenever lb does not increase for 30 successive iterations. The stopping tests used are:

a) $\pi \le 0.005$;

b) $ub - lb < 1$;

c) $\| g^{\lambda} \|^2 = 0$.

3. The Local Search Heuristics

The Lagrangean/surrogate approach described in section 2 is integrated with local search heuristics to make primal feasible a sequence of intermediate dual solutions. These heuristics will be described in the following.

The *CPMP* is known to be NP-hard. Some earlier approaches applying Lagrangean heuristics to *CPMP* are proposed in Koskosidis and Powell [7] and in [13]. Recent approaches apply metaheuristics, such as simulated annealing and tabu search (as in França, Sosa and Pureza [4] and in [15]), and genetic algorithms (Maniezzo, Mingozzi and Baldacci [11]). Good results are reported for a set of standard test problems (OR-Library - http://mscmga.ms.ic.ac.uk/info.html).

In this paper, the Lagrangean heuristic is revisited, now using the improved Lagrangean/surrogate version. New local search heuristics are combined with the dual process. One objective of our approach is to apply it on large scale real data obtained using Geographic Information Systems. Due to small computational times compared to metaheuristic approaches, our proposal seems to be indicated to this kind of data.

Solution x^l in procedure SubG is not necessarily feasible to *CPMP*, but the set J identifies median nodes that can be used to produce feasible solutions. In order to allocate the non-median nodes to the identified set of medians we approximately solve the following generalized assignment problem:

$$\textbf{\textit{Max}} \sum_{i \in N} \sum_{j \in J} p_{ij}\, x_{ij}^f \qquad (9)$$

(*GAP*) subject to:

$$\sum_{i \in N} q_i x_{ij}^f \le Q_j,\ j \in J \qquad (10)$$

$$\sum_{j \in J} x_{ij}^f = 1,\ i \in N \qquad (11)$$

$$x_{ij}^f \in \{0,1\},\ i \in N;\ j \in J \qquad (12)$$

where $p_{ij} = -d_{ij}$, $i \in N$; $j \in J$, is the profit of node i if assigned to median j.

The algorithm *MTHG* proposed in [12] is used to provide approximated solutions x^f to *GAP*. Solution x^f is further improved by an additional location-allocation heuristic (*LAH*) based on the observation that after the definition of x^f exactly p clusters can be identified, C_1, C_2, ..., C_p, corresponding to the p medians and their allocated non-medians. Solution x^f can be improved by searching for a new median inside each cluster, swapping the current median by a non-median and reallocating. If the set J changes we recalculate the x^f value on the new *GAP*, and if the new solution is better, we can repeat the reallocation process inside the new clusters, and all the process until no more improvements are reached.

Specifically, in order to improve solutions x^f we have used the following heuristic:

Location-Allocation Heuristic (LAH)

For each cluster C_j, j = 1, ..., p let $z_j = \sum_{k \in C_j} d_{kj}$;

Repeat

  For each cluster C_j, j = 1, ..., p do

    If (nchanges < Max_Changes) then

      If there exists a non-median node $i \in C_j$ such that $Q_i \geq Q_j$ then

        Interchange i with j and update the cluster C_j recalculating $z_i = \sum_{k \in C_j} d_{ki}$;

        If $z_i \leq z_j$ then nchanges = nchanges + 1 and update the set *J*,

        Else interchange j with i and update the cluster C_j ;

      End_if;

    End_if;

End_for;

Solve *GAP* considering the set J obtaining a new set of clusters $C_1, ..., C_p$.

Until (It is possible to improve the feasible solution).

In the computational tests we have used Max_Changes = 3. In addition, the following interchange heuristics are used, trying a further improvement to the feasible solution:

Interchange-Transfer Heuristic (ITH)

Let $D_j = \sum_{k \in C_j} q_k$ be the total demand of the cluster C_j, j = 1, ..., p.

For each cluster C_j, j = 1, ..., p do:

For each cluster C_i, i = 1, ..., p, i $\neq$ j do:

Try the following changes:

If there exist non-median nodes n $\in$ C_j and m $\in$ C_i such as:

$$q_m \leq Q_j - (D_j - q_n) \text{ and}$$

$$q_n \leq Q_i - (D_i - q_m) \text{ and}$$

$$(z_j - d_{nj} + d_{ni}) + (z_i - d_{mi} + d_{mj}) < (z_j + z_i)$$

then interchange m with n.

End_if;

If there exist non-median nodes n $\in$ C_j and m $\in$ C_i such as:

$$q_m \leq Q_j - D_j \text{ and}$$

$$(z_j + d_{ni}) + (z_i - d_{mi}) < (z_j + z_i)$$

then transfer m from cluster C_i to cluster C_j.

End_if;

End_for;

End_for.

4. Computational results

Two sets of instances are used in computational tests, a classical set frequently used in others papers, and a set of real data collected at the central area of the São José dos Campos city. The *Lagrangean/Surrogate Local Search Heuristic (LSLSH)* described in this paper is coded in C and the tests made on a SUN ULTRA30 machine.

The first set of instances was used before in [15] and is formed by 2 sets of 10 instances, with (50x5) and (100x10) vertices and medians, respectively (available on the OR-Library - http://mscmga.ms.ic.ac.uk/info.html).

Table 1 reports the *LSLSH* application to these instances. The results are compared to the ones of two metaheuristics, the *HSS.OC* heuristic that presented the best performance among those reported on [15], and the *ATS* heuristic of [4].

Columns in *table 1* are composed of: the problem identification, the best known solution and the gaps (%) to the best solutions. Heuristic *HSS.OC* is a simulated annealing probabilistic acceptance approach that makes use of a non monotonic cooling schedule, a systematic neighborhood search, and a termination condition based on the number of temperature resets performed without improving the best solution. Heuristic *ATS* is an adaptive tabu search algorithm that systematically perturbs selected tabu elements, promoting intensification of the search when some indicators identify promising regions, and diversification if improvements seem to be minimal.

The last line in *table 1* shows the average gaps for the instances. Results are very good and *LSLSH* seems to be better than the corresponding metaheuristics.

Table 2 reports the average running times for the heuristics. The times for *HSS.OC* were obtained with a VAX 8600, while the times for *ATS* were obtained on a SUN Sparc20. Although the tests were performed on different machines, it can be conjectured that the *LSLSH* is faster than the other approaches, as it obtains a smaller number of feasible solutions.

The second set of instances is composed of real data collected using the Geographical Information System *ArcView* (ESRI [3]), and reporting the central area of São José dos Campos city. Six instances (100x10), (200x15), (300x25), (300x30), (402x30) and (402x40) are created. Each point is located on a block which presents a demand and is also a possible place to locate medians. The demand was estimated considering the number of houses (apartments) at each block. An empty block receives value 1. Capacities are then estimated as $C = \left\lceil \frac{\sum demands}{number\ of\ medians} \right\rceil \times t$, where t is 0.9 or 0.8. These instances are available at http://www.lac.inpe.br/~lorena/instancias.html.

Table 3 presents the results. All the dual gaps are lower than 1% and results are obtained at reasonable computer times.

5. Conclusions

This work considers Lagrangean/surrogate local search heuristics for capacitated p-median problems. The Lagrangean/surrogate approach was able to generate as good approximate solutions as the obtained by metaheuristic approaches employing small computational times.

The use of location-allocation followed by interchange heuristics proved to be useful for the primal feasibility of intermediate dual solutions. Heuristic *LSLSH* has been shown flexible and fast for large-scale real data obtained using Geographical Information Systems. These data present a spatially distributed set of points where location-allocation based heuristics have best performance.

We hope that this feature can be explored for even large-scale problems to produce high quality approximate solutions at reasonable computational cost.

Table 1: Results for the first set of instances

Problem	Vertices	Medians	Best known solution	HSS.OC gap (%)	ATS gap (%)	LSLSH gap (%)
1	50	5	713	0	0	0
2	50	5	740	0	0	0
3	50	5	751	0	0	0
4	50	5	651	0	0	0
5	50	5	664	0	0	0
6	50	5	778	0	0	0
7	50	5	787	0	0	0
8	50	5	820	0	0	0
9	50	5	715	0	0	0
10	50	5	829	0	0	0
11	100	10	1006	0	0	0
12	100	10	966	0	0	0
13	100	10	1026	0	0	0
14	100	10	982	0.30	0.30	0
15	100	10	1091	0	0.27	0.09
16	100	10	954	0	0	0
17	100	10	1034	0.48	0	0
18	100	10	1043	0.19	0.19	0
19	100	10	1031	0	0.19	0
20	100	10	1005	0	0	0.39
Mean				0.049	0.047	0.024

Table 2: Average CPU times - comparison (seconds)

Medians	HSS.OC	ATS	LSLSH
50	23.23	13.89	3.81
100	338.19	304.67	37.1

Table 3: Results for São José dos Campos city set of instances

Problem	**Size**	**Bound LSLSH dual**	**Bound LSLSH primal**	**Gap (%)**	**Time (sec.)**
1	100 x 10	17252.12	17288.99	0.21	68.62
2	200 x 15	33223.66	33395.38	0.51	2083.37
3	300 x 25	45313.43	45364.30	0.11	2604.92
4	300 x 30	40634.91	40635.90	0.00	867.68
5	402 x 30	61842.49	62000.23	0.25	27717.11
6	402 x 40	52396.54	52641.79	0.46	4649.47

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